Physics 418
Final Exam - May 5-6, 2009

Rules of the exam: This is a 24-hour take-home exam (12 noon to 12 noon). Please drop the exam off in the Physics 418 box when you are finished. You are expected to work *by yourself* **on this, with no assistance of any kind from anyone else. You may use the class texts (Pathria and Callen), your class notes and a math/plotting resource (i.e., integration tables, Mathematica, etc.), but no other books, online sources, or materials of any kind.**

Problem 1 (33 pts): Consider a classical one dimensional wire attached between two fixed walls separated by a distance L and tightened to a tension τ. For simplicity, consider only motion in one transverse dimension, $y(x,t)$. The wire has a mass density (mass/unit length) μ and is in thermal equilibrium with the surrounding air at a temperature T. The temperature is sufficiently moderate that only the transverse degree of freedom is excited, describable by a linear wave equation with velocity $c = \sqrt{\tau/\mu}$. (a) (10 pts) Solve the wave equation and find all of its standing waves (modes) for the above situation, $y_n(x,t)$. (b) (20 pts) Find the root-mean-square displacement of the *center* of the string ($x = L/2$), $\sqrt{\langle y^2 \rangle}$. (c) (3 pts) Find this displacement for a violin A string at room temperature; it has a vibrating length of 32 cm, a fundamental frequency of 440 Hz and a mass of 0.35 g. Hint: The equipartition theorem may come in handy.

Problem 2 (33 pts): Consider blackbody radiation at temperature T in d spatial dimensions. (a) (11 pts) What is the mean energy per photon? (b) (11 pts) What is the entropy per photon? (c) (8 pts) What would the answers to these question be if photons obeyed classical Boltzmann statistics? (d) (3 pts) Plot results (a), (b), and (c) in dimensionless form versus d.

Problem 3 (33 pts): Consider the Ising model in one dimension where there is both exchange interaction $J > 0$ and applied magnetic field $\mathcal{H}$ for N spins. (a) (10 pts) Construct the matrix $\mathbf{O}$ that diagonalizes the transfer matrix $\mathbf{T}$. You will find it helpful to write down the matrix elements in terms of the variable ϕ given by

$$\cot(2\phi) = e^{2K} \sinh h, \tag{1}$$

where K, h are the rescaled Ising parameters introduced in class. (b) (10 pts) For periodic boundary conditions, calculate the magnetization of site i, $\langle S_i \rangle$, and express your answer in terms of ϕ in the limit $N \to \infty$. What happens when $h = 0$? (c) (13 pts) Now consider free (not periodic) boundary conditions, so S_1 does not interact with S_N. In this case, the partition function is not simply $\mathrm{Tr}\,\mathbf{T}^N$. Work out what the correct expression is (you will need to introduce a new matrix in addition to $\mathbf{T}$), and show that the free energy breaks into 3 contributions, $F = Nf_b + f_s + F_{fs}$, where Nf_b is the bulk contribution, f_s is the surface free energy due to the boundaries, and F_{fs} is an intrinsically finite size contribution that decreases exponentially with N.