

Physics 418
Homework 6 - Due Wed., April 21, 2010

Problem 1: Consider an ideal Bose gas composed of molecules with an internal degree of freedom. Assume that this internal degree of freedom can have one of two energy values, the ground state $\epsilon_0 = 0$, and an excited state, $\epsilon_1 > 0$. Determine the Bose-Einstein condensation temperature T_c of the gas as a function of ϵ_1 . Show in particular that for $\epsilon_1 \gg k_B T$,

$$\frac{T_c}{T_{c,0}} = 1 - \frac{2 \exp(-\epsilon_1/k_B T)}{3\zeta(3/2)} \quad (1)$$

where $T_{c,0}$ is the transition temperature when ϵ_1 is infinite, and $\zeta(x)$ is the Riemann zeta function.

Problem 2: Consider photons of a given energy $\epsilon = \hbar\omega$.

(a) If $\langle n \rangle$ is the average number of such photons in equilibrium at temperature T , show that the fluctuation in the number of photons is

$$\langle n^2 \rangle - \langle n \rangle^2 = -\frac{1}{\epsilon} (d\langle n \rangle / d\beta). \quad (2)$$

Using the formula for the equilibrium value of $\langle n \rangle$, apply the above result to determine the relative fluctuation in the number of photons,

$$(\langle n^2 \rangle - \langle n \rangle^2) / \langle n \rangle^2. \quad (3)$$

Is this large or small? Can it be smaller than 1?

Problem 3: Pathria 7.19