Hong-Ou-Mandel Interference: A method for determining time-time correlations of entangled photons

John C. Howell

Department of Physics and Astronomy,

University of Rochester, Rochester, NY 14627, USA

Since the early 70's, groups have tried to measure the time-correlation of downconverted photons. Hong, Ou and Mandel (HOM) showed that two photon interference gave an indirect method for measuring the correlations. The HOM method was to observe the fourth order interference at a beam splitter as two time degenergate correlated photons were made spacetime indistinguishable at a beam splitter.

A simplified longitudinal wavefunction, which neglects transverse fields and polarization is given by

$$|\Psi_0\rangle = \int\int dtdt' A(t,t')a_1^\dagger(t)a_2^\dagger(t')|0\rangle \tag{1}$$

where $A(t,t')$ is the nonseparable temporal amplitude correlation function, $a_1^\dagger(t)a_2^\dagger(t')$ are photon creation operators for modes 1 and 2 respectively and $|0\rangle$ is the vacuum state.

The photons from the EPR source are then each sent to two input ports of a beamsplitter. The beamsplitters, which will be assumed to be 50/50 beamsplitters, have the matrix form

$$B = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}. \tag{2}$$

The field operators at the detectors, after passing through the interferometer, are given by (it is assumed that the trasmitted mode through the beam splitter preserves the mode)

$$a_1^\dagger(t) \rightarrow \frac{1}{\sqrt{2}}(a_1^\dagger(t+\tau_1) + ia_2^\dagger(t+\tau_2)) \tag{3}$$

and

$$a_2^\dagger(t') \rightarrow \frac{1}{\sqrt{2}}(ia_1^\dagger(t') + a_2^\dagger(t')) \tag{4}$$

where the subscripts denote the spatial modes, and $\delta\tau$ represents the change in relative optical path length by manual adjustment of the position of the beam splitter. The wavefunction is then given by

$$|\Psi_1\rangle = \frac{1}{2}\int\int dtdt' A(t,t')(a_1^\dagger(t+\tau_1)a_2^\dagger(t') - a_1^\dagger(t')a_2^\dagger(t+\tau_1))|0\rangle \tag{5}$$

The fourth order coherence between the two detectors can be mathematically represented by

$$P_{1,2} \propto \langle\Psi_1|a_1^\dagger(t_1)a_2^\dagger(t_1-\tau_2)a_2(t_1-\tau_2)a_1(t_1)|\Psi_1\rangle \tag{6}$$

where the narrow bandwidth approximation has been assumed (i.e., the bandwidth of the downconversion is much smaller than the radial frequency of the light) and τ_2 represents the detector response time. An observation of this coherence function shows that there are four terms of eight operators. We first simplify this correlation function by multiplying by a closure relation in the number operators:

$$P_{1,2} \propto \sum_{n=0}^{\infty}\langle\Psi_1|a_1^\dagger(t_1)a_2^\dagger(t_1-\tau_2)|n\rangle\langle n|a_2(t_1-\tau_2)a_1(t_1)|\Psi_1\rangle \tag{7}$$

which reduces to

$$P_{1,2} \propto (\langle 0|a_2(t_1-\tau_2)a_1(t_1)|\Psi_1\rangle)^2 \tag{8}$$

The effort is to remove the operators from the function by normally ordering the operators. When normally ordered $a|0\rangle = 0$ which means that the operators can be removed from the calculation. The ordering is achieved using the equal time boson commutator relation $[a(t), a^\dagger(t')] = \delta(t-t')$. After a somewhat lengthy, but straightforward calculation, the fourth order coherence is found to be

$$\begin{aligned} P_{1,2} &\propto \frac{1}{2}\int\int dtdt' A(t,t')(\delta(t_1-\tau_2-t')\delta(t_1-t-\tau_1) - \delta(t_1-t')\delta(t_1-\tau_2-t-\tau_1))^2 \\ &\propto (A(-t_1+\tau_1, -t_1+\tau_2) - A(-t_1+\tau_2+\tau_1, t_1))^2 \\ &\propto (A(\tau_2-\tau_1) - A(\tau_2+\tau_1))^2 \end{aligned} \tag{9}$$

where we assume that the function is of the form $A(t,t') = A(t-t')$. Now, we integrate over the detector response time τ_2. Since the detector response time is typically much larger than the correlation time, the limits of integration can be assumed to be taken over all time. Lastly, we will assume that the correlation function is real. Making these assumptions, the two-photon detection rate is then

$$R_{1,2} \propto \int_{-\infty}^{\infty} d\tau_2|A(\tau_2-\tau_1)|^2 + \int_{-\infty}^{\infty} d\tau_2|A(\tau_2+\tau_1)|^2 - 2\int_{-\infty}^{\infty} d\tau_2 A(\tau_2-\tau_1)A(\tau_2+\tau_1) \tag{10}$$

This rate function determines the relative time-time correlations of the photons from downconversion even though the detector response times are several orders of magnitude less sensitive to time measurements.

The rate equation in eqn. 10 is a general statement about the relative time correlations of photon emissions. For downconversion, the temporal correlation function is primarily attributed to the gaussian spectral passbands of interference filters. We therefore find $A(t-t') = e^{-(t-t')^2\Delta^2/2}$, where Δ is the spectral bandwidth of the filters. The normalized rate equation is then simplified:

$$R_{1,2} = \frac{1+e^{-\tau_1^2\Delta^2}}{2}. \tag{11}$$

Hence, when $\tau_1 = 0$ or in other words, two photon counts will vanish as both photons leave the beamsplitter together. When $\tau_1 >> 1/\Delta$ there will be a constant rate for all relative delays rate in two photon counts.