Physics 418
Homework 4 - Due Mon. March 15, 2010

Problem 1: Some systems are adequately described by a one-dimensional potential energy in the form of an asymmetric double well. To good accuracy each well can be assumed to be harmonic with potential energies

$$V_L(x) = (1/2)k_L x^2, \text{ in the left well,} \qquad V_R = \epsilon + (1/2)k_R(x-a)^2, \text{ in the right well.} \tag{1}$$

Here, $\epsilon = V_R(a) > 0$. N classical particles of mass m are brought into thermal equilibrium in this potential.
(a) At temperature T, what is the average number of particles in each well?

(b) What conditions need to be imposed on the parameters of the potential in order to have equal populations at temperature T^*?

(c) Calculate the difference $u_R - u_L$ between the internal energy of two particles in the two wells. Explain why your result is not inconsistent with the way the particles are distributed between the wells for $T \geq T^*$ (when the conditions found in (b) hold.)

Problem 2: Patheria 3.31

Problem 3: Suppose that a complete description of a physical process is not possible due to some missing information. We can think of this missing information as characterizing our ability to predict an event. Consider, for example, a loaded die. If the die is strongly weighted, then it will always land on 6 (for example), so we have no missing information. However, if the weighting is imperfect, sometimes it will land on another side (say 1) (and we are not given the detailed trajectory that the die took) with small probability, and we will be surprised. Our ability to predict that event is quite low, so our missing information is very high. It is useful for shifty gamblers (and therefore physicists) to define a missing information function I that depends on the probability of an event P. This missing information function may also be thought of as the surprisal value of an event. The missing information function I of seperate events $E_1, E_2, \ldots, E_N$ which are described by the probabilities $\{P_j\}, j = 1, \ldots, N$ should have these properties:

- $I(P = 1) = 0$: if there is only one possible event, there is no missing information.
- $I(P)$ is a smooth function of the probability P.
- $I(PQ) = I(P) + I(Q)$. In other words the missing information of two *independent* events (with probability P and probability Q) occurring is the sum of the missing information of both of those events.

(a) What is the missing information I if an event E of probability P occurs?
(b) What is the average missing information of all possible events $E_1, E_2, \ldots, E_n$? Show this quantity is the Shannon entropy mentioned in class.

Problem 4: Starting with the Shannon entropy, $S = -k \sum_j P_j \ln P_j$, apply this entropy to a statistical mechanical system where the only information we are given about the system is that the total internal energy is U, and the total particle number is N. The constant k is taken to be the Boltzmann constant. Find the probability distribution $\{P_j\}$ that maximizes the average missing information S, subject only to the information that we are given above. Show that the grand canonical probabilities are recovered.