Physics 418
Homework 7 - Due, Friday, April 30, 2010

Problem 1: For the 1D Ising model solved in class, calculate and plot (a) the internal energy, (b) the specific heat, and (c) the magnetization versus both magnetic field and temperature for different values of the interaction constant. Verify the specific heat has a Schottky anomaly, and that the magnetic susceptibility obeys Curie's law for large temperature.

Problem 2: Consider the Ising model in one dimension where there is both exchange interaction $J > 0$ and applied magnetic field $\mathcal{H}$ for N spins. (a) Construct the matrix $\mathbf{O}$ that diagonalizes the transfer matrix $\mathbf{T}$. You will find it helpful to write down the matrix elements in terms of the variable ϕ given by

$$\cot(2\phi) = e^{2K} \sinh h, \tag{1}$$

where K, h are the rescaled Ising parameters introduced in class. (b) For periodic boundary conditions, calculate the magnetization of site i, $\langle S_i \rangle$, and express your answer in terms of ϕ in the limit $N \to \infty$. What happens when $h = 0$? (c) Now consider free (not periodic) boundary conditions, so S_1 does not interact with S_N. In this case, the partition function is not simply $\mathrm{Tr}\,\mathbf{T}^N$. Work out what the correct expression is (you will need to introduce a new matrix in addition to $\mathbf{T}$), and show that the free energy breaks into 3 contributions, $F = Nf_b + f_s + F_{fs}$, where Nf_b is the bulk contribution, f_s is the surface free energy due to the boundaries, and F_{fs} is an intrinsically finite size contribution that decreases exponentially with N.