Thermal machines at the single trajectory level & stochastic excursions

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Cooling on-demand





Aamir, M. A. et al. "Thermally driven quantum refrigerator autonomously resets superconducting qubit" arXiv.2305.16710 (2023).

3-body resonant interaction

 $\omega_h + \omega_t = \omega_c$

Steady-state picture: currents J_H and J_C

On-demand picture: If an excitation suddenly appears in green, extract it as fast as possible.

Changes the questions:

- How long does it take to cool?
- How many things can go wrong before it works?
- What is the entropy production of a single cooling event?



Cooling on-demand





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3-body resonant interaction

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What is stochastic quantum thermodynamics? We cannot see quantum systems... All we see is data ...111000010001001110011101100... How can we study the stochastic thermodynamics of quantum devices?

- To measure a system we must send in a **probe** (or **ancilla**).
 - S+A interaction encodes information about S on A.
 - Extract information by measuring A.
- Information-back action trade-off: the the the system.



iid outcomes

robe (or **ancilla**). out S on A.



Information-back action trade-off: the more information we want, the more we disturb



Correlated outcomes

A simple example

- Start in $|\psi_0\rangle$.
 - 1. Sample first outcome x_1 from $p(x_1) = 1$ Update state to $|\psi_1\rangle = |x_1\rangle$.
 - 2. Sample second outcome x_2 from $p(x_2)$ Update state to $|\psi_2\rangle = |x_2\rangle$.
- Generates a **bitstring of emitted symbols**

• Qubit: apply unitary U then measure in the computational basis $P_x = |x\rangle\langle x|$ where x = 0, 1.

$$\langle x_1 | U | \psi_0 \rangle |^2$$

$$x_1) = |\langle x_2 | U | x_1 \rangle|^2$$

$$x_{1:n} = (x_1, \dots, x_n).$$

• Probability of a sequence forms a Markov chain: $P(x_1, ..., x_n) = p(x_n | x_{n-1}) ... p(x_2 | x_1) p(x_1)$.

Non-projective measurements lead to long memory

- Apply a set of Kraus operators $\sum_{x} F_{x}^{\dagger}F_{x} = 1$. Starting at ρ_{0} :
 - 1. Sample first outcome x_1 from $p(x_1) = tr\{F_{x_1}\rho_0 F$

$$p(x_{n+1} | x_{1:n}) = \operatorname{tr} \{ F_{x_{n+1}} \rho_{x_{1:n}} F_{x_{n+1}}^{\dagger} \} \quad \text{and} \quad \rho_{x_{1:n+1}} = \frac{F_{x_{n+1}} \rho_{x_{1:n}} F_{x_{n+1}}^{\dagger}}{p(x_{n+1} | x_{1:n})}$$

- - Evolution of the system is Markovian. But output data is not.
- Looks like a Hidden Markov Model (HMM):
 - Quantum system is hidden.
 - Measurement outcomes (what we see) = emitted symbols

GTL "Patterns in the jump-channel statistics of open quantum systems," arXiv 2305.07957

$$\{F_{x_1}^{\dagger}\}$$
. Update state to $\rho_{x_1} = \frac{F_{x_1}\rho_0 F_{x_1}^{\dagger}}{p(x_1)}$.

2. Sample second outcome x_2 from $p(x_2 | x_1) = \text{tr}\{F_{x_2}\rho_{x_1}F_{x_2}^{\dagger}\}$. Update state to $\rho_{x_{1:2}} = \frac{F_{x_2}\rho_{x_1}F_{x_2}^{\dagger}}{p(x_2 | x_1)}$

• String probability is now $P(x_{1:n}) = p(x_n | x_{1:n-1}) p(x_{n-1} | x_{1:n-2}) \dots p(x_2 | x_1) p(x_1)$ which is highly non-Markovian.



Instruments: simplify and generalize

• Instruments = superoperators:

$$M_x \rho = F_x \rho F_x^{\dagger}$$

• Update rules become:

$$p(x_{n+1} | x_{1:n}) = \operatorname{tr} \{ M_{x_{n+1}} \rho_{x_{1:n}} \}$$

and

$$\rho_{x_{1:n+1}} = \frac{M_{x_{n+1}}\rho_{x_{1:n}}}{p(x_{n+1} | x_{1:n})}$$

 $P(x_1)$

 ho_{χ_1}

Wiseman, H. M. & Milburn, G. J. Quantum Measurement and Control. (Cambridge University Press, New York, 2009)



Prob. of a string:

$$m_{:n}) = \operatorname{tr}\{M_{x_N} \dots M_{x_1}\rho_0\}$$

Conditional state

$$= M_{x_N} \dots M_{x_1} \rho_0 / P(x_{1:n})$$

Instruments represent the most general kind of measurement possible.

Also encompass inefficient measurements

$$M_x \rho = \sum_{k \in x} F_k \rho F_k^{\dagger}$$



Unconditional dynamics

- If we measure but don't record the outcome the state of the system still changes (measurement back action)
- Ex: collision model or master equation.

$$\rho' = \sum_{x} p_x \rho'_x = \sum_{x} M_x \rho = \mathcal{M}\rho$$

- *M* is a quantum channel.
- After *n* steps: $\rho_n = \mathcal{M}^n \rho_0$.
- Describes the average impact that the interaction with the ancilla causes in the system.



Connection to Hidden Markov Models

- $P(x, \sigma | \sigma')$ = prob. that system goes from $\sigma' \rightarrow \sigma$ while emitting a symbol x.
 - If HMM state is $\pi(\sigma')$ the prob. that we observe symbol x is

$$p(x) = \sum_{\sigma,\sigma'} P(x,\sigma \,|\, \sigma') \pi(\sigma$$

• If outcome was x_i bayesian update the state of the hidden layer:

$$\pi(\sigma \mid x) = \frac{P(x, \sigma)}{p(x)} = \frac{\sum_{\sigma'} P(x, \sigma)}{p(x)}$$

• Define substochastic matrices: $(M_x)_{\sigma,\sigma'} = P(x,\sigma | \sigma')$ and $\langle 1 | = (1,...,1)$. Then

$$p(x) = \langle 1 | M_x | \pi \rangle \qquad \text{and} \qquad$$

Milz, S. & Modi, K. "Quantum Stochastic Processes and Quantum non-Markovian Phenomena". PRX Quantum 2, 030201 (2021)

 $\sigma \sigma' \sigma' \pi(\sigma')$

$$\pi_x \rangle = \frac{M_x | \pi \rangle}{p(x)}$$







Prediction

- Mixed state representation & unifilar models: if we know $ho_{x_{1}\cdot n}$ and we

- observe x_{n+1} we know with certainty that the system evolved to $\rho_{x_{1:n+1}}$.
- Usefulness: data compression

$$p(x_{n+1} | x_{1:n}) = p(x_{n+1} | \rho_{x_{1:n}})$$

If we can know the internal state, we can make statistical predictions of future outcomes.

• Example: figuring out the internal state of a large language model.

F. Binder, J. Thompson, M. Gu, "Practical unitary simulator for non-Markovian complex processes," Phys. Rev. Lett. **120** 240502 (2018).

Quantum jumps

GTL, Michael J. Kewming, Mark T. Mitchison, Patrick P. Potts **"Current fluctuations in open quantum systems: Bridging the gap between quantum continuous measurements and full counting statistics,"** PRX Quantum 5, 020201 (2024)

GTL "Patterns in the jump-channel statistics of open quantum systems," arXiv 2305.07957



Mark Mitchison





Michael Kewming

Patrick Potts

Consider a quantum master equation

$$\frac{d\rho}{dt} = \mathscr{L}\rho = -i[H,\rho] + \sum_{x=1}^{r} L_x \rho L_x^{\dagger} - \frac{1}{2} \{L_x^{\dagger} L_x,\rho\}$$

• The infinitesimal evolution can be written as a set of instruments:

$$\rho_{t+dt} = e^{\mathscr{L}dt}\rho_t =$$

(jump)
$$M_x \rho = dt \ L_x \rho L_x^{\dagger} = dt \ \mathcal{J}_x \rho$$

(no jump) $M_0 \rho = \rho + dt \mathscr{L}_0 \rho$

• $p_x = tr\{M_x \rho\} = dttr\{L_x^{\dagger}L_x \rho\}$ is infinitesimal: most of the time the system evolves with no jump.

GTL, Michael J. Kewming, Mark T. Mitchison, Patrick P. Potts "Current fluctuations in open quantum systems: Bridging the gap between quantum continuous measurements and full counting statistics," PRX Quantum 5,020201 (2024)





for x = 1, 2, ..., r

where
$$\mathscr{L}_{0}\rho = -i[H,\rho] - \frac{1}{2}\sum_{x=1}^{r} \{L_{x}^{\dagger}L_{x},\rho\}$$

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	I	
i.	I.	
i	Т	
i	Т	
i	Т	
i	Ι	
i	Т	
i	Ι	
i	Ι	
i		

The t and the N ensembles

- *t*-ensemble: t_f fixed. \hat{N} is random.
 - Instruments: $M_0 \rho = (1 + dt \mathscr{L}_0) \rho$ and $M_x \rho = dt L_x \rho L_x^{\dagger}$ for x = 1, 2, ..., r
 - Trajectory: $0000x_1000000x_200000...$
- *N*-ensemble: *N* is fixed. \hat{t}_f is random.
 - Instruments: $M_{x,\tau}\rho = \mathcal{J}_x e^{\mathcal{L}_0 \tau}\rho$
 - Trajectory: $(x_1, \tau_1), (x_2, \tau_2), \dots, (x_N, \tau_N)$
- Quantum jumps without time tags: we know a jump happened, but do not know when
 - Instruments: $M_x = -\mathcal{J}_x \mathcal{L}_0^{-1}$.
 - Trajectory: x_1, x_2, \ldots

Statistical Mechanics: Theory and Experiment 2014 (3): P03012 (2014)



$\tau_j = t_j - t_{j-1}$

A. A. Budini, R. M. Turner, and J. P. Garrahan. "Fluctuating Observation Time Ensembles in the Thermodynamics of Trajectories." Journal of



Quantum jumps without time tags

- Lattice with L sites, each of which can have 0 or 1 particles.
 - excitations can be injected on the left (I_I)
 - or extracted on the right (E_R) .
 - And they can tunnel back and forth through the chain: not monitorable.
- All we would observe are symbols: $I_L I_L E_R$.

Prob. of a string:

$$P(x_{1:n}) = \operatorname{tr} \{ M_{x_N} \dots M_{x_1} \rho_0 \}$$
Conditional state

$$\rho_{x_{1:n}} = M_{x_N} \dots M_{x_1} \rho_0 / P(x_{1:n})$$

GTL "Patterns in the jump-channel statistics of open quantum systems," arXiv 2305.07957







Direct and indirect observation of quantum jumps

Quantum jumps = observable clicks in the environment



Fink et. al., "Signatures of a dissipative phase transition in photon correlation measurements"

Nature Physics **14** 365-369 (2018)

Quantum jumps observed indirectly through continuous measurements of the system





Hofmann, et. al. "Measuring the Degeneracy of Discrete Energy Levels Using a GaAs / AlGaAs Quantum Dot," Phys Rev. Lett **117**, 206803 (2016)







Double quantum dot - 3-level system

- Two dots + Coulomb blockade \rightarrow 3 levels only: $|0\rangle$, $|L\rangle$, $|R\rangle$
- One-to-one mapping between system transition and jump channel
 - e.g. $|0\rangle \xrightarrow{I_H} |L\rangle$ or $|R\rangle \xrightarrow{E_C} |0\rangle$, etc.





QPC asymmetrically placed

Unpublished data - Natalia Ares' group





Stochastic operation of thermal machines

Abhaya S. Hegde, Patrick P. Potts, GTL, "Time-resolved Stochastic Dynamics of Quantum Thermal Machines," arXiv:2408.00694



Patrick Potts





- Double quantum dot
 - Engine process: uses thermal gradient to extract chemical work .) $\widehat{}$
 - Refrigerator process: uses chemical work to make heat flow from $\sum_{k=1}^{E_h} \int_{-\infty}^{I_c} (I_k) dk$ cold to hot.



- There can also be "idle cycles" (bounces)
 - "Hot bounce")____ (
 - "Cold bounce") (

Can we identify individual cycles solely from a bitstring?

Manzano, Gonzalo, and Roberta Zambrini "Quantum Thermodynamics under Continuous Monitoring: A General Framework," AVS Quantum Science 4 (2): 025302 (2022).



Impossible in general, if excitations are indistinguishable

 $I_c I_h E_h E_c = \begin{cases} I_c I_h E_h E_c \\ I_c I_h E_h E_c \end{cases}$



Single excitation assumption

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_{n} D[H]$$

$$\bigcup_{n} Unitary \qquad W mork \qquad reserved}$$

• Result: for cycles to be identifiable the string must always have injections followed by extractions.

$$\dots I_\bullet E_\bullet I_\bullet E_\bullet I_\bullet E_\bullet I_\bullet E_\bullet \dots$$

- Condition: Hilbert space must be split in 2.
 - $L_{\alpha j}^{\dagger}$ injects \rightarrow post-injection subspace.
 - $L_{\alpha i}$ extracts \rightarrow post-extraction subspace.

 $K_n] \rho + \sum_{\alpha \in \{h,c\}} \sum_{j} \gamma_{\alpha j}^{-} D[L_{\alpha j}] \rho + \gamma_{\alpha j}^{+} D[L_{\alpha j}^{\dagger}] \rho$

Vork ervoirs Extraction to bath α

Injection from bath α



Bitstrings of jumps \rightarrow bitstrings of cycles

$\dots I E I E I E I E \dots = \dots X X X X \dots$

- We can use this to answer the following questions:
 - What is the probability that the next cycle is of type X and takes a time τ ?
 - How are cycles correlated with each other?
 - What is the average time required to complete each cycle?
 - How many idle cycles happen between two useful cycles?
- Define instruments

$$M_{X\tau} = \int_{0}^{\tau} dt \, \mathscr{J}_{E_X} e^{\mathscr{L}_0(\tau-t)} \mathscr{J}_{I_X} e^{\mathscr{L}_0 t}$$









Cycle probabilities

- Then prob. a cycle is of type X and takes a time τ : $p_{X,\tau} = \operatorname{tr}\{M_{X\tau}\pi_E\}$.
- If we don't care about how long a cycle takes, we just need to marginalize the instrument:

$$M_X = \int_0^\infty d\tau \ M_{X\tau}$$

• Prob. of obtaining each cycle type

$$p_X = \operatorname{tr}\{M_X \pi_E\}$$

• Conditional cycle times: if cycle is of type X, how long it takes?

$$E(\tau | X) = \int_{0}^{\infty} d\tau \ \tau \frac{p_{X,\tau}}{p_X}$$

 π_E = Jump Steady-State

Correct state to get long-time statistics

Relation to steady-state currents: $I = \frac{p_1 - p_2}{E(\tau)}$

Correlations between cycles: $P(X_1, \tau_1, \dots, X_n, \tau_n) = \operatorname{tr}\{M_{X_n\tau_n} \dots M_{X_1\tau_1}\pi_E\}$











(a-d) Statistics of cycles in three-level maser from Fig. 2. (a) Probability of observing a cycle X within a duration τ FIG. 3. [Eq. (9)] at resonance $\omega_d = \omega_h - \omega_c$ and $T_h/T_c = 10$. (b) Total probability of observing a cycle X [Eq. (10)] and (c) expectation values for cycle duration [Eqs. (11), (12)] as a function of the ratio of bath temperatures. A vertical line at $T_h/T_c = \omega_h/\omega_c$ separates the refrigerator and engine regimes. The inset shows all expectation values nearly converge at resonance. (d) Mean of intervening idle cycles between useful cycles and ratios of fraction of idle-to-useful times against bath gradient. The parameters are fixed (in units of $T_c = 1$) at $\gamma_h = \gamma_c \equiv \gamma = 0.05$, $\omega_h = 8$, $\omega_c = 2$, $\omega_d = 4$, $\epsilon = 0.5$ unless mentioned otherwise.

Double quantum dot or 3-level system









Stochastic excursions

See also poster by Guilherme Fiusa this afternoon!



Guilherme Fiusa



Pedro Harunari





From quantum to classical master equation

Any quantum master equation

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_{k} D[L_{k}]\rho$$

Can be converted to a classical rate equat

$$\frac{dp_x}{dt} = \sum_{\substack{y \neq x}} \left\{ W_{xy}(t)p_y - W_{yx}(t)p_x \right\},$$

• If eigenbasis of $\rho(t)$ does not change in times the set of the

tion. Define
$$\rho_t = \sum_x p_x(t) |x_t\rangle \langle x_t|$$
. Then
 x

$$W_{xy}(t) = \sum_k |\langle x_t | L_k | y_t\rangle|^2$$

k

• Also useful as an approximation technique. Ex:

$$\frac{d\rho}{dt} = -i[g(c_1^{\dagger}c_2 + c_2^{\dagger}c_1), \rho] + \gamma_h(1 - f_h).$$

• In the basis $|0\rangle$, $|L\rangle$, $|R\rangle$

$$W = \begin{pmatrix} 0 & \gamma_h (1 - f_h) & \gamma_c (1 - f_c) \\ \gamma_h f_h & 0 & \frac{4g^2}{\gamma_h + \gamma_c} \\ \gamma_c f_c & \frac{4g^2}{\gamma_h + \gamma_c} & 0 \end{pmatrix}$$

Derived using perturbation theory

K. Prech, P. Johansson, E. Nyholm ,GTL, C. Verdozzi , P. Samuelsson , P. P. Potts "Entanglement and thermokinetic uncertainty relations in coherent mesoscopic transport". Phys. Rev. Res. 5, 023155 (2023).



 $D[c_1] + \gamma_h f_h D[c_1^{\dagger}] + \gamma_c (1 - f_c) D[c_2] + \gamma_c f_c D[c_2^{\dagger}]$



Counting observables

- Steady-state = ensemble thing.
- At the stochastic level, system is always jumping up and down.



Counting variables $\hat{N}_{xy}(t)$

Build physical currents through counting observables

$$\hat{Q}(t) = \sum_{x,y} \nu_{x,y} \hat{N}_{xy}(t)$$

Weights u_{xy} determine the physics. Example: current to the left bath $\nu_{0L} = -\nu_{L0} = 1$

Beyond physics: ν_{xy} appends contextual meaning





Full Counting Statistics

- FCS deals with the long-time statistics of counting observables.
- Define the tilted matrix (ξ = counting field)

$$\mathbb{W}_{xy}^{\xi} = \begin{cases} W_{xy}e^{i\xi\nu_{xy}}\\ -\sum_{z}W_{zx} \end{cases}$$

• And define the generalized master equation

Then

$$\frac{dp_x^{\xi}}{dt} = \sum_y W_{xy}^{\xi} p_y^{\xi}$$
$$P_t(\mathcal{Q}) = \int_{-\infty}^{\infty} \frac{d\xi}{2\pi} e^{-i\xi\mathcal{Q}} \left(\sum_x p_x^{\xi}\right)$$

$$x \neq y$$
$$x = y$$

All cumulants are extensive: E(Q) = Jt (J = current) $\operatorname{var}(\mathcal{Q}) = Dt$ D =noise/diffusion coefficient/scaled variance. (not an actual variance)

D is the thing in a TUR.



Stochastic excursions

- Starts when system leave A.
- Ends when system first comes back to A.
- Generalizes the notion of cycles.
- Excursion time \hat{T} is random: first passage time. Well known and extensively studied.





• Our question: statistics of counting observables within a single excursion $\hat{Q}(\hat{T})$.

Ongoing work

- counting observable has a given value. Can also compute $P(Q_1, Q_2, ..., T)$.
- Exchange FT: if $\hat{Q} = \hat{\Sigma}$ = entropy production then $P(\Sigma) = e^{-\Sigma}P(-\Sigma)$.
 - Valid at the level of a single excursion.
- Connection with steady-state quantities: let $\hat{T}_{tot} = \hat{T} + \hat{T}_{res}$. Then

$$J = \frac{E(\hat{Q})}{\mu} \quad \text{where} \quad \mu = E(\hat{T}_{\text{tot}})$$
$$\text{var}(Q) \quad E(\hat{Q})\Delta^2 \quad 2E(\hat{Q}) \quad \hat{Q} \quad \hat{$$

$$D = \frac{\mu}{\mu} + \frac{L(Q)\Delta}{\mu^3} - \frac{L(Q)\Delta}{\mu} \operatorname{cov}(\hat{Q}, \mu)$$

• We have found a way to compute P(Q, T): joint prob. that excursion takes a given time and



Conclusions

- Sequential quantum measurements = time-series of correlated stochastic outcomes.
 - Bayesian inference of the quantum state, given outcomes.
 - Unveiling the thermodynamics from measurement data.
 - Stochastic operation of a thermal machine.
 - Open question: machine intermittency vs. current fluctuations?
 - Stochastic excursions: so far classical, but very exciting.

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