Physics 227 Homework 8 - Due April 17, 2009

Problem 1: Callen, 16.1-1

Problem 2: Reinvestigate the 'two-state' system of section 15-3, but from the point of view of the canonical ensemble. Find the canonical partition function, the Helmholtz free energy, and the average internal energy, assuming that there are N 'atoms'. Are your results consistent with the microcanonical results?

Problem 3: In lecture we derived the canonical partition function Z(V, T, N) for the ideal gas using the factorization method. One can also directly compute it by taking the Laplace transform of the microcanonical partition function $\Omega(E, V, N)$. Using our result from lecture,

$$\Omega = \frac{1}{N!} \left[\frac{V(2\pi mE)^{3/2}}{h^3} \right]^N \frac{\Delta}{E[(3N/2) - 1]!},\tag{1}$$

compute directly the Laplace transform,

$$Z(V,T,N) = \int_0^\infty \frac{dE}{\Delta} \Omega(E,V,N) e^{-\beta E},$$
(2)

and show that you get the same result as found in lecture.

Problem 4: The potential energy between atoms of a Hydrogen molecule is given by the (semi-empirical) *Morse* potential

$$V(r) = V_0 \left\{ e^{-2(r-r_0)/a} - 2e^{-(r-r_0)/2} \right\},$$
(3)

where $V_0 = 7 \times 10^{-12}$ erg, $r_0 = 8 \times 10^{-9}$ cm, and $a = 5 \times 10^{-9}$ cm. Calculate the low-lying vibrational and rotational quanta of energy, and estimate the temperatures at which the rotational and vibrational modes of the molecules would begin to contribute toward the specific heat of the Hydrogen gas.