nature physics

ARTICLES

https://doi.org/10.1038/s41567-020-0990-x

Check for updates

A strong no-go theorem on the Wigner's friend paradox

Kok-Wei Bong[1,4], Aníbal Utreras-Alarcón[1,4], Farzad Ghafari[1], Yeong-Cherng Liang[2], Nora Tischler[1] ✉, Eric G. Cavalcanti[3] ✉, Geoff J. Pryde[1] and Howard M. Wiseman[1]

Nature Physics 16, 1199–1205 (2020)

Journal Club - By Gabriel T. Landi

Wigner’s friend paradox

The measurement problem

- Closed quantum systems evolve *deterministically* and *unitarily*:

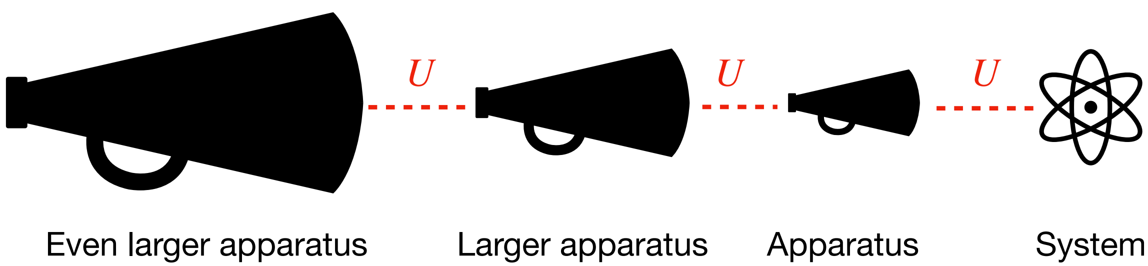
$$\psi\rangle \to U \;\; \psi\rangle$$

- But if we measure a quantum system, the outcome is random (Born's rule):

$$p_k = \;\; \langle k \;\; \psi\rangle \;\; ^2 \qquad\qquad \text{and} \qquad\qquad \psi\rangle \to \;\; k\rangle$$

Quantum mechanics should apply at all scales

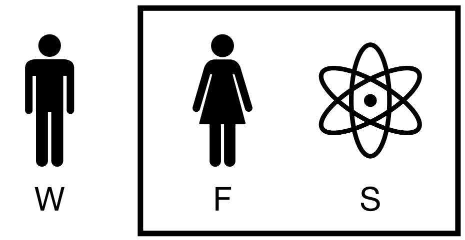
- Classical physics is a limiting case of quantum theory.
- **How can Born's rule *emerge* from unitary evolution?**
- To measure we must interact a system with an apparatus (**amplification process**).



Somehow in this amplification process a deterministic evolution turns into a random outcome.

Measurement problem

Wigner's friend paradox



- System is a spin 1/2 in the initial state $\psi_0\rangle_S = \frac{1}{\sqrt{2}}\left(\uparrow \rangle_S + \downarrow \rangle_S\right)$
- Wigner's friend F measures S in the z-basis.
  - Obtains $\uparrow, \downarrow$ with probability 1/2.
- **Wigner's perspective:** to make a measurement, F must interact with S with some unitary U_{FS}
  - The result is an entangled state:

$$\Psi\rangle_{FS} = \frac{1}{\sqrt{2}}\left(\mathrm{A}\rangle_F \otimes \uparrow \rangle_S + \mathrm{B}\rangle_F \otimes \downarrow \rangle_S\right)$$

  - *Not a statistical mixture of* $\mathrm{A}\rangle_F \otimes \uparrow \rangle_S$ *and* $\mathrm{B}\rangle_F \otimes \downarrow \rangle_S$
- Contradiction:
  - Friend obtains outcomes with probability 1/2.
  - Wigner obtains an entangled state.
- This contradiction *is* the measurement problem.

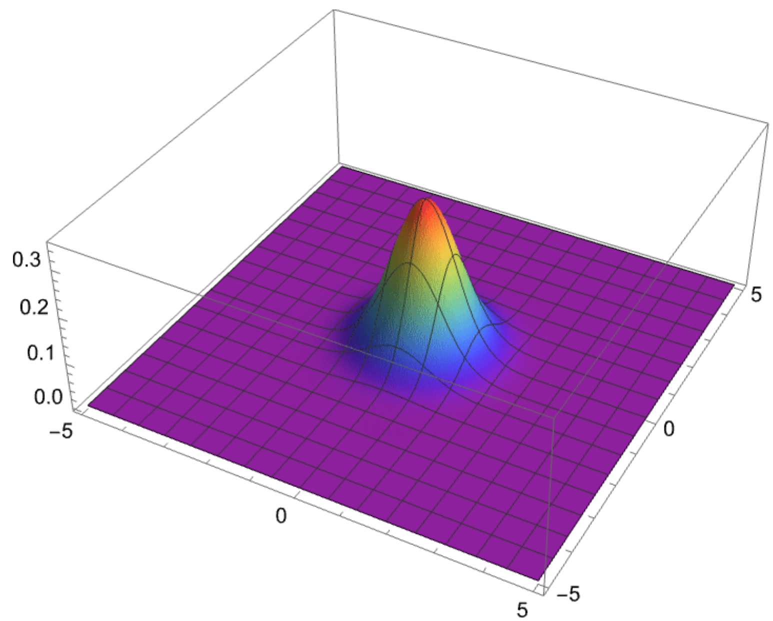
Wigner, E. P. Remarks on the mind-body question. In Good, I. J. (ed.) The Scientist Speculates, 284–302 (Heinemann, London, 1961).

Example: von Neumann measurement model

- System is a spin 1/2 in the initial state $\psi_0\rangle_S = \frac{1}{\sqrt{2}}\left(\uparrow\rangle_S + \downarrow\rangle_S\right)$
- Suppose F is a harmonic oscillator: $U_{FS} = e^{-iHt}$ with $H = ig\sigma_z(a^\dagger - a)$. Then

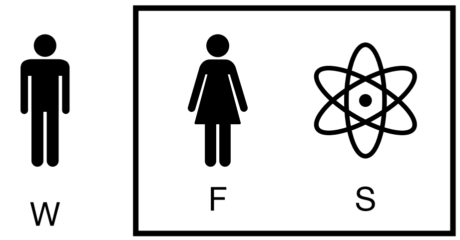
$$\Psi(t)\rangle_{FS} = \frac{1}{\sqrt{2}}\left(gt\rangle_F \otimes \uparrow\rangle_S + -gt\rangle_F \otimes \downarrow\rangle_S\right)$$ where $\alpha\rangle_F$ = coherent state.

Wigner function of F as a function of time.



F is mixed, can be in either left or right blob.

Wigner’s friend paradox



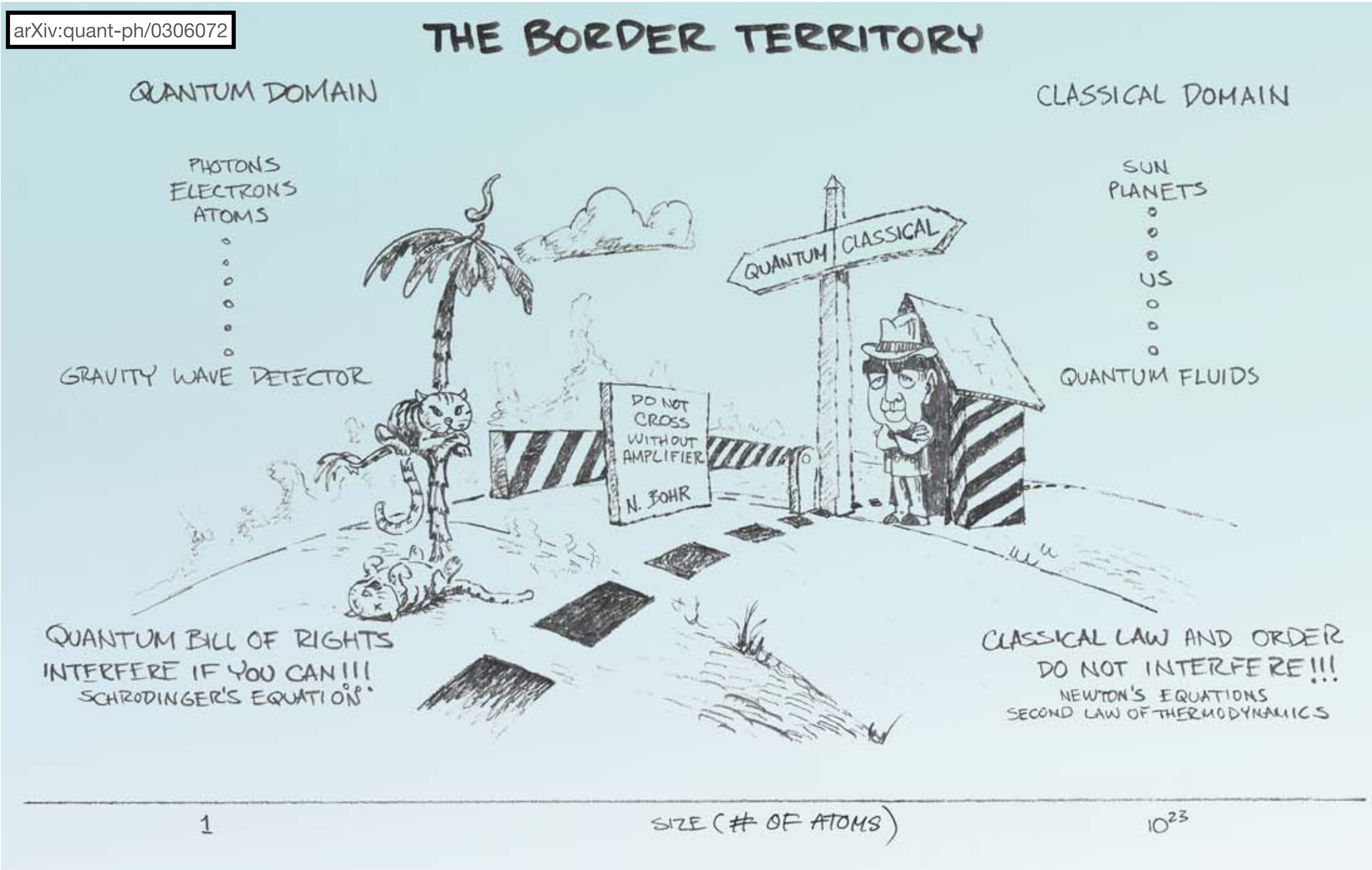
Contradiction arises from 2 assumptions:

1. Quantum theory is universal and can be applied at any scale, even to a macroscopic observer.

2. There is an *objective* collapse after a measurement.

- There is no contradiction if:
  - Quantum mechanics does not apply to conscious observers, or
  - Collapse is not an objective physical process affecting the wave function described by Wigner.

Wigner, E. P. Remarks on the mind-body question. In Good, I. J. (ed.) The Scientist Speculates, 284–302 (Heinemann, London, 1961).


THE BORDER TERRITORY
QUANTUM DOMAIN
CLASSICAL DOMAIN
PHOTONS
ELECTRONS
ATOMS
GRAVITY WAVE DETECTOR
SUN
PLANETS
US
QUANTUM FLUIDS
QUANTUM
CLASSICAL
DO NOT CROSS WITHOUT AMPLIFIER
N. BOHR
QUANTUM BILL OF RIGHTS
INTERFERE IF YOU CAN!!!
SCHRODINGER'S EQUATION
CLASSICAL LAW AND ORDER
DO NOT INTERFERE!!!
NEWTON'S EQUATIONS
SECOND LAW OF THERMODYNAMICS
1
SIZE (# OF ATOMS)
10^23

Local Friendliness
&
Extended Wigner's friend scenario

Local Friendliness (LF)

1. Freedom of choice.
2. Locality.
3. Absoluteness of Observed Events (AOE): an observed event is a real single event, and not relative to anything or anyone.

Theorem: *If a superobserver (Wigner) can perform arbitrary quantum operations on an observer and its environment, then no physical theory can satisfy Local Friendliness.*

The proof of the Theorem is similar in spirt to Bell's inequalities:

- See what restrictions these 3 hypotheses entail
- Then show that a quantum mechanical experiment can violate it.

Frauchiger, D. & Renner, R. Quantum theory cannot consistently describe the use of itself. Nat. Commun. 9, 3711 (2018).

- 2 entangled spin 1/2 particles, one goes to Charlie, one to Debbie:

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_{S_A} \otimes |\downarrow\rangle_{S_B} - |\downarrow\rangle_{S_A} \otimes |\uparrow\rangle_{S_B}\right)$$

- What the experiment cares about is a joint probability distribution:

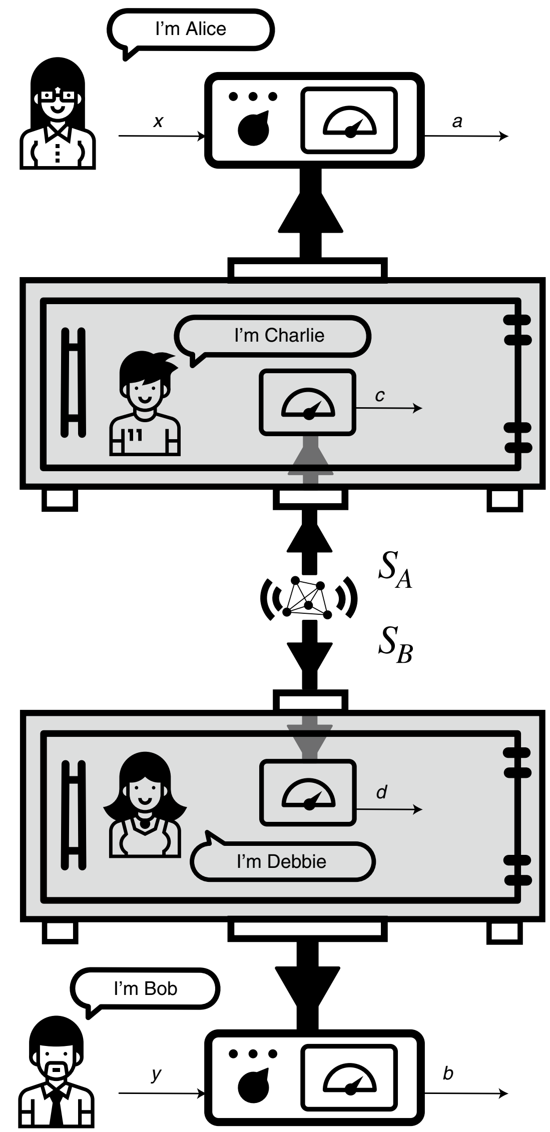
$$P(a, b \,|\, x, y)$$

- where

  a, b = outcomes of the experiments performed by Alice and Bob.

  x, y = choices in the experimental setup made by Alice and Bob.

- Proof:
  - The 3 assumptions in Local Friendliness impose constraints on the possible correlations of $P(a, b \,|\, x, y)$
  - It is possible to find quantum experiments which violate these constraints.



Example of probabilities - Bell & CHSH

- Example probability for 2 entangled spins:

$$P(a,b\ \ x,y) = \frac{1}{4}\{1 - ab\cos(\theta_x - \theta_y)\},$$

where $a, b = \pm 1$

  - $x, y =$ choices. In this case, angles θ_x, θ_y.
  - $a, b =$ random outcomes.

- Define averages:

$$\langle A_x B_y \rangle = \sum_{a,b=\pm 1} abP(ab\ \ xy)$$

- Bell studied the restrictions that appear from a **Local Hidden Variable (LHV)** model:

$$P(ab\ \ xy) = \sum_{\lambda} P(a\ \ x\lambda)P(b\ \ y\lambda)P(\lambda)$$

where λ is an arbitrary hidden variable.

- This restriction leads to the CHSH inequality

$$\langle A_1 B_1 \rangle + \langle A_1 B_2 \rangle + \langle A_2 B_1 \rangle - \langle A_2 B_2 \rangle - 2 \overset{\text{LHV}}{\leqslant} 0$$

- Choose $\theta_x = \{0, \pi/2\}, \quad \theta_y = \{-3\pi/4, 3\pi/4\}$ and we get 0.82. Violates the inequality.

Choices of Alice and Bob

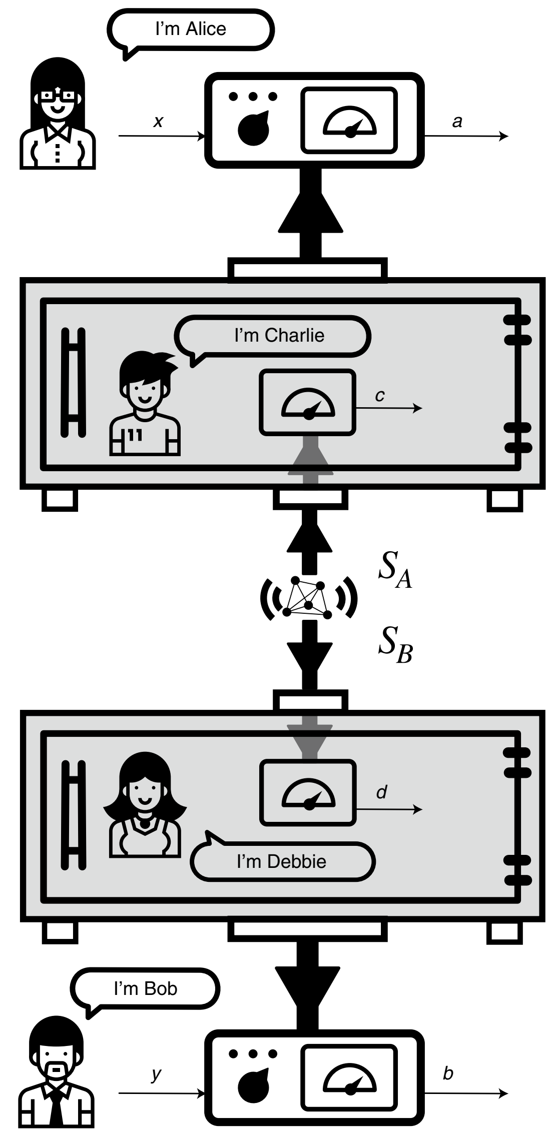
- Alice has one of 3 choices:

  $x = 1$: *open the box and ask Charlie what he saw.*

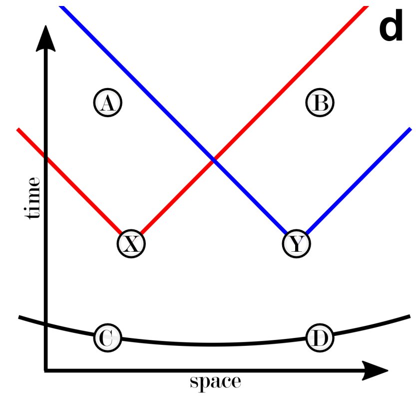
  $x = 2{,}3$*: apply a unitary in* CS_A*, disregard Charlie and measure* S_A *directly.*

  $x = 2{,}3$ represent two possible ways of measuring S_A.

- Bob will do something analogous with $y = 1{,}2{,}3$.



Local Friendliness



Absoluteness of Observed Events

- Is the assumption that there exists a well-defined value for the outcome of each observation:

  $P(abcd \;\; xy)$

- We want to marginalize over c, d:

$$P(ab \;\; xy) = \sum_{c,d} P(abcd \;\; xy)$$

- Consistency:

  $P(a \;\; cd, x = 1, y) = \delta_{a,c}$
  $P(b \;\; cd, x, y = 1) = \delta_{b,d}$

  (when Alice asks Charlie what he saw, her outcome is the same as that of Charlie)

Freewill

- $P(cd \;\; xy) = P(cd)$
  - Alice and Bob's choices x, y are independent of those of Charlie and Debbie.

Locality (no-signaling)

- $P(a \;\; cdxy) = P(a \;\; cdx)$
- $P(b \;\; cdxy) = P(b \;\; cdy)$
  - Because Alice and Bob are space-like separated, Bob's choice cannot affect the outcomes of Alice.

LF inequalities

- The LF conditions then impose certain inequalities: e.g.

$$\langle A_1 B_1 \rangle - \langle A_1 B_3 \rangle - \langle A_2 B_1 \rangle - \langle A_2 B_3 \rangle - 2 \overset{\text{LF}}{\leqslant} 0$$

or

$$-\langle A_1 \rangle - \langle A_2 \rangle - \langle B_1 \rangle - \langle B_2 \rangle - \langle A_1 B_1 \rangle - 2\langle A_1 B_2 \rangle - 2\langle A_2 B_1 \rangle + 2\langle A_2 B_2 \rangle - \langle A_2 B_3 \rangle - \langle A_3 B_2 \rangle - \langle A_3 B_3 \rangle \overset{\text{LF}}{\leqslant} 0$$

- There are 932 inequalities in total.
- This is what the LF assumptions predict.
- The trick is now to see if they can be violated by a quantum experiment.
  - *If they can, then nature is not LF.*

Quantum mechanical protocol:

- Charlie interacts with S_A via entangling unitary U_{CS_A} *(e.g. CNOT)*

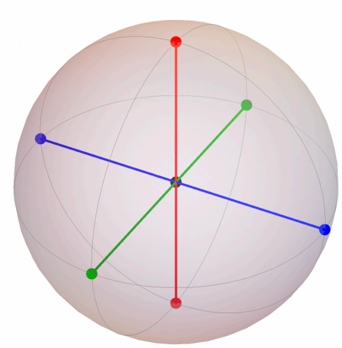
- Alice's actions:

  $x = 1$: generalized measurement of Charlie
  ("*open the box and ask Charlie what he saw*"):

$$M_{a|x=1} = |a_1\rangle\langle a_1|_C \otimes I_{S_A}$$

  $x = 2{,}3$: undo U_{CS_A} then measure S_A:

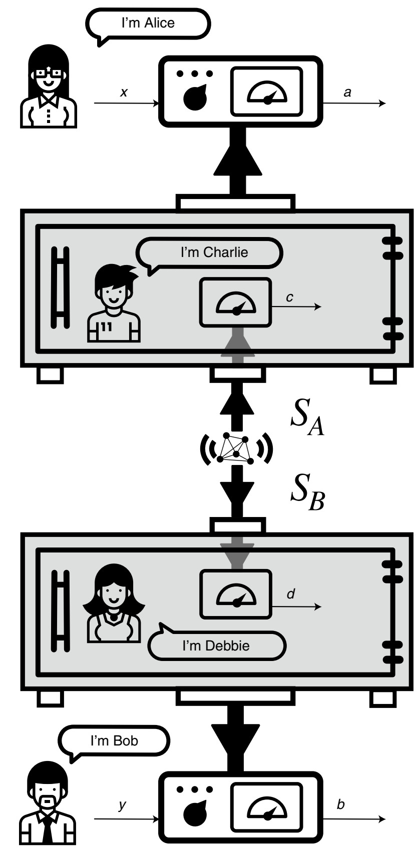
$$M_{a|x=2,3} = (I_C \otimes |a_x\rangle\langle a_x|)U^\dagger_{CS_A}$$



- Here $|a_x\rangle$ are always the eigenvectors of $A_x = e^{-i\phi_x}\sigma_+ + e^{i\phi_x}\sigma_-$ with different angles $\phi_{x=1,2,3}$.

- Bob does the same, but with $B_y = e^{-i(\beta-\phi_x)}\sigma_+ + e^{i(\beta-\phi_x)}\sigma_-$

- They fix: $\phi_1 = 168^\circ,\ \ \phi_2 = 0,\ \ \phi_3 = 118^\circ,\ \ \beta = 175^\circ$



Result is the probability distribution

$$P(a,b|x,y) = \langle\psi_0|M^\dagger_{a|x}M_{a|x} \otimes M^\dagger_{b|x}M_{b|x}|\psi_0\rangle$$

Violations of LF ineqs.

- They actually choose as initial state

$$\rho_\mu = \mu \;\; \psi_0\rangle\langle\psi_0 \;\; +$$

$$\frac{1-\mu}{2}\left(\;\uparrow\downarrow\rangle\langle\uparrow\downarrow \;\; + \;\; \downarrow\uparrow\rangle\langle\downarrow\uparrow \;\; \right)$$

where

$$\psi_0\rangle = \frac{1}{\sqrt{2}}\left(\;\uparrow\downarrow\rangle - \;\downarrow\uparrow\rangle\right)$$

This interpolates between a maximally entangled state when $\mu = 1$ to a purely classical mixture when $\mu = 0$.

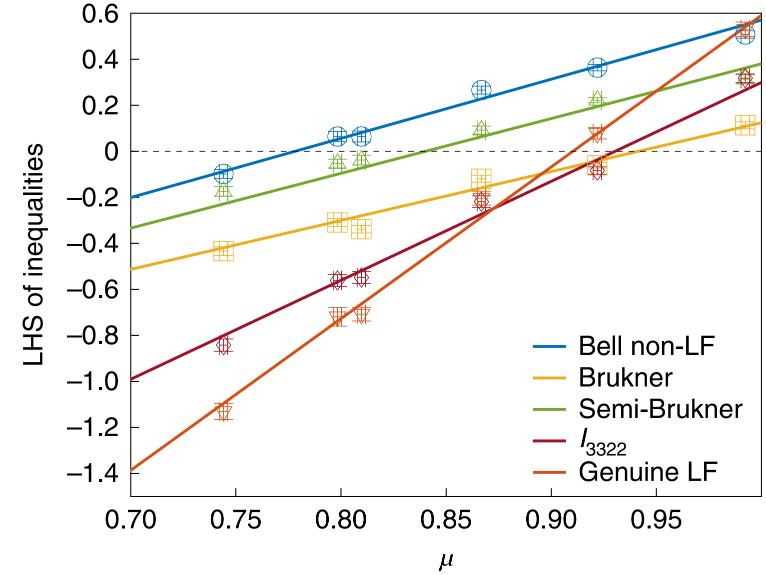


Fig. 4 | Results for the left-hand sides of Bell and LF inequalities for different quantum states. The parameter μ is the pure-state fraction of the quantum state in equation (1). The measurements and inequalities considered are provided in the Methods, using the labels introduced in Table 1. The dashed line in the plot represents the bound above which a violation occurs. The solid lines are theory predictions and the symbols represent experimental data. The uncertainties for the data points represent ±1 standard deviations, calculated from Monte Carlo simulations using 100 samples of Poisson-distributed photon counts. Figure reproduced with permission from ref. [40], SPIE.

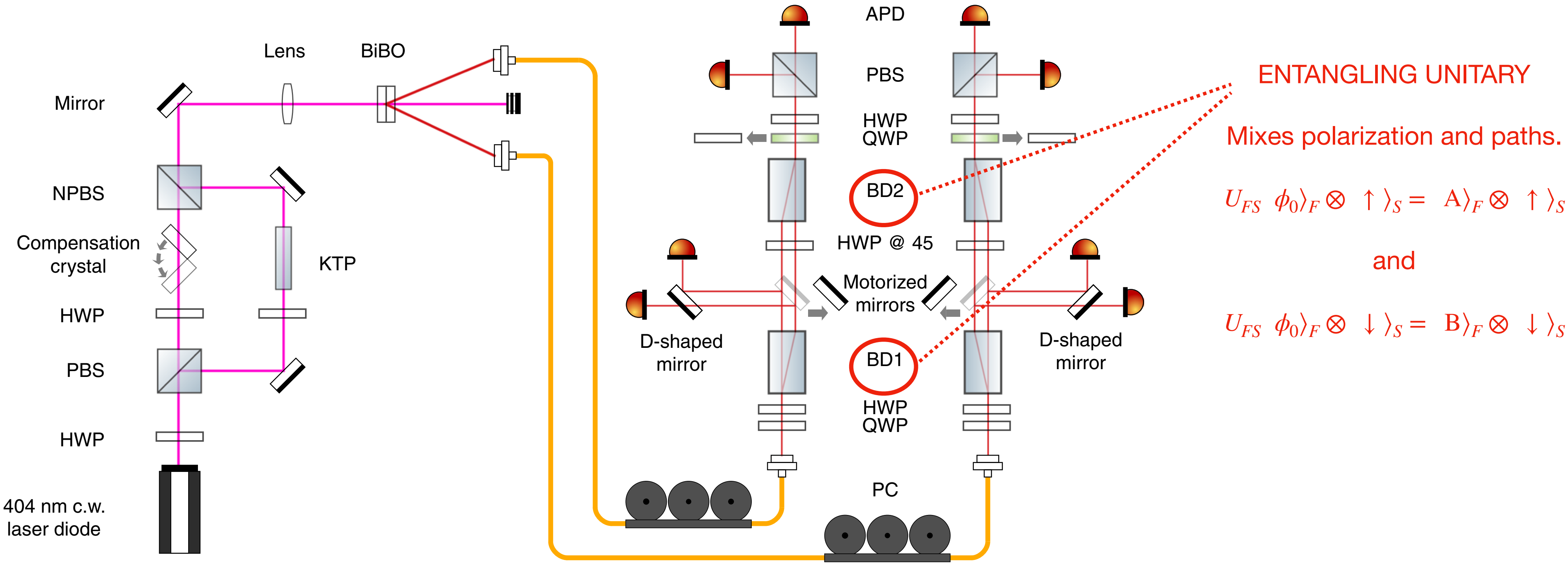


Fig. 5 | Experimental set-up. The source is depicted on the left-hand side, and the measurement section on the right-hand side. The desired quantum state is generated via type-I spontaneous parametric downconversion using two orthogonally oriented bismuth triborate (BiBO) crystals. The pump beam for the downconversion process is a mixture of a decohered state that is obtained from the long arm of the interferometer and a diagonally polarized state from the short arm. The measurement section allows for tomography to be carried out when the motorized mirrors are removed and the photons traverse the beam displacer (BD) interferometers. Alice and Bob perform projective measurements when the quarter-wave plates (QWPs) of the tomography stages are removed. Alternatively, they can ask Charlie and Debbie for their respective measurement outcomes by sliding in the motorized mirrors, using the fact that the projective measurements of their friends correspond to the beam paths inside the interferometers. NPBS, non-polarizing beamsplitter; KTP, potassium titanyl phosphate; HWP, half-wave plate; APD, avalanche photodiode; PC, polarization control; PBS, polarizing beamsplitter. Figure reproduced with permission from ref. [40], SPIE.

What did I learn from this?

Wigner's friend paradox: how to reconcile unitary dynamics with the randomness of measurements.

- Propose 3 “reasonable” assumptions and show that Nature cannot satisfy them.
    - Assumptions are weaker then Bell, so conclusions are stronger.
- But don't really address the Wigner's friend paradox.
    - In the experiment the friend is just a qubit.

A fully convincing demonstration would require a strong justification for the attribution of a ‘fact’ to the friend's measurement. This, of course, depends on what counts as an ‘observer’ (and as a ‘measurement’). Because conducting this kind of experiment with human beings is physically impractical, what do we learn from experiments with simpler ‘friends’? Wigner's own conclusion from his thought experiment was that the collapse of the wave function should happen at least before it reaches the level of an ‘observer’. The concept of an ‘observer’, however, is a fuzzy one.

Born’s rule is an emergent property

- Measurement problem *emerges* when we have a large number of degrees of freedom.
- This might be difficult to probe experimentally.
    - But I cannot imagine any resolution to the Wigner’s friend paradox that does not involve this.

More Is Different

Broken symmetry and the nature of the hierarchical structure of science.

P. W. Anderson

Suggestion for next time: arXiv:2305.15497

Observers in superposition and the no-signaling principle

Veronika Baumann,[1,2,*] and Caslav Brukner[1,3,†]

[1] *Institute for Quantum Optics and Quantum Information (IQOQI), Austrian Academy of Sciences, Boltzmanngasse 3, 1090 Vienna, Austria*

[2] *Atominstitut, Technische Universität Wien, 1020 Vienna, Austria*

[3] *University of Vienna, Faculty of Physics, Boltzmanngasse 5, 1090 Vienna, Austria*

(Dated: May 26, 2023)

The Wigner's friend experiment is a thought experiment in which a so-called superobserver (Wigner) observes another observer (the friend) who has performed a quantum measurement on a physical system. In this setup Wigner treats the friend the system and potentially other degrees of freedom involved in the friend's measurement as one joint quantum system. In general, Wigner's measurement changes the internal record of the friend's measurement result such that after the measurement by the superobserver the result stored in the observer's memory register is no longer the same as the result the friend obtained at her measurement, i.e. before she was measured by Wigner. Here, we show that any awareness by the friend of such a change, which can be modeled by an additional memory register storing the information about the change, conflicts with the no-signaling condition in extended Wigner-friend scenarios.