On Position-Momentum Entanglement, Nonlocality, and Measurement

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To The Ones who Made this Possible

John Howell
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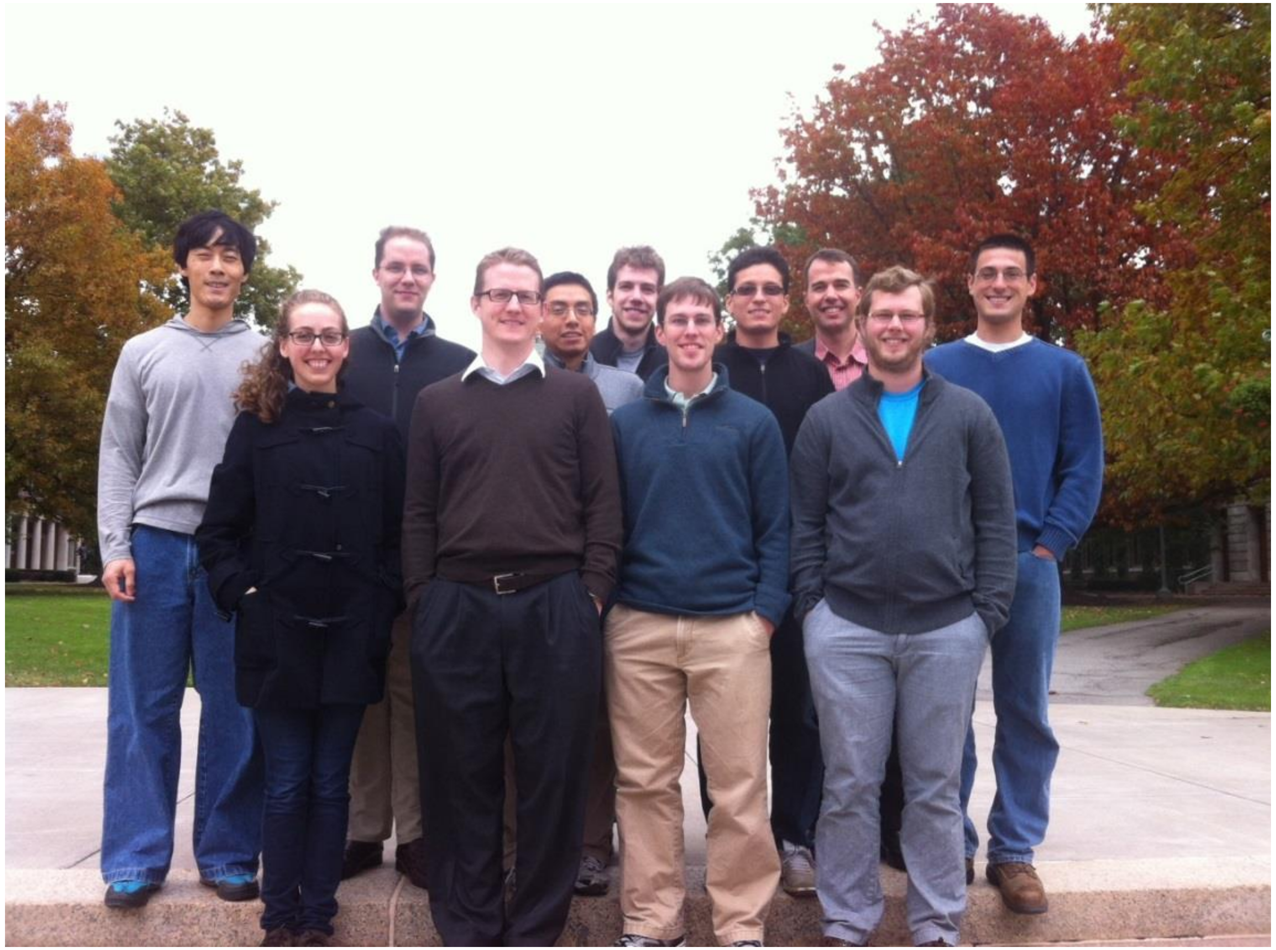
Justin Winkler



Thank You!!

Outline:

- Measurement
  - The uncertainty principle
  - What position measurements do to momentum
  - Notions of uncertainty
- Entanglement
  - How to prove quantum entanglement experimentally
  - Witnessing entanglement with EPR-steering inequalities
  - How to demonstrate *continuous-variable* entanglement with discrete measurements
- Nonlocality
  - The Position-momentum CHSH Bell inequality
  - Bell's strategy for demonstrating position-momentum Bell nonlocality
  - Bringing Bell's proposal to the lab
  - Challenges and prospects

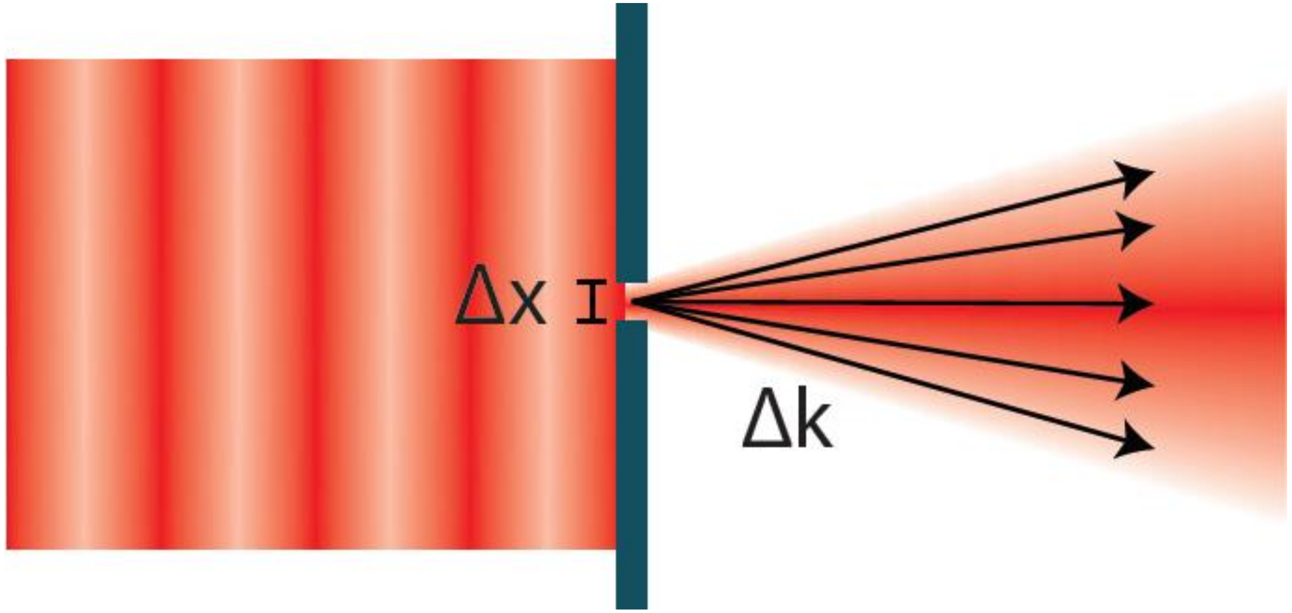
Part One: Measurement

(and reimagining the uncertainty principle)

The Uncertainty Principle

In quantum mechanics:

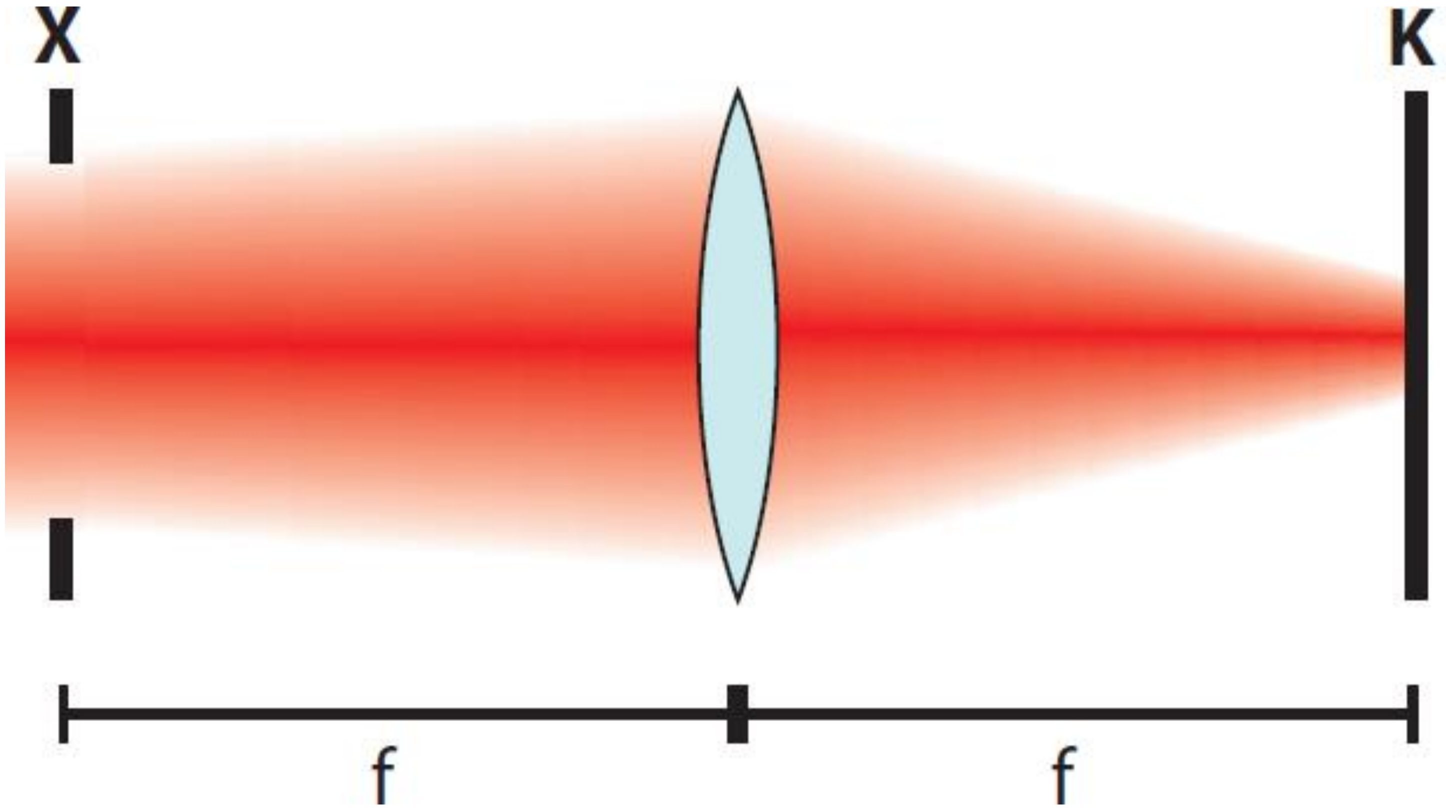
- Position $\boldsymbol{x}$ and momentum $\boldsymbol{k}$ are "complementary" observables
  - There are many other such pairs
- You can make (prepare the state of) a particle with near-definite position..
  - ..but not also with a near-definite momentum.
- The narrower you make the position of a wave, the wider its momentum spread must be.



$$\Delta \boldsymbol{x} \cdot \Delta \boldsymbol{k} \geq \frac{1}{2}$$

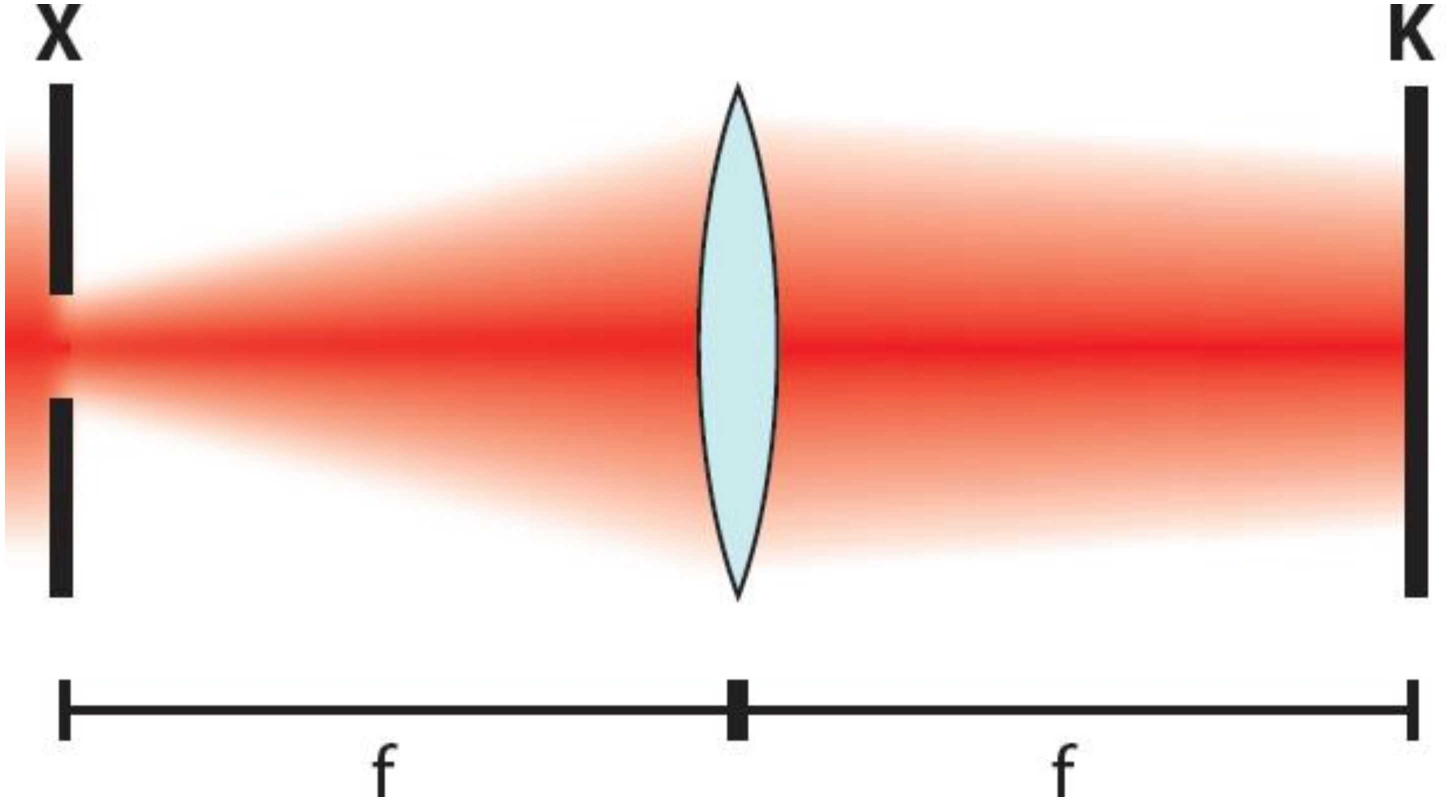
The effect of a Position measurement on Momentum

- A particle passing through a pinhole, definitely had a position within that pinhole.
- How much can we know about its momentum?



The effect of a Position measurement on Momentum

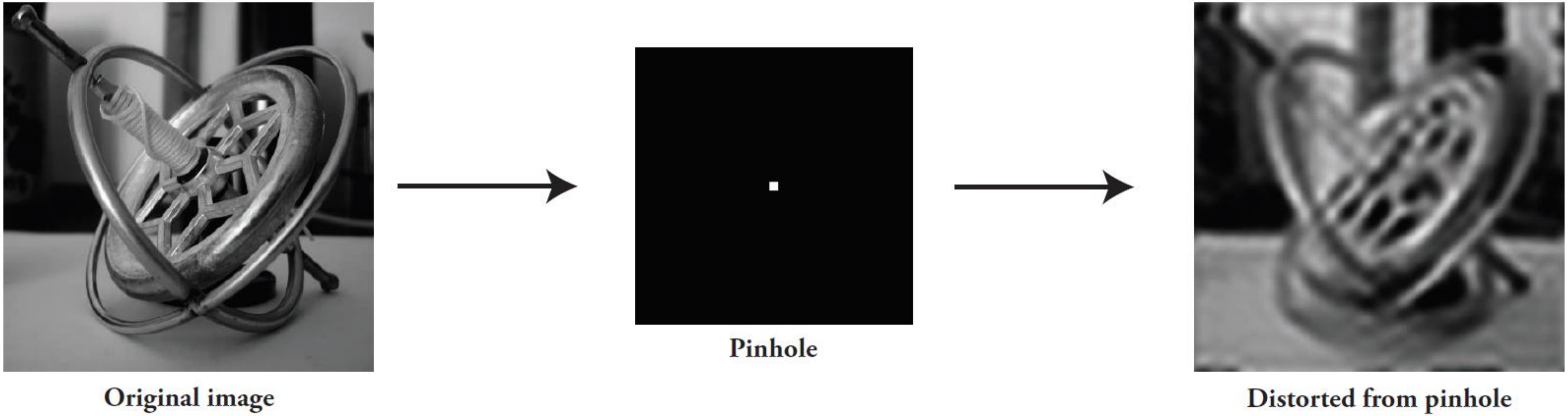
- A particle passing through a pinhole, definitely had a position within that pinhole.
- The smaller the pinhole, the better we know the position of the particle.
- But what does this do to the momentum of the particle?



The effect of a Position measurement on Momentum

Every position amplitude is a sum over momentum frequency components.

- A pinhole in momentum space excludes many high frequency components
- The image after too-small a pinhole will be significantly blurred.
- Similarly, a pinhole in position blurs the momentum amplitude distribution.



The effect of a Position measurement on Momentum

- Do sharper position measurements mean blurrier momentum measurements?
  - Not necessarily!
- Using random screens of pinholes, the momentum distribution is not blurred.
- The effect is instead seen as low level noise.

Original image

Random pattern

Distorted from random pattern

The effect of a Position measurement on Momentum

- A thin pinhole in position space, is a broad function in momentum space.
- The distortion in momentum is modeled as the *convolution* of the field with this broad transformed pinhole function.

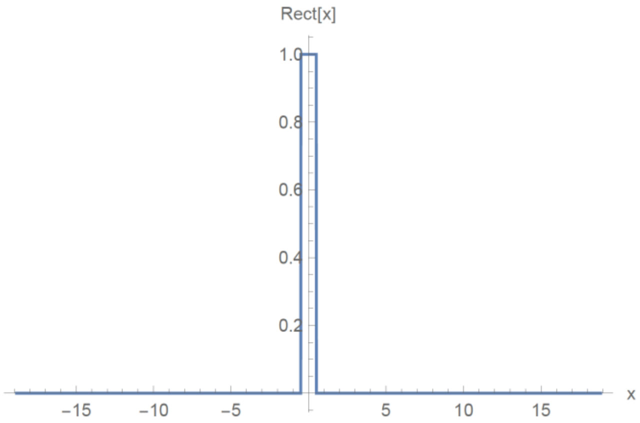
- Convolving with broad functions makes for a blurry image.
  - Convolving with narrower functions makes less blurry images

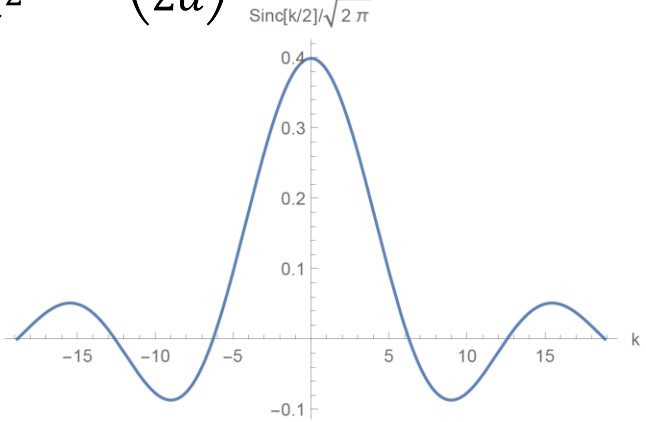
$\psi(x)$ = position amplitude of field
$f(x)$ = binary pinhole function
$\bar{\psi}(x)$ = distorted position amplitude

$$\bar{\psi}(x) = f(x)\psi(x)$$
$$\bar{\psi}(k) = f(k) * \psi(k)$$

For a pinhole:

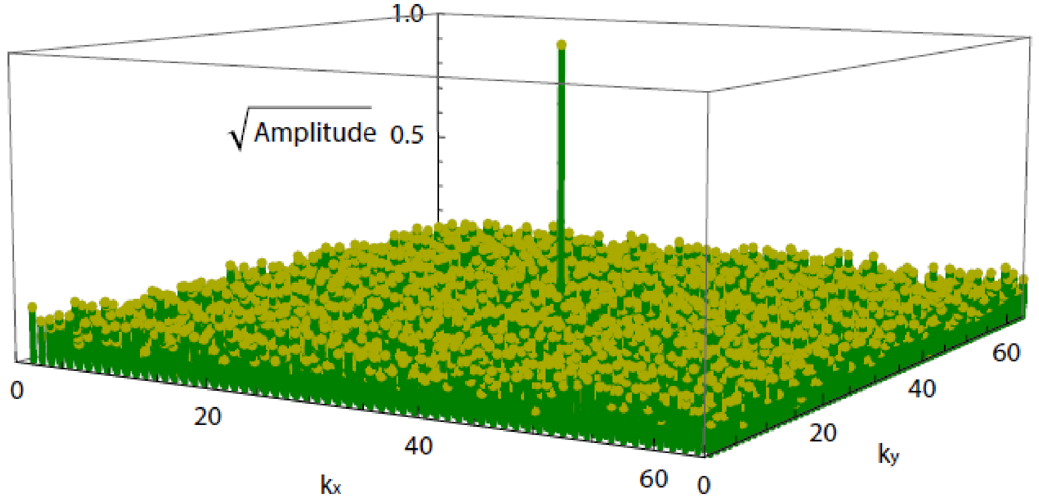
$$f(x) = \mathrm{rect}(a\,x)$$
$$f(k) = \frac{1}{\sqrt{2\pi a^2}}\mathrm{sinc}\left(\frac{k}{2a}\right)$$





The effect of a Position measurement on Momentum

- Convolving with broad functions makes for a blurry image.
  - Convolving with narrower functions makes less blurry images
- A random array of many pinholes in position space is a sharp narrow function with low level noise in momentum space.



- For a pinhole:

$$f(k) \propto \mathrm{sinc}\left(\frac{k}{2a}\right)$$

- For a random pinhole array of N total pixels:

$$f(k) \propto \mathrm{sinc}\left(\frac{k}{2a}\right)\left(\delta(k) + \frac{b}{\sqrt{N}}\phi(k)\right)$$

  - ..where $\phi(k)$ is a unit-variance Gaussian complex random variable for each value k.

The perturbed momentum amplitude is then:

$$\bar{\psi}(k) \approx \mathcal{N}\left(\psi(k) + \frac{b}{\sqrt{N}}(\psi(k) * \phi(k))\right)$$

The effect of a Position measurement on Momentum

- With N random patterns (the same as the number of pixels)…
- …we can retrieve the position distribution *without also blurring* the momentum
  - With say, least-squares optimization or compressive sensing

- What does this say about the uncertainty principle?

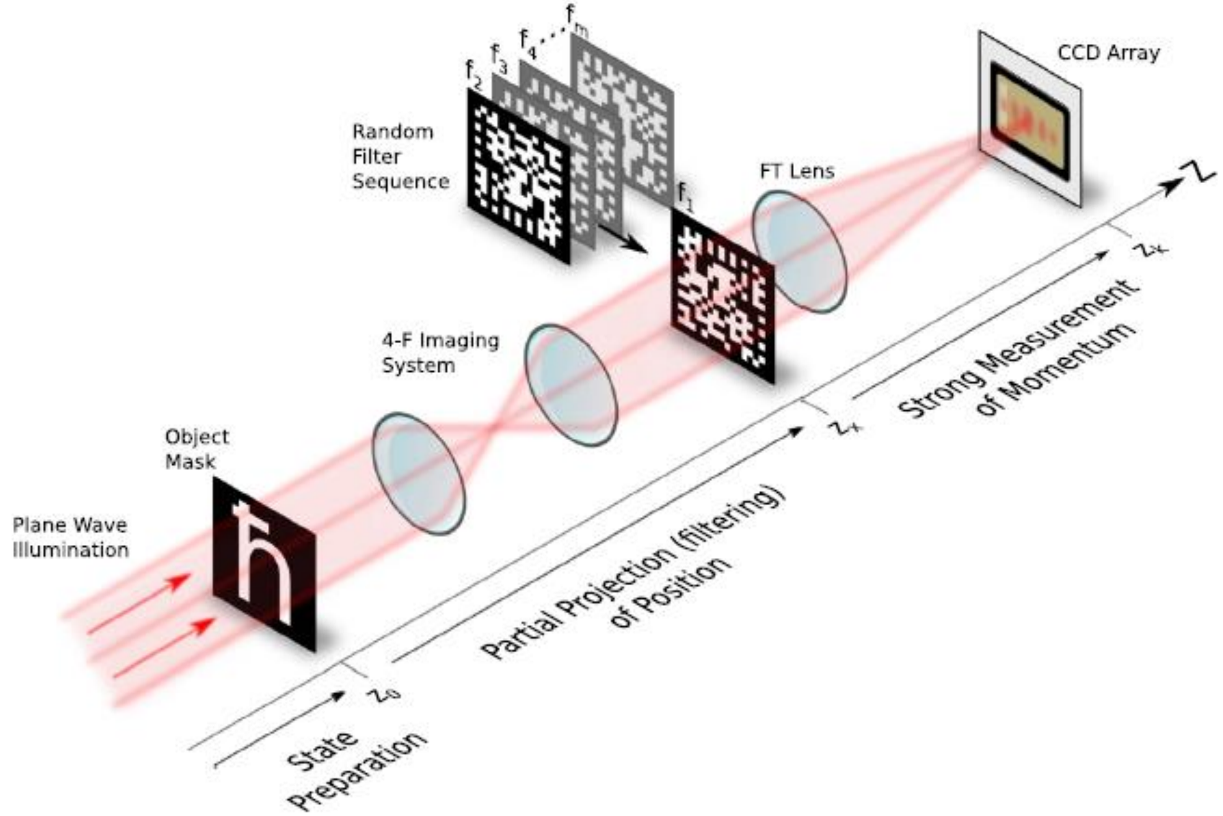


Diagram is first figure of:
Phys. Rev. Lett **112**, 253602 (2014).

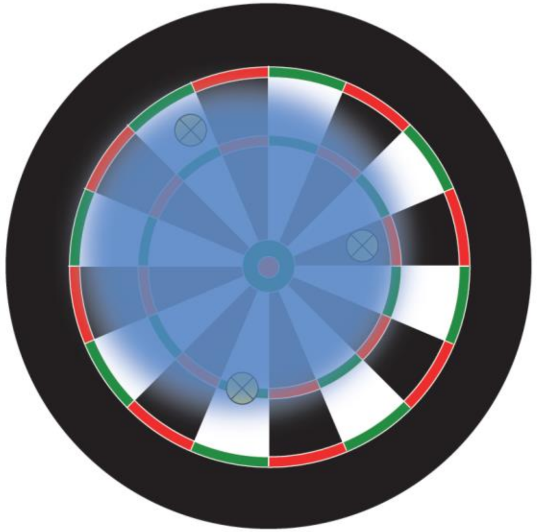
Notions of "Quantum" Uncertainty

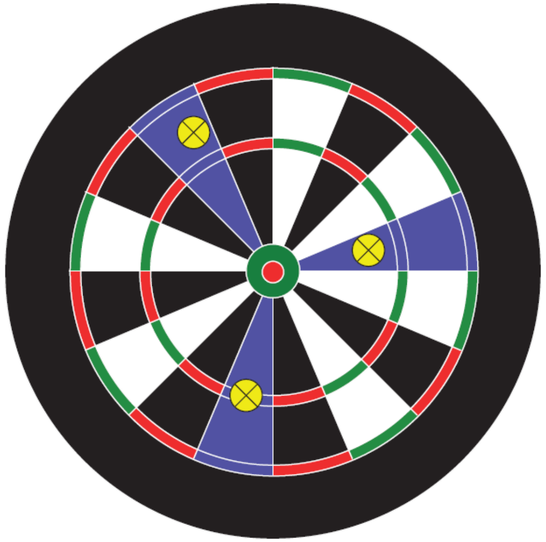
Localization

- Within what tolerance can you reliably predict the outcome?
- How tightly are the random outcomes clustered about a single peak?

Information

- What is the size the set of likely outcomes?
- How many bits do you need to communicate the outcome?





Notions of *Quantum* Uncertainty

Localization

- Within what tolerance can you reliably predict the outcome?
- How tightly are the random outcomes clustered about a single peak?

$$\sigma_{\boldsymbol{x}} = \sqrt{\langle(\boldsymbol{x} - \langle\boldsymbol{x}\rangle)^{\mathbf{2}}\rangle}$$

$$\sigma_{\boldsymbol{x}} \cdot \sigma_{\boldsymbol{k}} \geq \frac{1}{2}$$

Information

- What is the size the set of likely outcomes?
- How many bits do you need to communicate the outcome?

$$\mathrm{h}(\boldsymbol{x}) = -\int dx\, \rho(\boldsymbol{x}) \log\big(\rho(\boldsymbol{x})\big)$$

$$\mathrm{h}(\boldsymbol{x}) + \mathrm{h}(\boldsymbol{k}) \geq \log(\pi e)$$

Part Two: Entanglement

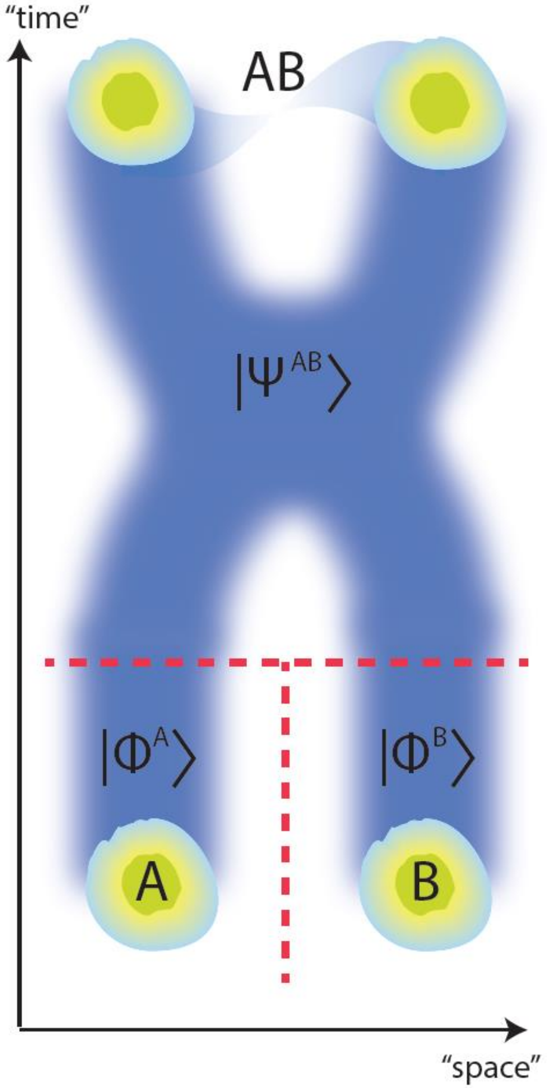
(through the EPR paradox and EPR steering inequalities)

Entanglement in a Nutshell.

(It's defined by what it isn't)

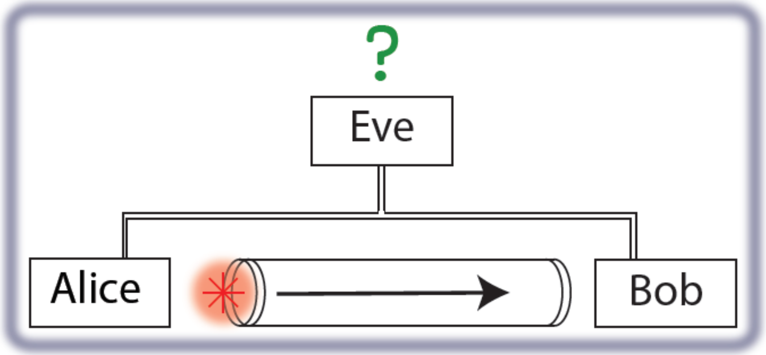
Entanglement is created through the interaction of quantum systems.

- The quantum state of a pair of non-interacting independent particles is separable:
  - E.g. $|\psi^{AB}\rangle = |\phi^{A}\rangle \otimes |\phi^{B}\rangle$.
- If the quantum state of a system can be made out of such non interacting, independent pairs, that state must also be separable:
  - E.g. $\hat{\rho}^{AB} = \sum_i p_i \left(\hat{\rho}_i^{A} \otimes \hat{\rho}_i^{B}\right)$.
- All other states are entangled



What's entanglement good for?

Quantum Cryptography



Quantum computing

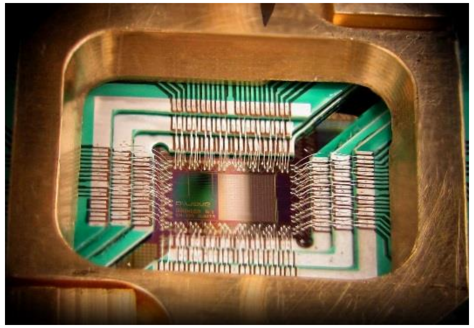
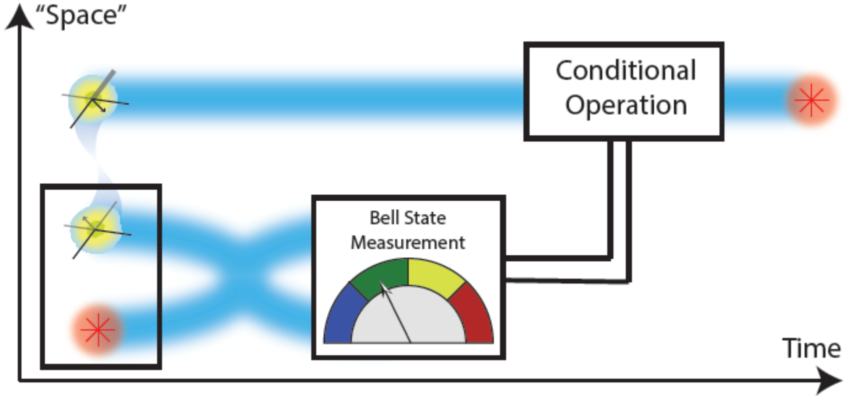


Image of D-wave chip from:
https://en.wikipedia.org/wiki/File:DWave_128chip.jpg

Quantum Teleportation



Enhanced measurement

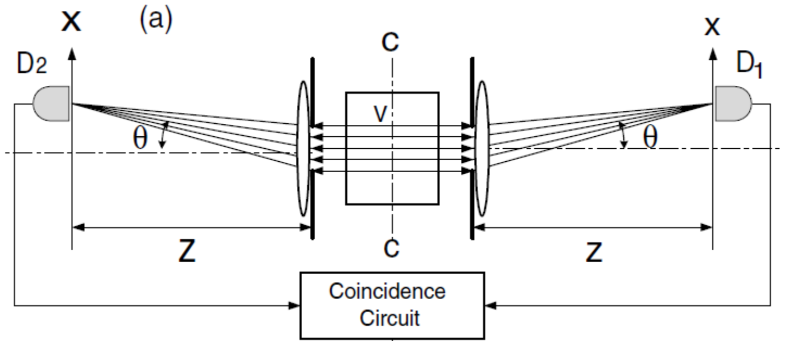


Diagram from:
Phys. Rev. Lett. **87**, 013602 (2001)

Proving Entanglement in the Lab

The **hard** way:

- Determine the joint quantum state through exhaustive tomography.
- Calculate a measure of entanglement for the given state.
  - (NP-hard, in general)

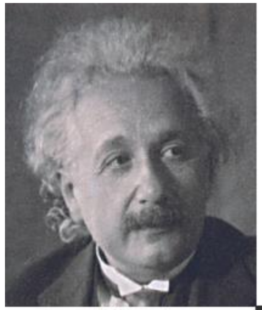
The **easy** way:

- Test an entanglement witness:
  - A statistical criterion all separable states satisfy.
- If the entanglement witness is violated,
  - Entanglement is certified.
- If the witness is not violated,
  - Entanglement is not certified

Witnessing Entanglement with EPR-Steering Inequalities

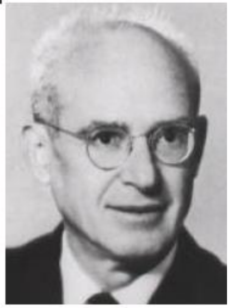
EPR steering: The explicitly nonlocal manipulation of a quantum state through actions on an entangled partner.

- If a pair of particle's statistics violate an EPR-steering inequality…
  - …they demonstrate the EPR paradox
  - …their state must be entangled.



Albert Einstein

Boris Podolsky

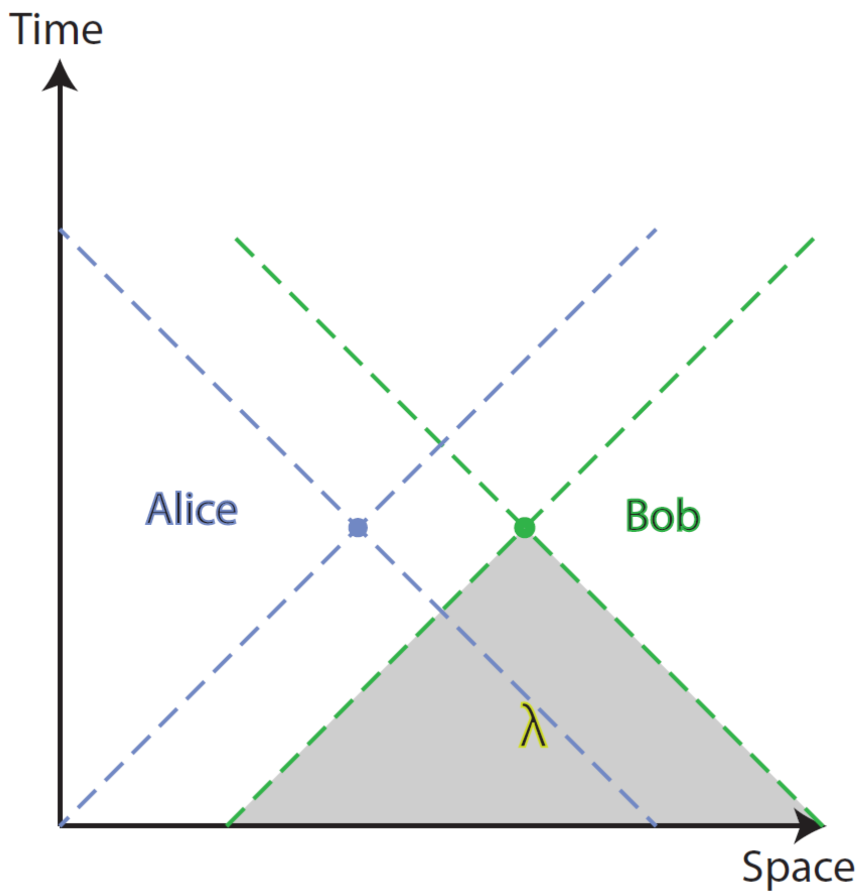


Nathan Rosen

From the EPR paradox to EPR-steering inequalities

- **The situation:** Alice and Bob share a pair of particles A and B entangled in position and momentum.
  - A and B are space-like separated from each other at the time of measurement.

- **Locality:** The effect of measurement cannot travel faster than light.
- **Completeness:** The uncertainty principle fundamentally limits our knowledge of a quantum system.
  - Knowing everything that could locally affect a particle's history wouldn't change this.



From the EPR paradox to EPR-steering inequalities

- Entangled pairs of particles can have *arbitrarily strong* correlations in position and in momentum.

- The seeming paradox:
- All "possible" information about x_B or k_B would be in Bob's past light cone λ.
- Alice's measurements couldn't possibly give you more information about x_B or k_B than knowing everything in λ.

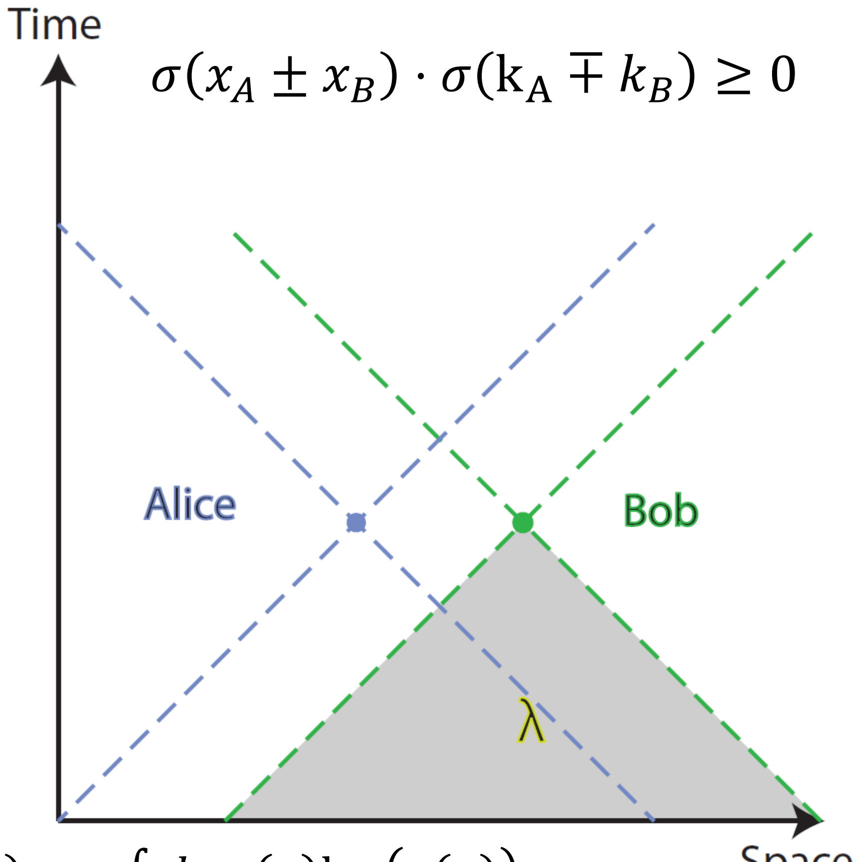
$$h(x_B|x_A) \geq \int d\lambda\, \rho(\lambda) h(x_B|\lambda)$$

$$h(k_B|k_A) \geq \int d\lambda\, \rho(\lambda) h(k_B|\lambda)$$

$$\sigma(x_A) \cdot \sigma(k_A) \geq \frac{1}{2}$$

but…

$$\sigma(x_A \pm x_B) \cdot \sigma(\mathrm{k_A} \mp k_B) \geq 0$$



$$\mathrm{h}(\boldsymbol{x}) = -\int dx\, \rho(\boldsymbol{x}) \log(\rho(\boldsymbol{x}))$$

From the EPR paradox to EPR-steering inequalities

$$h(x_B|x_A) \geq \int d\lambda\, \rho(\lambda) h(x_B|\lambda)$$

$$h(k_B|k_A) \geq \int d\lambda\, \rho(\lambda) h(k_B|\lambda)$$

- Using the entropic uncertainty relation

$$h(x_B) + h(k_B) \geq \log(\pi e)$$

- We find that in a local universe, Alice and Bob's measurement correlations must be limited by the (EPR-steering) inequality:

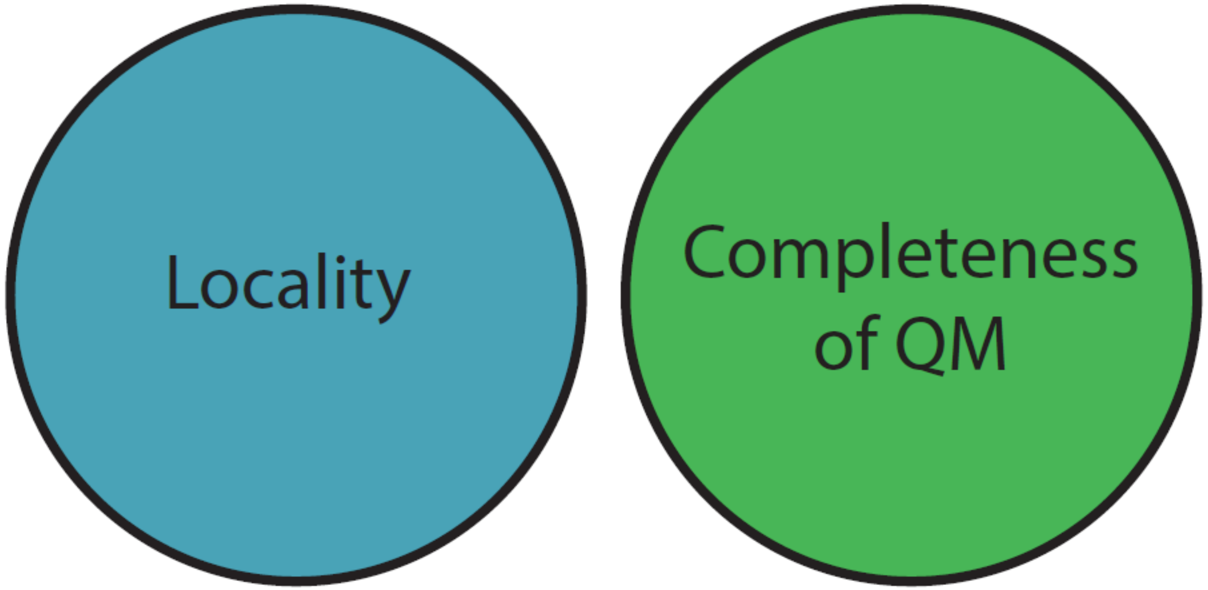
$$h(x_B|x_A) + h(k_B|k_A) \geq \log(\pi e)$$

- QM predicts there are no limits to these correlations!
  - EPR-steering inequalities can be violated!

$$\sigma(x_A)\sigma(k_A) \geq \frac{1}{2}$$

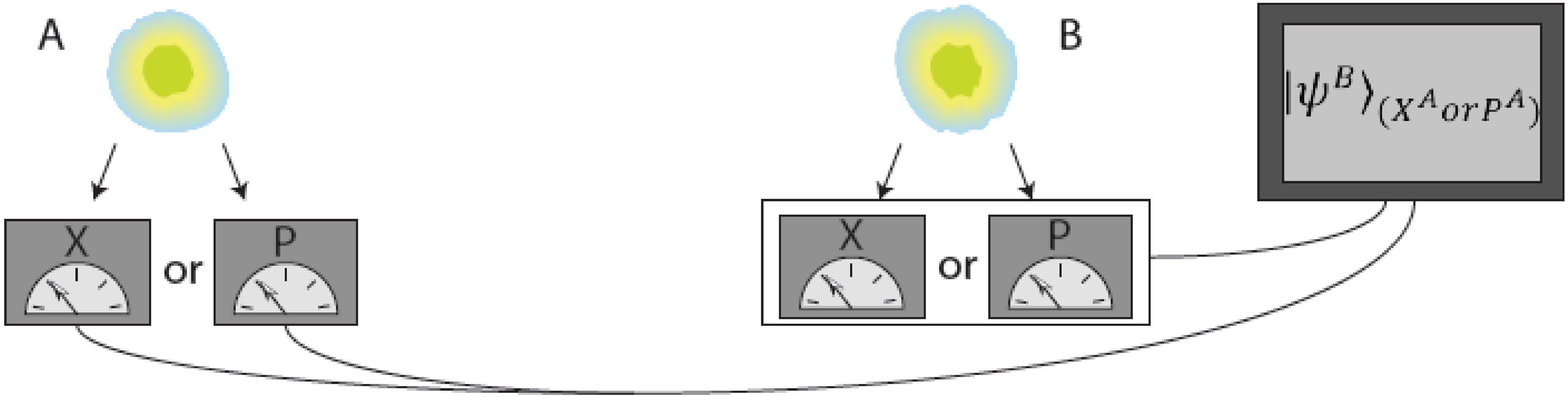
but…

$$\sigma(x_A \pm x_B)\sigma(k_A \mp k_B) \geq 0$$



(A Venn Diagram of the EPR paradox)

Where's the steering?



The setup:

- When Alice measures **X**:
  - Bob finds a state *well defined in position* when conditioning on Alice's outcome
- When Alice measures **K**:
  - Bob finds a state *well defined in momentum* when conditioning on Alice's outcome

How does EPR-steering ***prove*** entanglement?

Problem:

- Lots of correlations can be explained classically
  - Alice and Bob could be receiving a classically correlated ensemble of states
  - Alice or Bob could have an untrusted measurement device (a “black box”)

- How do you rule out this possibility?
  - i.e., the possibility of a model of **local hidden states** for Alice or for Bob.

Local Hidden ***States***?

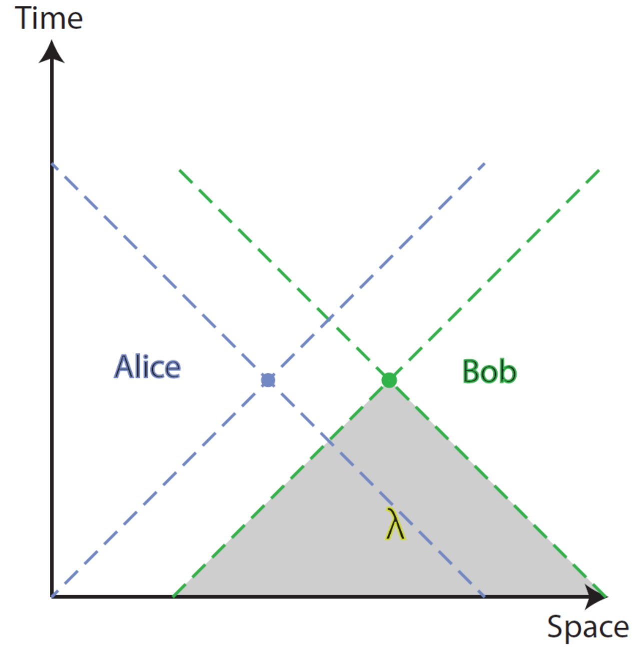
- Local hidden variables (LHV):
  - Information existing in past light cone
- LHV models:

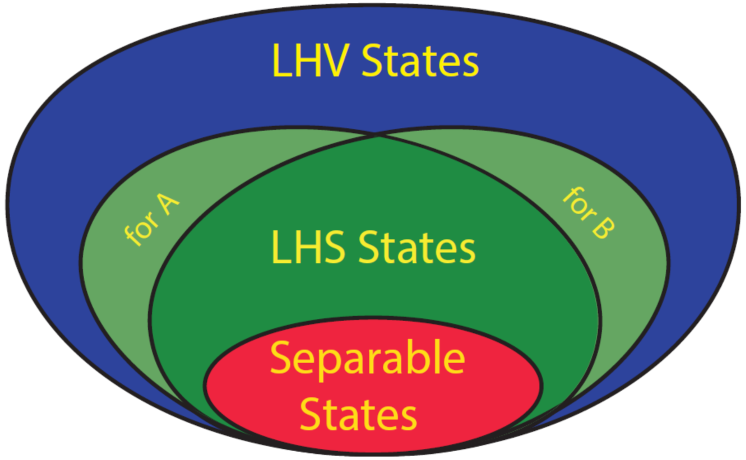
$$\rho(x_A, x_B) = \int d\lambda\ \rho(\lambda)\rho(x_A|\lambda)\rho(x_B|\lambda)$$

  - Ruled out by violating a Bell Inequality
- Local hidden states (LHS):
  - States determined by local hidden variables
- LHS model (for Bob):

$$\rho(x_A, x_B) = \int d\lambda\ \rho(\lambda)\rho(x_A|\lambda)Tr[\hat{\Pi}_x^B\ \hat{\rho}_\lambda^B]$$

  - **Ruled out by violating an EPR-steering inequality (proving you can do it)**





Position-Momentum EPR-steering inequalities

- Reid (1989)

$$\sigma(x_B|x_A) \cdot \sigma(k_B|k_A) \geq \frac{1}{2}$$

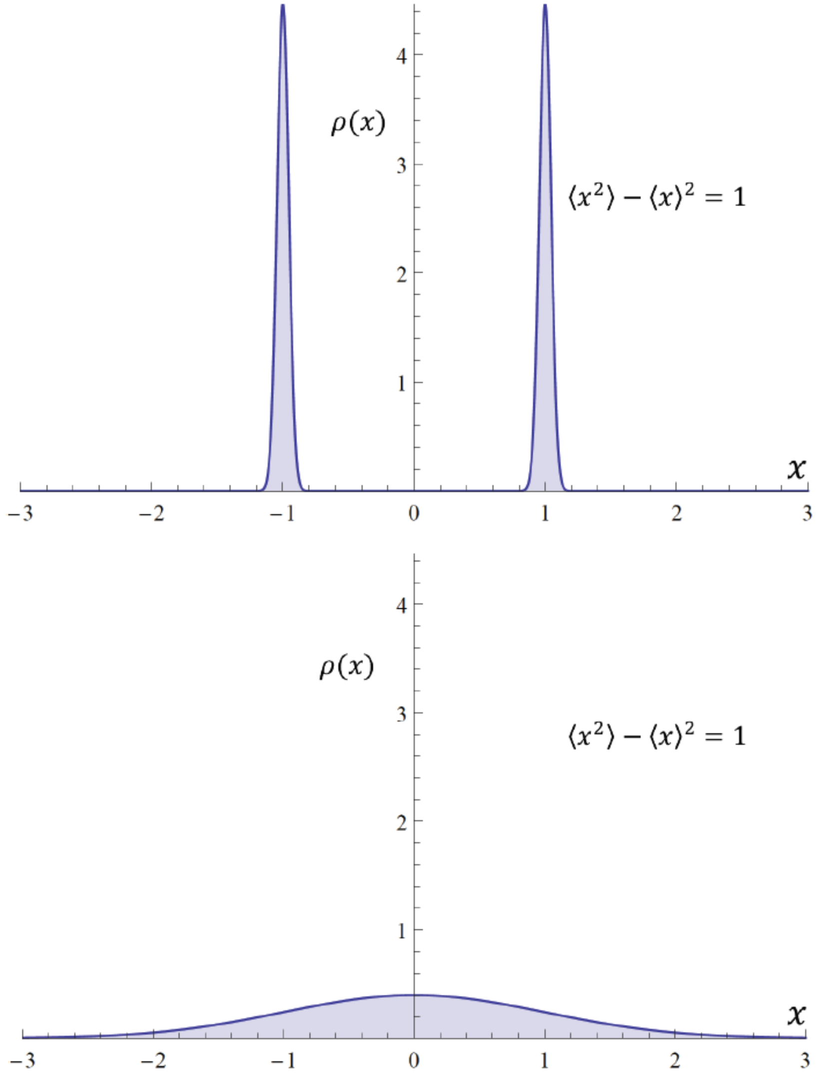
- Walborn et al (2011)

$$h(x_B|x_A) + h(k_B|k_A) \geq \log(\pi e)$$

Why use Walborn et al's steering inequality?

$$h(x_B|x_A) + h(k_B|k_A) \geq \log(\pi e)$$

- Entropy is a more sensitive measure of uncertainty than variances.
- The entropic uncertainty relation is tighter than the Heisenberg uncertainty relation (more states are closer to the threshold)
- Information-based uncertainty relations are easier to apply in quantum information
- You need the same information either way



How to experimentally demonstrate steering with Walborn at al's inequality?

- Problem:
  - You need to know $\rho(x_A, x_B)$ to find $h(x_B|x_A)$.

$$h(x) \equiv -\int dx\rho(x)\log(\rho(x))$$

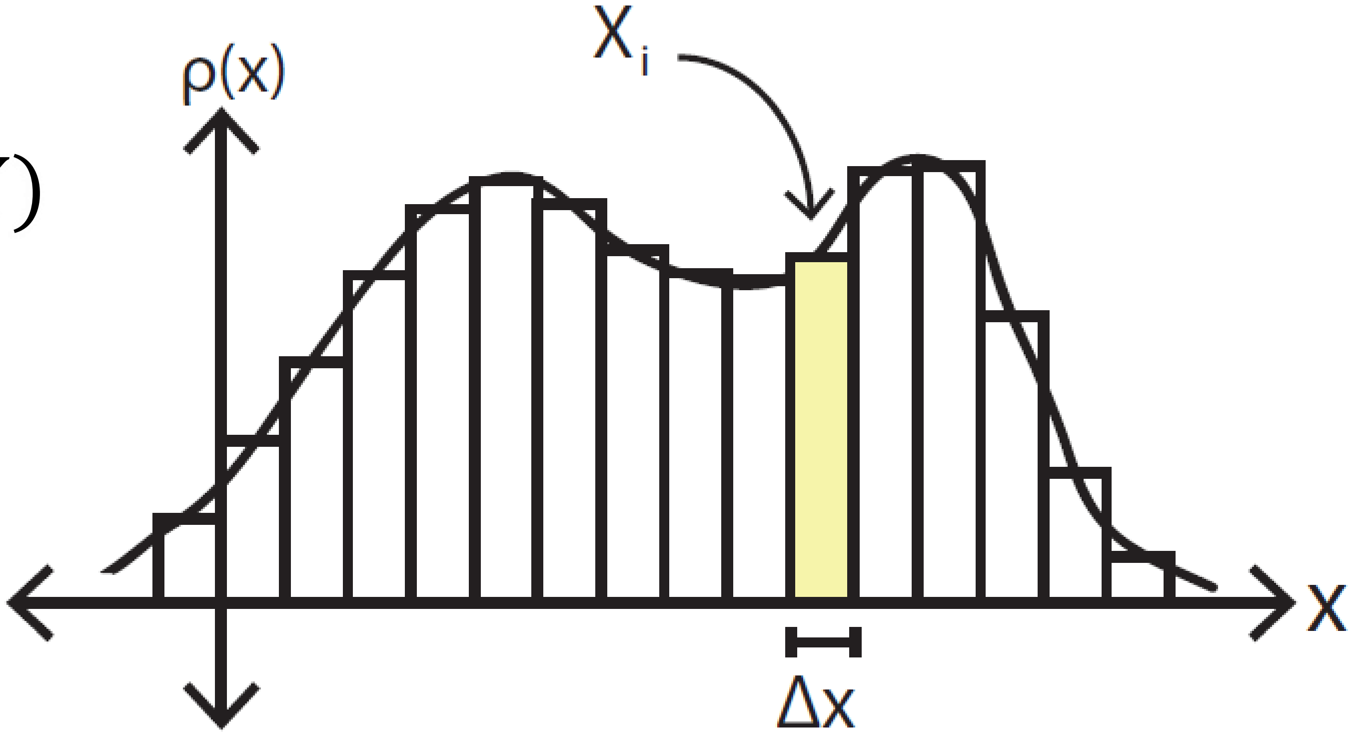
- Solutions:
  - (hard) Elaborate density function estimation algorithms
  - (Easy) Just use the discrete distribution!
    - How?
    - **Discrete approximation never decreases the entropy!**

Relating discrete to continuous entropy

$$H(X) \equiv -\sum_i P(X_i)\log(P(X_i))$$

$$\mathrm{h}(\boldsymbol{x}) = -\int dx\, \rho(\boldsymbol{x})\log(\rho(\boldsymbol{x}))$$

$$h(x) = \sum_i P(X_i)\, h_i(x) + H(X)$$



$$h_i(x) \leq \log(\Delta x)$$

$$h(x) \leq H(X) + \log(\Delta x)$$

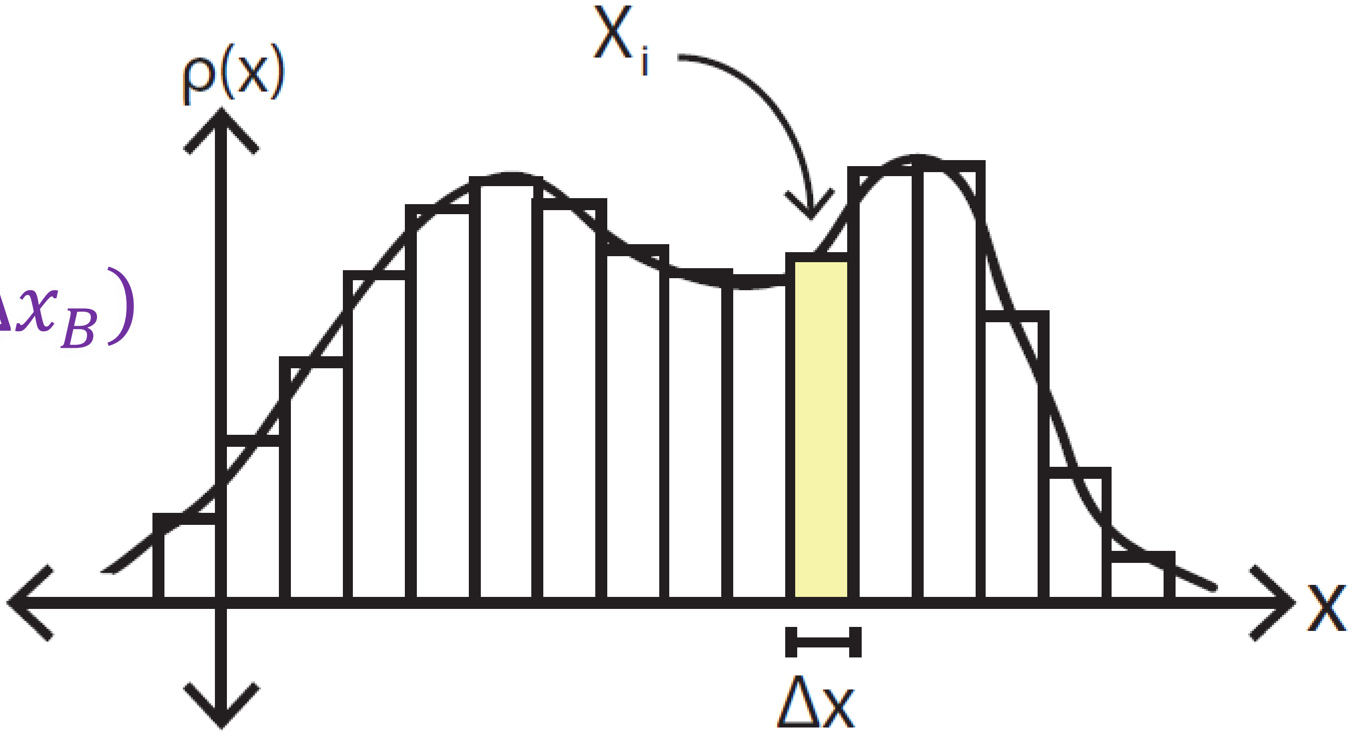
Relating discrete to continuous entropy

- The entropy of the discrete approximation of ρ is never smaller than the entropy of ρ, itself.

$$h(x) \leq H(X) + \log(\Delta x)$$

$$h(x_B|x_A) \leq H(X_B|X_A) + \log(\Delta x_B)$$

$$h(x_A{:}x_B) \geq H(X_A{:}X_B)$$



$h(x_B|x_A) = h(x_A, x_B) - h(x_A)$

A continuous variable steering inequality for discrete measurements!

- Since

$$H(X_B|X_A) \geq h(x_B|x_A) - \log(\Delta x_B)$$

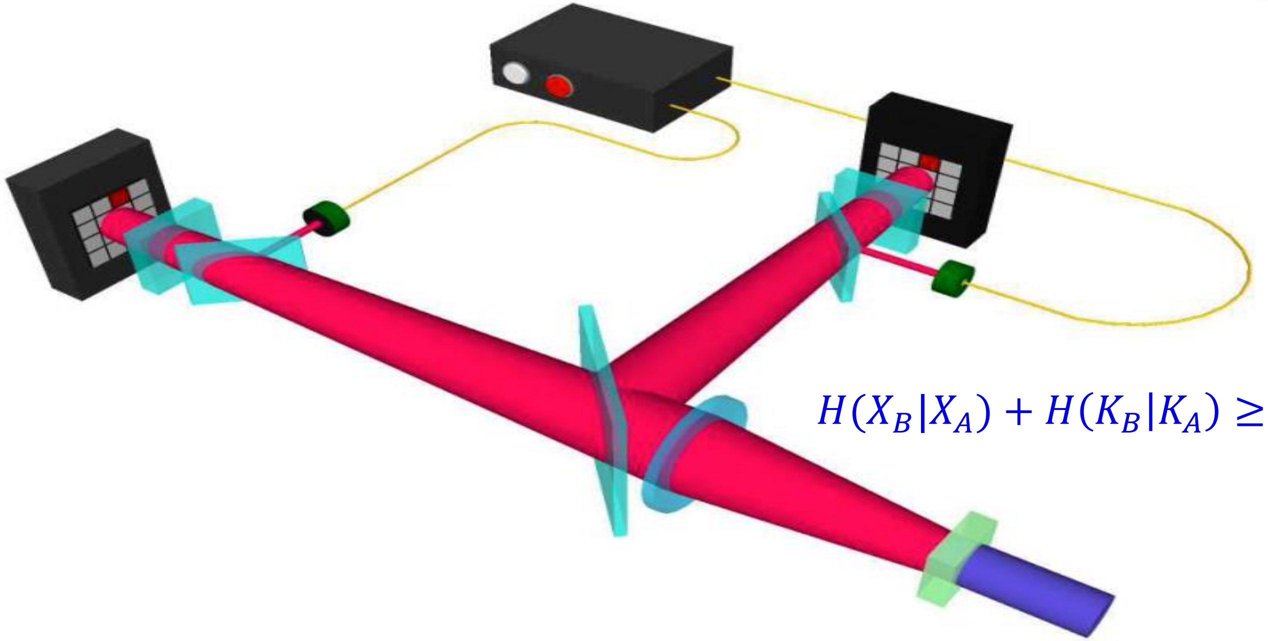
- We can use Walborn's inequality:

$$h(x_B|x_A) + h(k_B|k_A) \geq \log(\pi e)$$

- To get our first result:

$$H(X_B|X_A) + H(K_B|K_A) \geq \log\left(\frac{\pi e}{\Delta x_B \Delta k_B}\right)$$

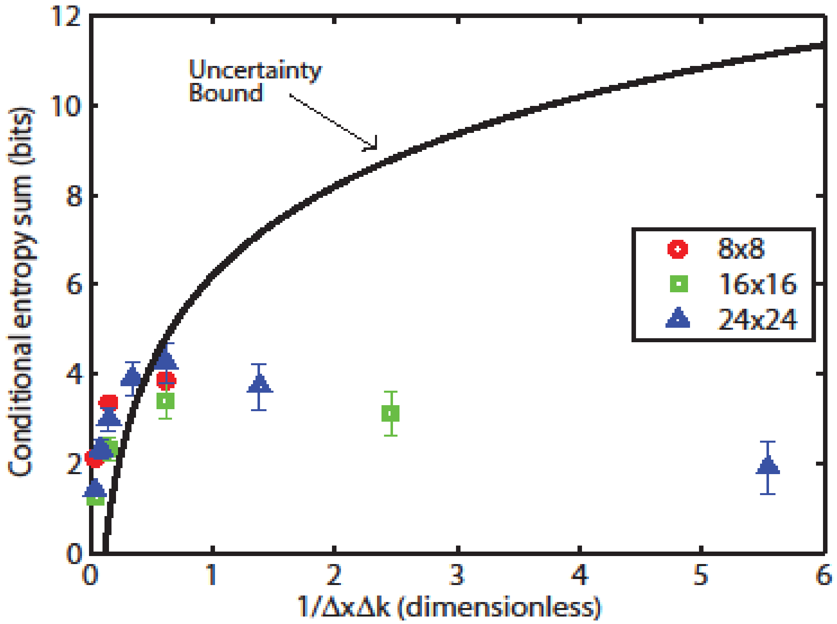
Experimental success 1



$$H(X_B|X_A) + H(K_B|K_A) \geq \log\left(\frac{\pi e}{\Delta x_B \Delta k_B}\right)$$

Results		
Resolution	Minimum $N\sigma$	Maximum $N\sigma$
8×8	3.65	5.9
16×16	8	11.2
24×24	12.3	16.4

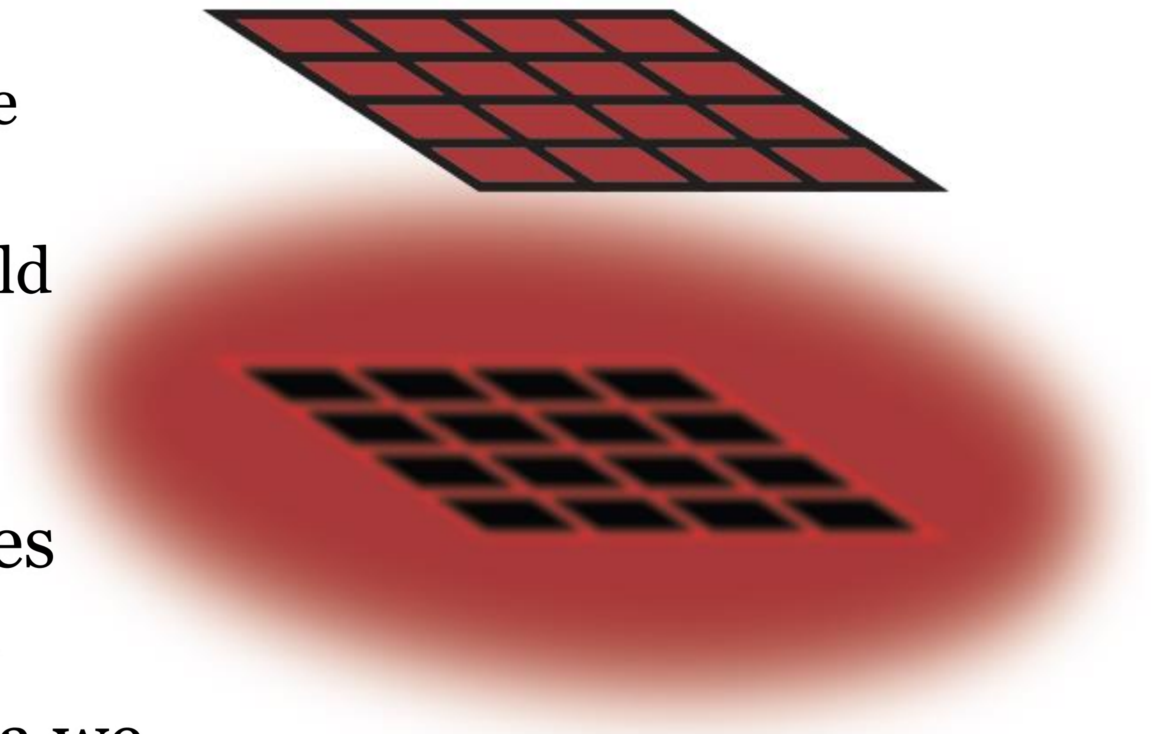
Experimental diagram and data from Phys. Rev. Lett. **108**,142603 (2012).



- Used down-converted 325 → 650 nm light from BBO nonlinear crystal.
- Measured joint coincident detections to get joint probability distributions in both image and Fourier planes of the crystal.
  - Recorded at different resolutions
- Successful violation at 8×8 through 24×24 resolutions

Lingering loopholes

- There's a lot of an infinite distribution experimenters don't have access to.
  - Even if we knew the exact probabilities we measure,
    - Any remaining probability outside could skew the entropy to infinity



- We cannot measure *all* the probabilities needed to get $H(X_B|X_A)$ and $H(K_B|K_A)$.
- But… we can bound these with the data we do have.

The Fano Inequality

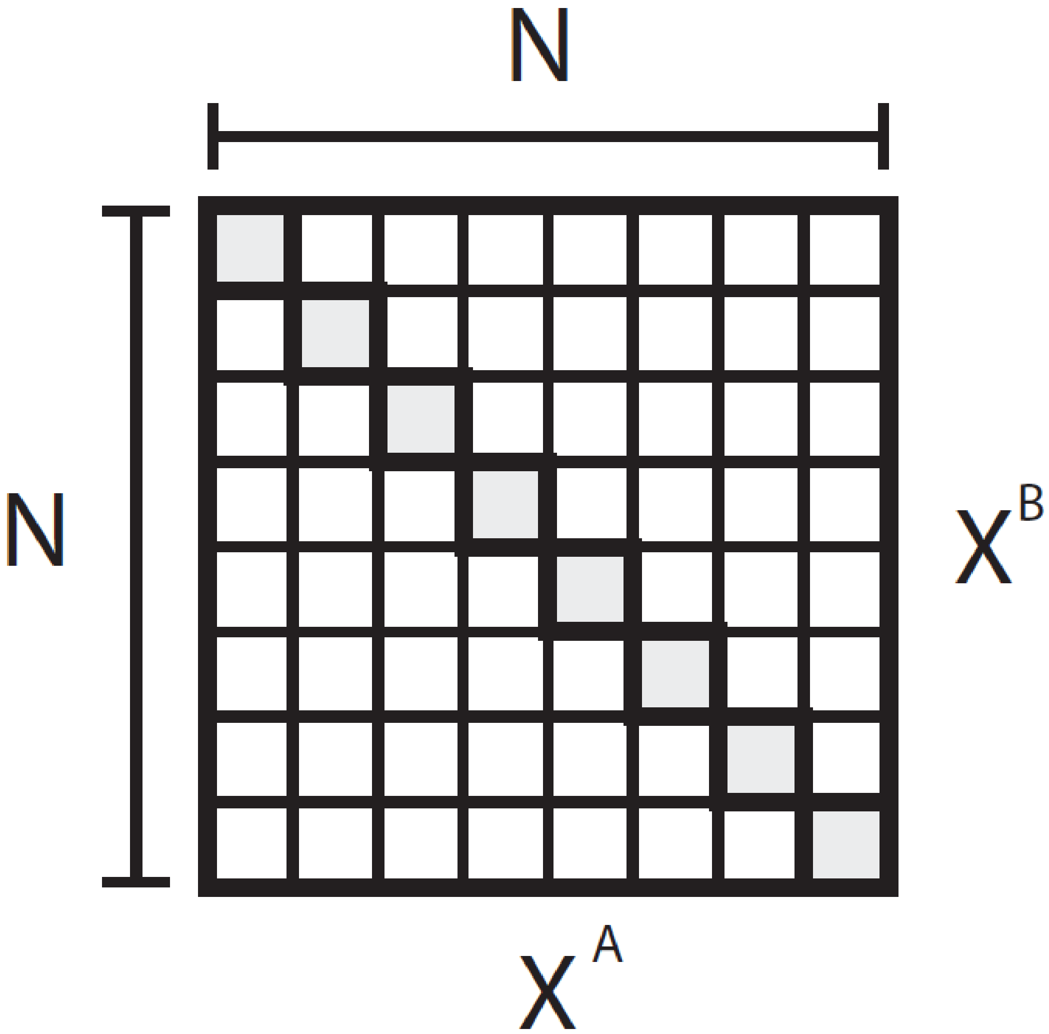
- An upper bound for $H(X_B|X_A)$ with the probability $\boldsymbol{\eta} \equiv P(X_A = X_B)$.

$$\boldsymbol{H(X_B|X_A) \leq h_2(\eta) + (1-\eta)\log(N-1)}$$

$$h_2(\eta) \equiv -\log_2(\eta) - (1-\eta)\log_2(1-\eta)$$

  - For continuous variables, Fano's inequality isn't helpful.

$$N \to \infty \implies H(X_B|X_A) \leq \infty$$



Making a continuous-variable Fano Inequality

$$\boldsymbol{H(X_B|X_A) \leq h_2(\eta) + (1-\eta)\log(N-1)}$$

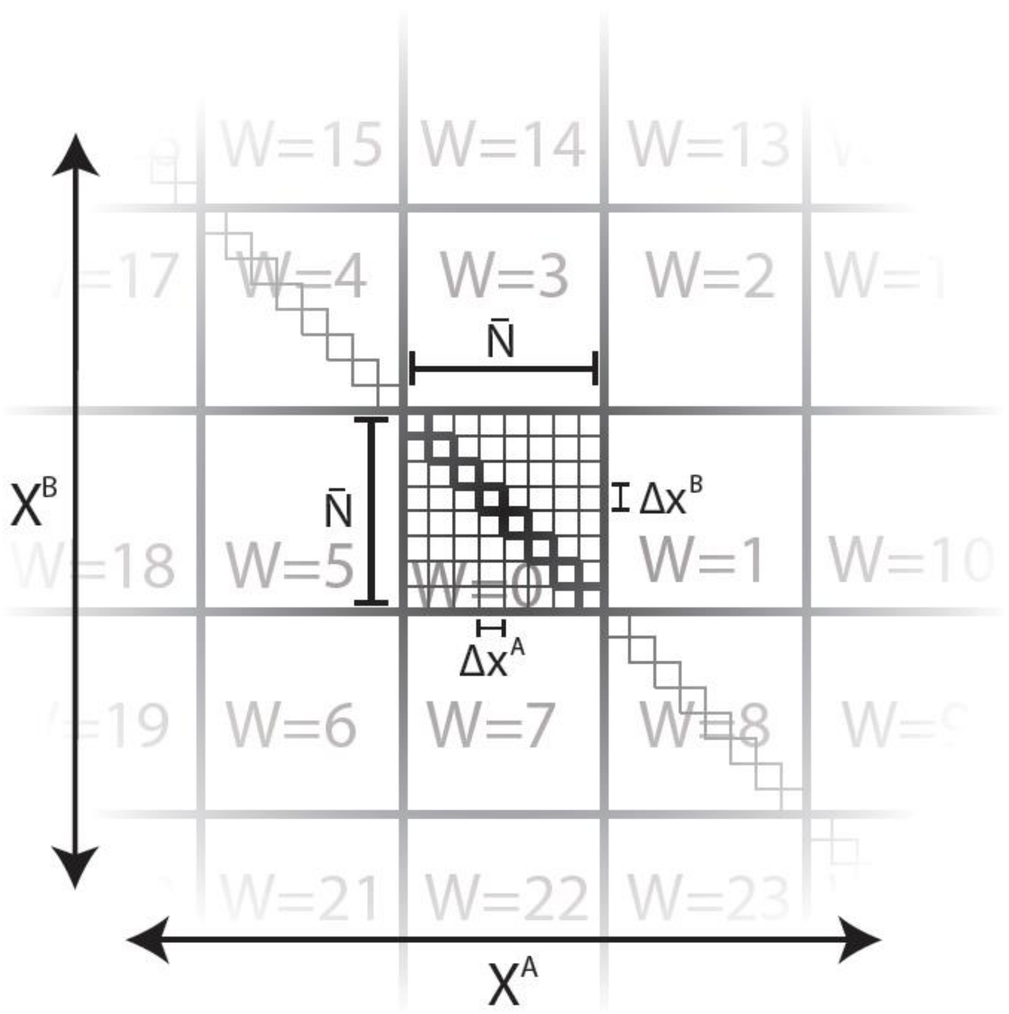
- Add a new window variable W
  - $W = \{0,1,2,3,\dots\}$, (infinite number of $\bar{N}$ pixel windows).

$$H(X_B|X_A) \leq h_2(\eta) + H(W) + (1-\eta)\log(\bar{N}-1)$$

- But.. $H(W) \leq \infty$
- However, if the mean $\langle W \rangle$ is finite…
  - We get a (useful) continuous-variable Fano inequality!

$$H(X_B|X_A) \leq h_2(\eta) + \frac{h_2(\mu)}{\mu} + (1-\eta)\log(\bar{N}-1)$$

Here, $\boldsymbol{\mu} = P(W=0)$, the domain probability



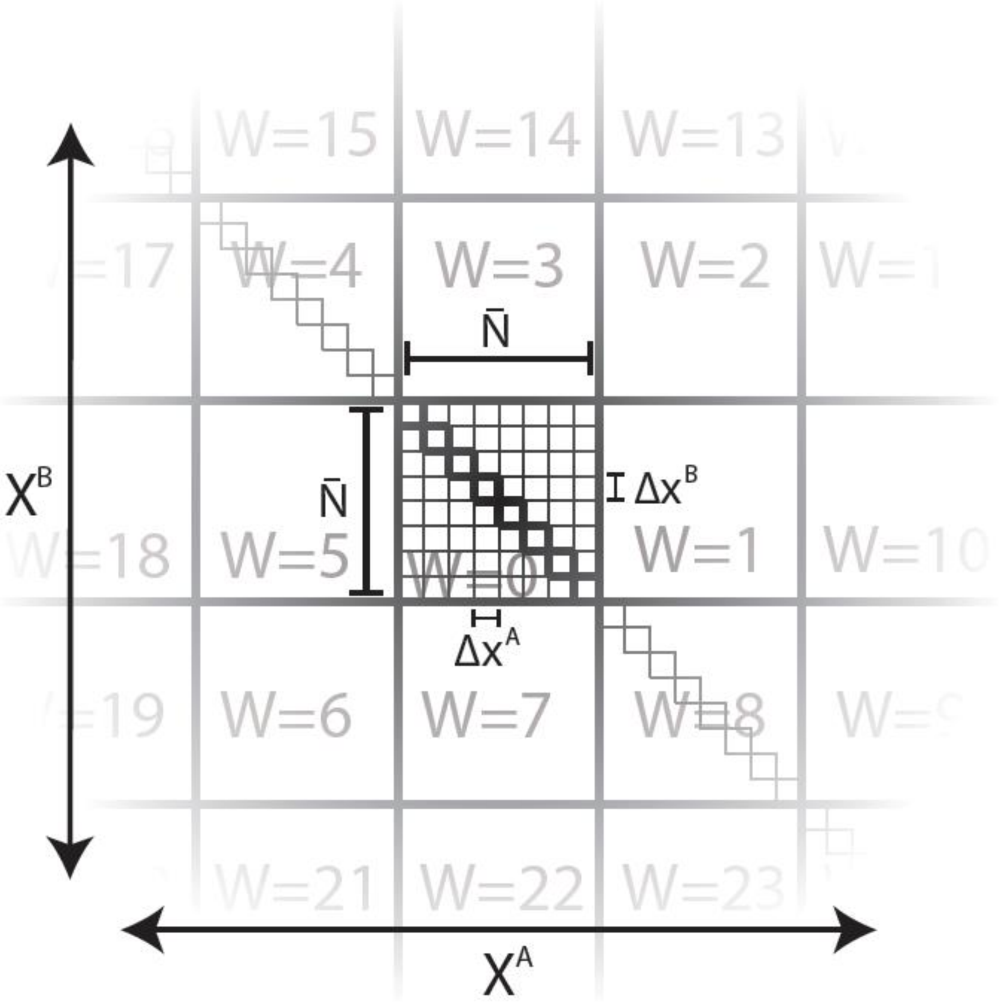
Steering with Fano's Inequality

$$H(X_B|X_A) \leq h_2(\eta) + \frac{h_2(\mu)}{\mu} + (1-\eta)\log(\bar{N}-1)$$

- To use this, we need to know $\boldsymbol{\mu}$ and $\boldsymbol{\eta}$
  - We can estimate $\boldsymbol{\mu}$ with fitting
  - Estimating $\boldsymbol{\eta}$ is more difficult
    - (you need probabilities outside your viewing window)
- So, we introduce the *measured* agreement probability $\bar{\boldsymbol{\eta}}$:

$$\bar{\eta}_x \equiv P(X_A = X_B | W = 0)$$

$$\bar{\eta}_x \leq \frac{\eta_x}{\mu_x}$$



Position-momentum EPR-steering with Fano steering bounds

$$\bar{\eta}_x \mu_x \leq \eta_x$$

- For $\bar{\boldsymbol{\eta}}_{\boldsymbol{x}} \boldsymbol{\mu}_{\boldsymbol{x}} < \frac{1}{2}$, (and similarly for momentum) we get the steering inequality

$$h_2(\bar{\eta}_x \mu_x) + h_2(\bar{\eta}_k \mu_k) + \frac{h_2(\mu_x)}{\mu_x} + \frac{h_2(\mu_k)}{\mu_k} + (2 - \bar{\eta}_x \mu_x - \bar{\eta}_k \mu_k) \log(\bar{N} - 1) \geq \log\left(\frac{\pi e}{\Delta \mathrm{x_B} \Delta k_B}\right)$$

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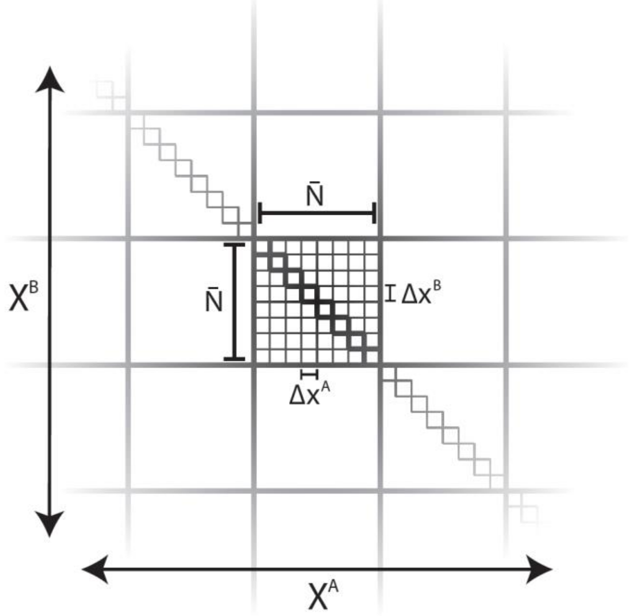
- What's it good for?
  - You need less information
    - (only that the agreement probabilities are big enough)
  - You can compensate for finite limitations with sufficiently good data
    - e.g., finite viewing area, dead space between pixels, etc
- Tradeoff
  - Only works well for highly correlated systems
  - But… down-converted photon pairs work well for this



Experimental success 2!

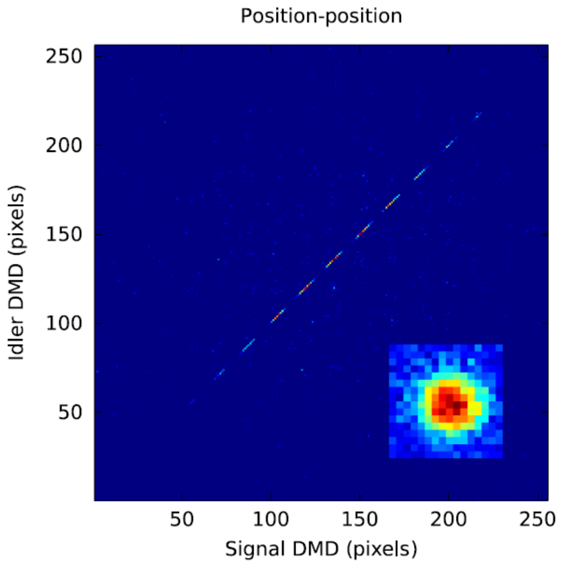
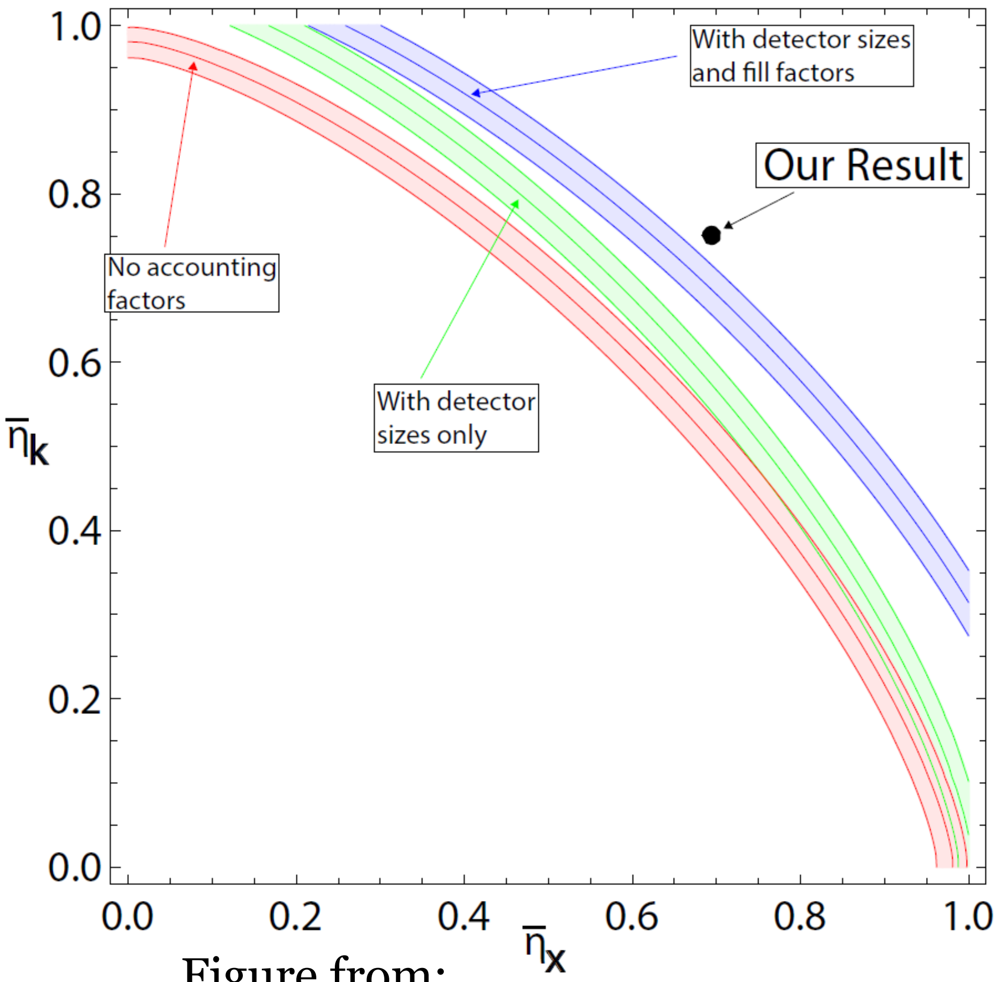


Figure from:
Phys. Rev. X, 3, 011013 (2013).

Table 2	
$\bar{\eta}_x$	69.4%
$\bar{\eta}_k$	75.1%
μ_x	99.7%
μ_k	95.2%
Position fill factor	92%
Momentum fill factor	100%

Successful violation of steering bound even accounting for these limitations!

- Used same source of 325nm -> 650 nm down-converted photon pairs.
- Measured joint position and momentum distributions using compressive sensing techniques (faster than raster scanning)
- Obtained detector fill factors from equipment manuals.
- Obtained domain probabilities from Gaussian fitting



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Part Three: Nonlocality

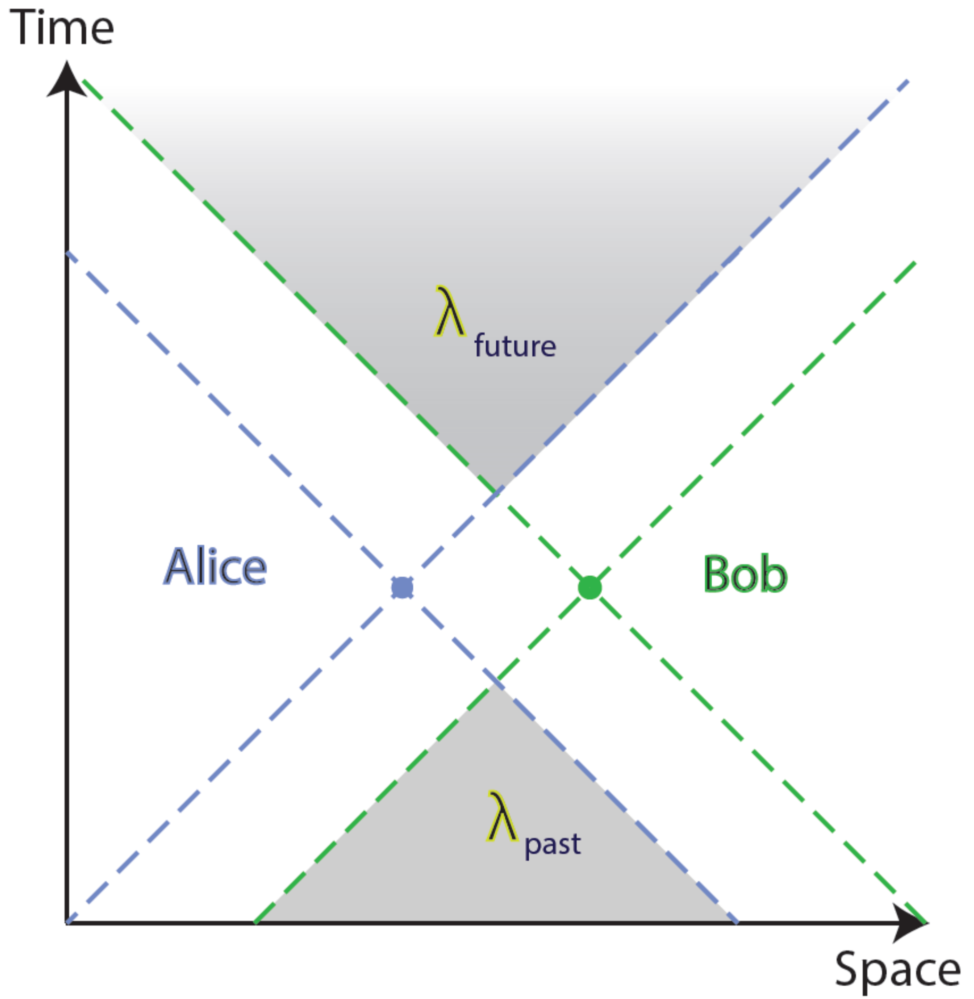
(Experimental hurdles and possibilities in position-momentum)

Non-locality In a nutshell

- Alice and Bob share a spacelike-separated pair of particles A and B.
- **Locality**: information travels no faster than light.
  - What Alice and Bob's measurements both affect is only in λ_{future}
  - What can affect both Alice and Bob's measurements is only in λ_{past}.
- If the Universe is local, measurement correlations can be "explained" locally:

$$\rho(x_A, x_B) = \int d\lambda\, \rho(\lambda)\, \rho(x_A|\lambda)\, \rho(x_B|\lambda)$$

(a model of Local Hidden Variables)



The CHSH-Bell inequality
(for position-momentum)

- If $\rho(x_A, x_B)$ factors this way:

$$\rho(x_A, x_B) = \int d\lambda\, \rho(\lambda)\, \rho(x_A|\lambda)\, \rho(x_B|\lambda)$$

- Then, the measurement statistics do as well:

$$\langle f(x_A) f(x_B)\rangle_{(\alpha,\beta)} = \int d\lambda\ \rho(\lambda)\ \langle f(x_A)\rangle_{(\alpha,\lambda)} \langle f(x_B)\rangle_{(\beta,\lambda)}$$

  - α is Alice's measurement setting
  - β is Bob's measurement setting

$$E(\alpha,\beta) \equiv \langle f(x_A) f(x_B)\rangle_{(\alpha,\beta)}$$

- With the right choice of function $f(x)$ (bounded between -1 and 1), we can get the CHSH inequality:

$$|E(\alpha,\beta) - E(\alpha,\beta')| \mp (E(\alpha',\beta) + E(\alpha',\beta')) \leq 2$$

Sign Binning:

$$f(x) = \begin{Bmatrix} 1 & \text{when} & x > 0 \\ -1 & \text{when} & x \leq 0 \end{Bmatrix}$$

Alternatives:

$$f(x) = \tanh(a\, x)$$
$$f(x) = \text{sgn}(x)$$

Remarks from Bell:

- Maximally entangled states don't have to violate this Bell inequality.
- The EPR state:

$$|\psi^{(EPR)}\rangle = \mathcal{N} \int dx_A dx_B \, \delta(x_A - x_B) |x_A, x_B\rangle$$

...is maximally entangled

- But its Wigner function:

$$W^{EPR}(x_A, x_B, k_A, k_B) = \mathcal{N} 2\pi \, \delta(x_A - x_B) \delta(k_A + k_B)$$

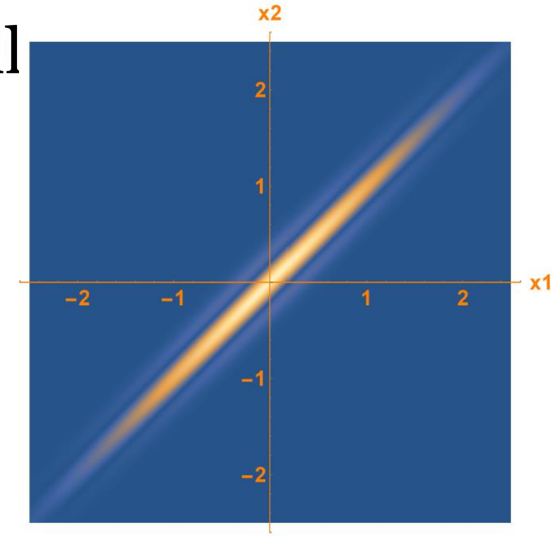
...is a valid probability distribution (and a local hidden variable model)

$$\rho(x_A, x_B) = \int d\lambda \left(\rho(\lambda) \, \rho(x_A|\lambda) \, \rho(x_B|\lambda)\right)$$

Remarks from Bell:

- However, there is an entangled state that does violate the CHSH-Bell inequality:
- We call it, Bell's wavefunction:

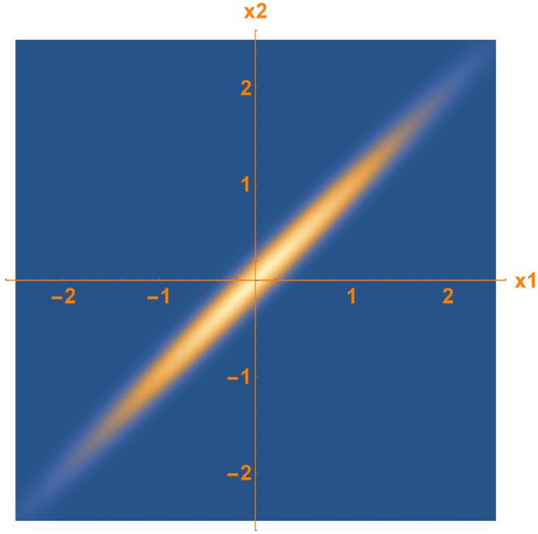
$$\psi^{BV}(x_A.x_B) = \mathcal{N}\left((x_A - x_B)^2 \ - 8\sigma_-^2\right)e^{-\left(\frac{(x_A+x_B)^2}{8\sigma_+^2}\right)}e^{-\left(\frac{(x_A-x_B)^2}{8\sigma_-^2}\right)}$$



- It's not unlike the Double Gaussian state:

$$\psi^{DG}(x_A.x_B) = \mathcal{N}'e^{-\left(\frac{(x_A+x_B)^2}{8\sigma_+^2}\right)}e^{-\left(\frac{(x_A-x_B)^2}{8\sigma_-^2}\right)}$$

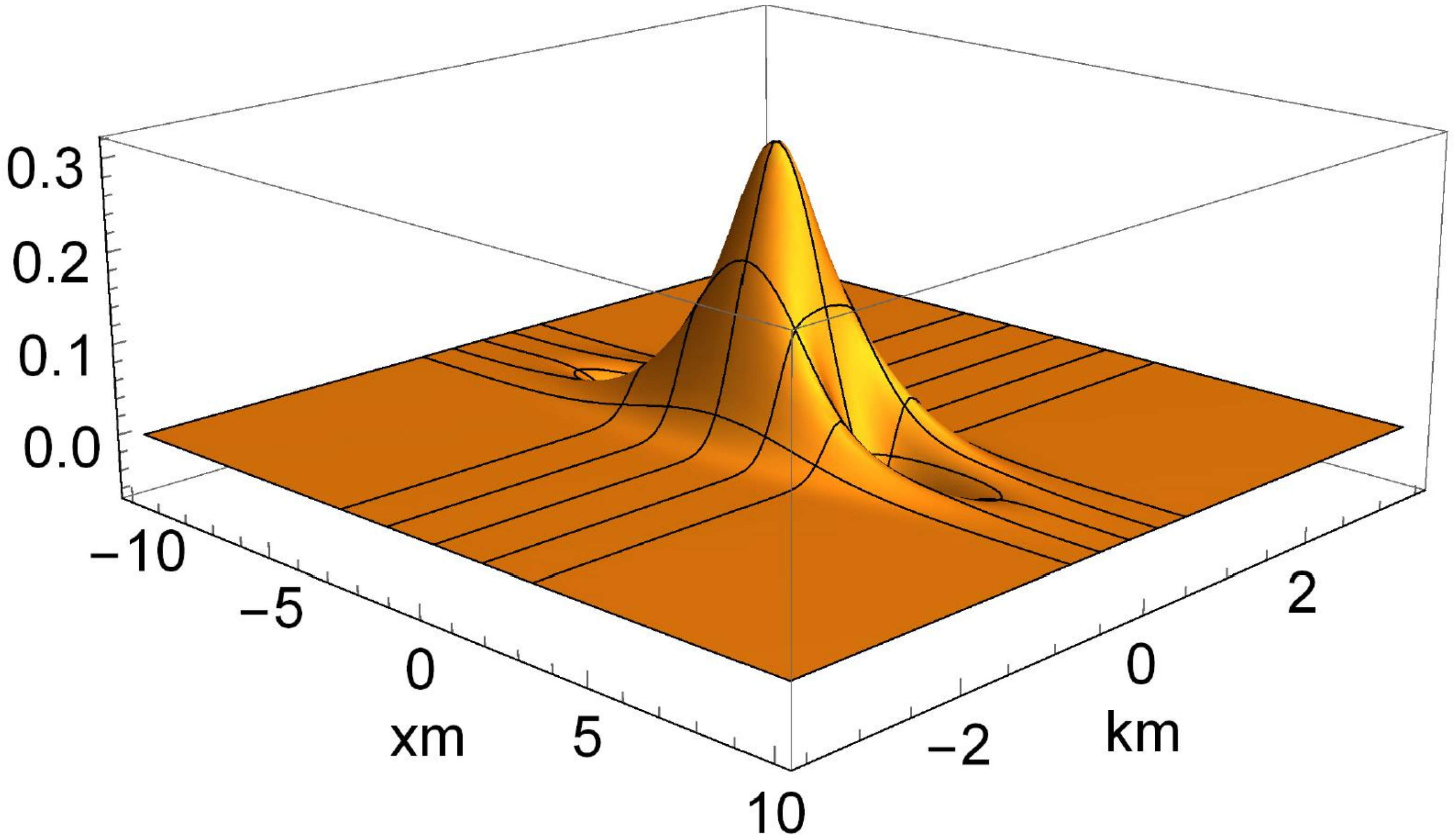
(gives EPR state as limiting case)



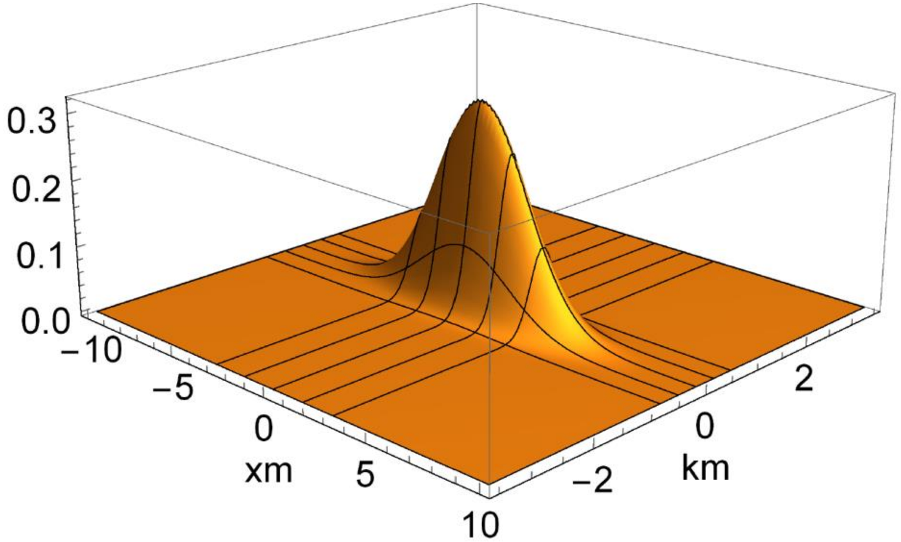
- Except…

Bell’s Wavefunction

- …it has a Wigner function with large regions of negativity.



- The corresponding Double-Gaussian state does not..



Bell's strategy:

- Let $\psi^{BV}(x_1, x_2)$ describe a pair of entangled (massive) particles (that no longer interact).
- Time-evolve the pair with the free particle Hamiltonian

$$\hat{H}_{\text{free}} = \frac{\hat{p}_1^2}{2\,m_1} + \frac{\hat{p}_2^2}{2\,m_2} = \hat{H}_1 + \hat{H}_2$$

- Measurement settings are the times each particle is measured.

$$(\alpha, \beta) \to (t_1, t_2)$$

In approximation ($\sigma_+ \to \infty$), the optimal correlation measurements violate the CHSH inequality.

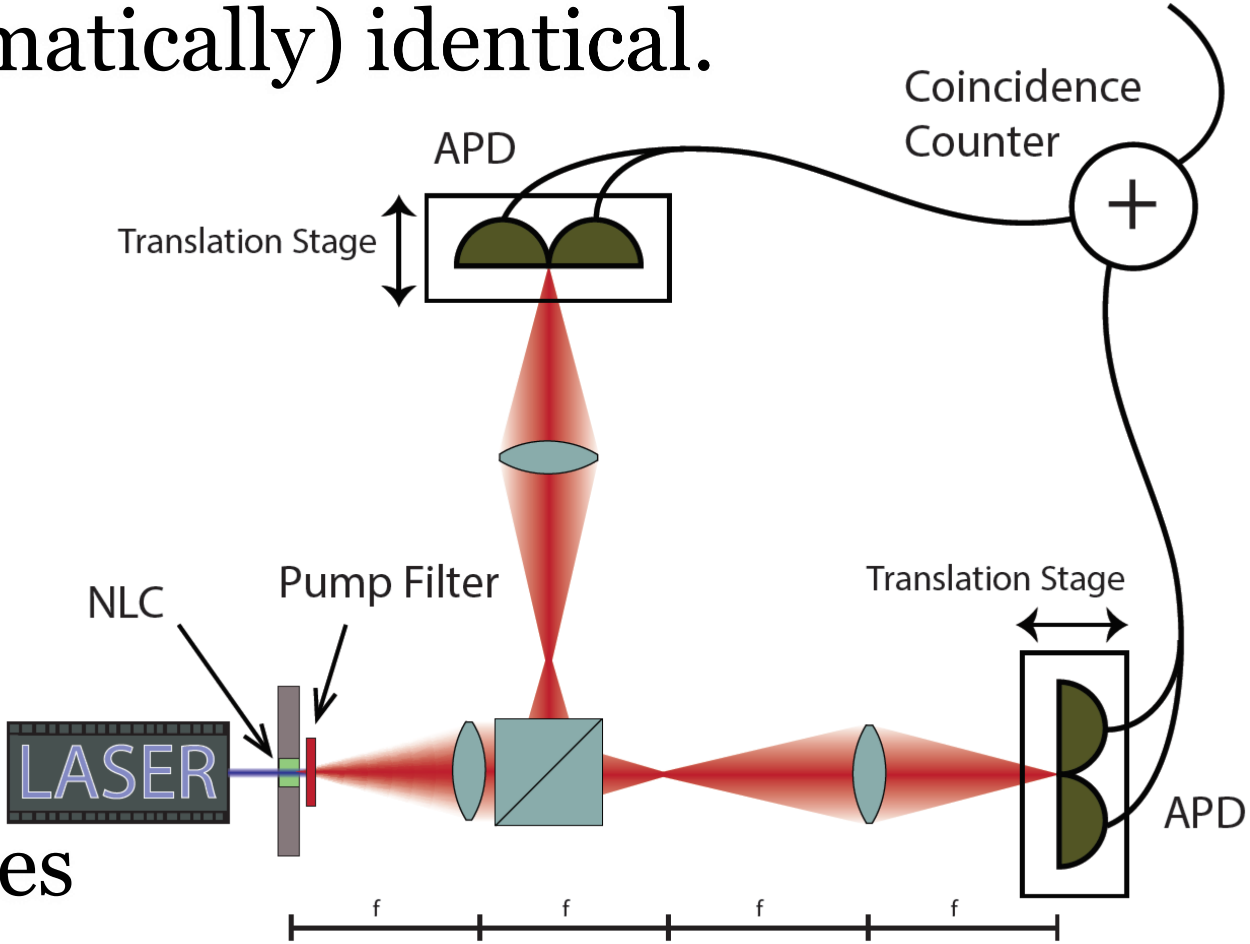
Bringing Bell's strategy to the Lab:

- The 2D free-particle Schrödinger equation, and paraxial Helmholtz equation are (mathematically) identical.

$$-\frac{\partial^2 A}{\partial x^2}-\frac{\partial^2 A}{\partial y^2}=ik_p\frac{\partial A}{\partial z} \quad \sim \quad -\frac{\partial^2 \Psi}{\partial x^2}-\frac{\partial^2 \Psi}{\partial y^2}=i\frac{2m}{\hbar}\frac{\partial \Psi}{\partial t}$$

Bringing Bell's strategy to the Lab:

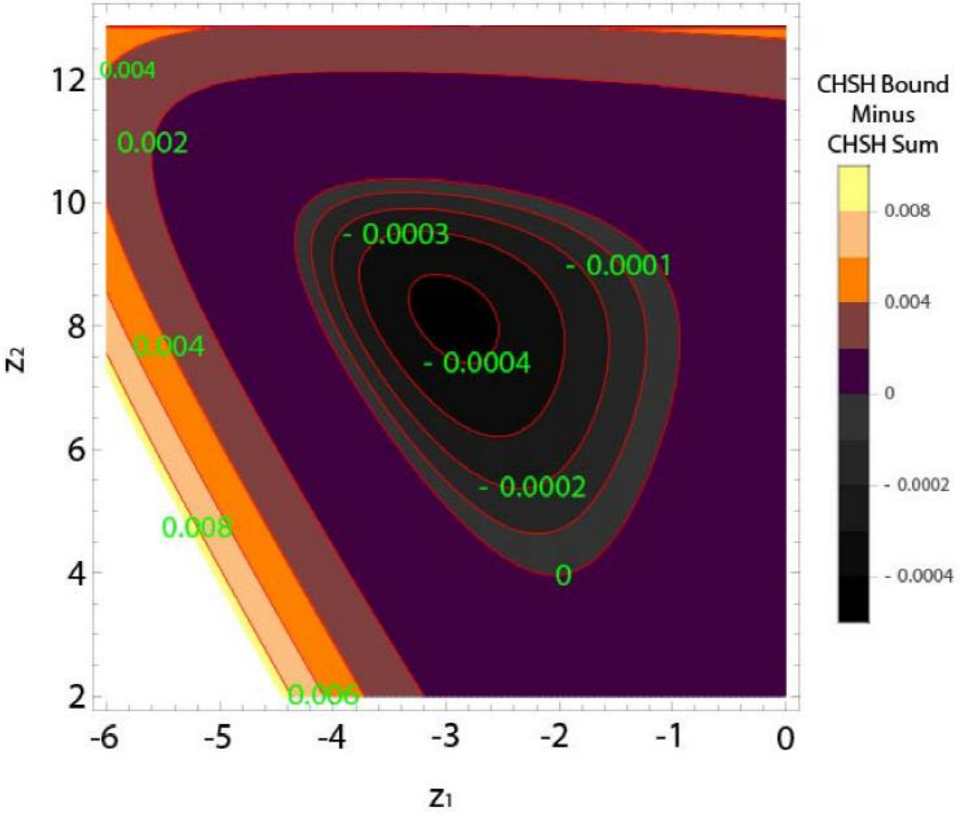
- The 2D free-particle Schrodinger equation, and paraxial Helmholtz equation are (mathematically) identical.
- If Bell's wavefunction could be mapped onto the biphoton state from SPDC…
  - i.e.,(Spontaneous Parametric Down-Conversion)
- We could measure the sign correlations at different signal/idler propagation distances



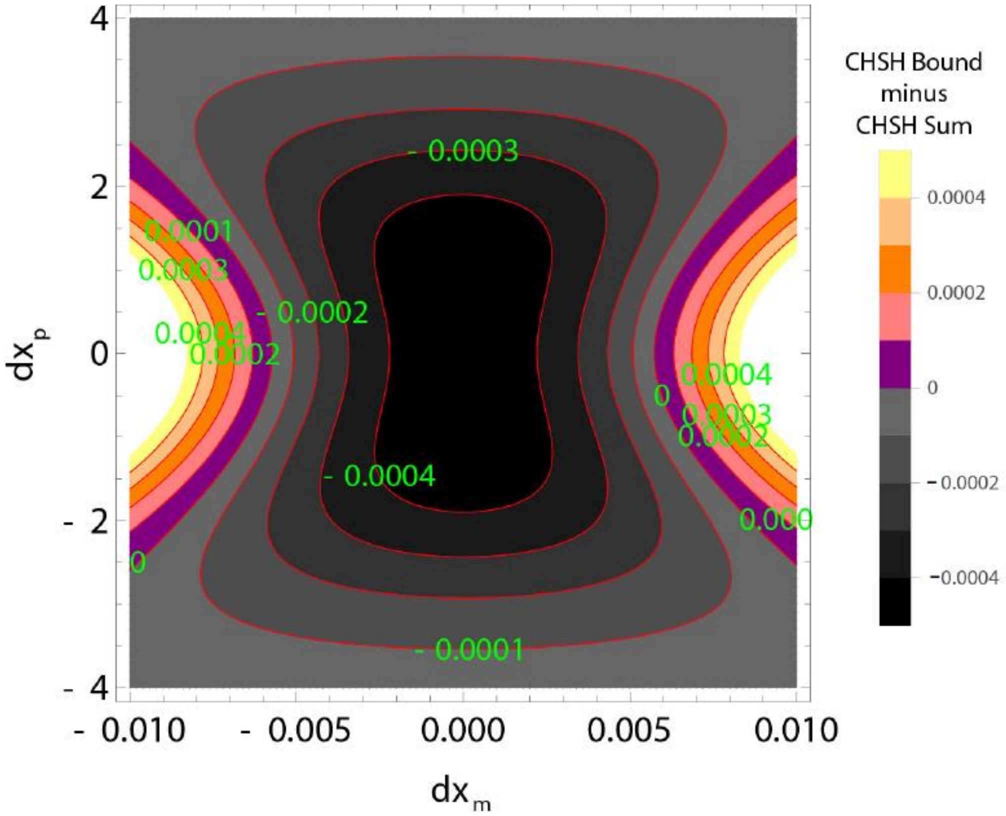
Theoretical results:

- If Bell's wavefunction *could* be mapped to the Biphoton state from SPDC…
- Then there would be (minute) violations!
  - Note: $(\alpha, \beta, \alpha', \beta') = (z_1, z_1, z_2, z_2)$

a)



b)



Challenges and hurdles:

- Minute violations need a LOT of coincidence counts to resolve…

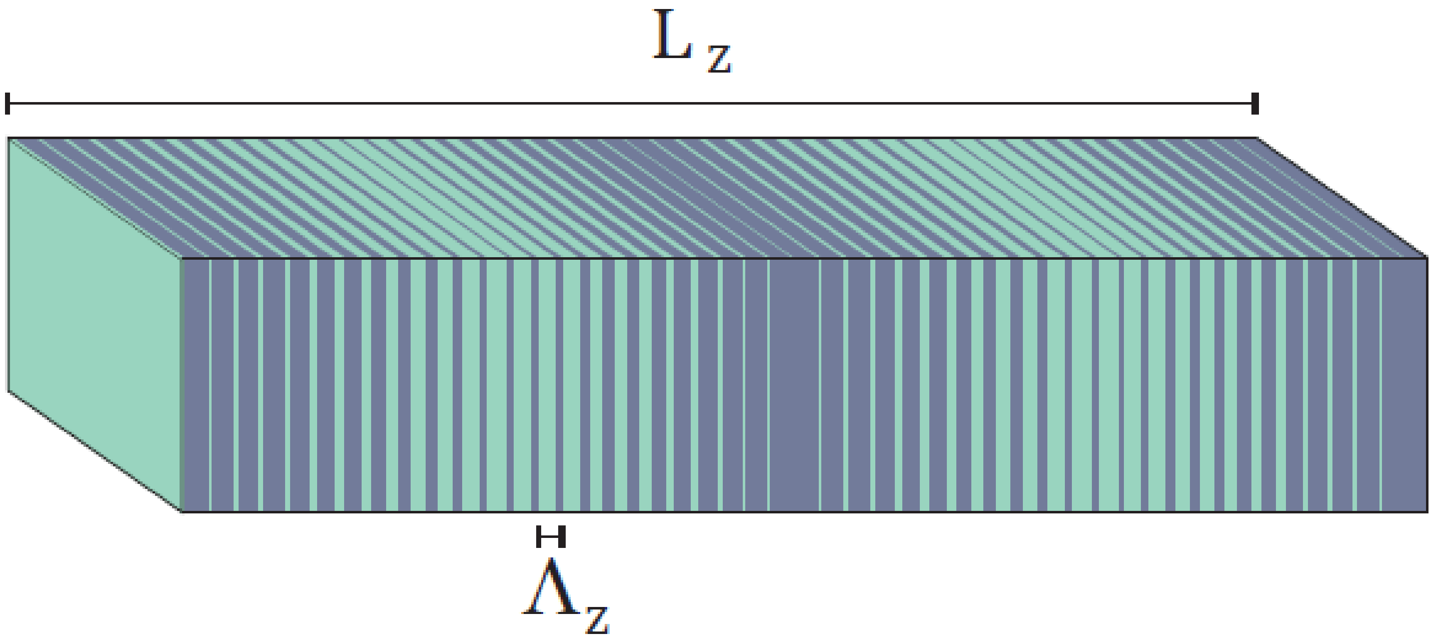
- Coincidence counts are Poisson Distributed…
  - You need 10^8 coincidence counts for the uncertainty in the count rate to be one part in 10^4
- With good count rates at 10^4/s…
- It would take several days to get enough data to violate the CHSH Bell inequality

Challenges and hurdles:

- Minute violations need a LOT of coincidence counts to resolve.

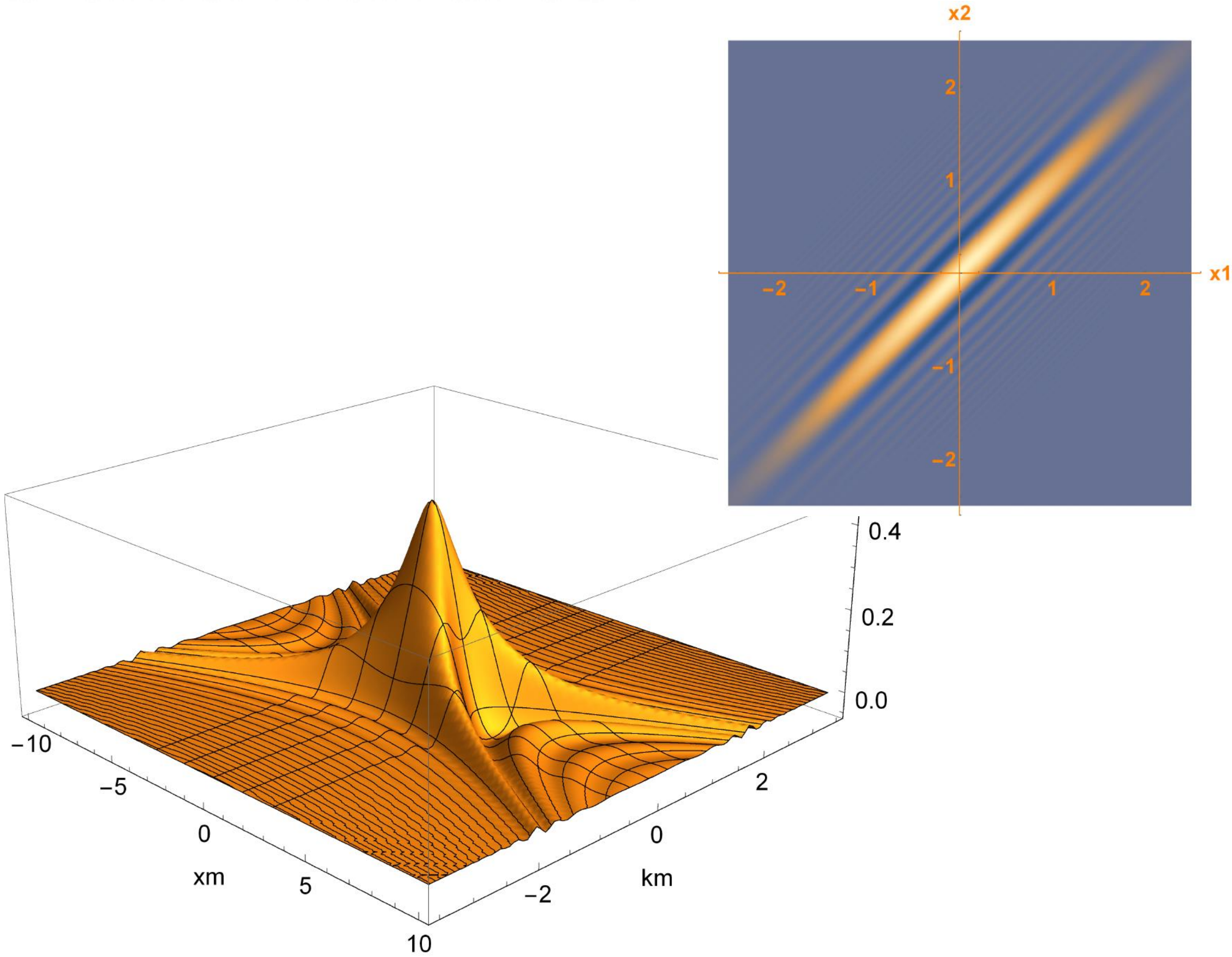
- The biphoton state still needs to resemble Bell's wavefunction
  - How to engineer the biphoton state to resemble Bell's wavefunction?

- If we could continuously vary $\chi^{(2)}$ in a nonlinear crystal..
  - We could engineer the biphoton wavefunction however we want.
- The next best thing…
  - Vary the duty cycle in a periodically poled crystal



Challenges and hurdles:

- Minute violations need a LOT of coincidence counts to resolve.
- The biphoton state still needs to resemble Bell's wavefunction
  - How to engineer the biphoton state to resemble Bell's wavefunction?
- Can the actual biphoton state violate the CHSH Bell Inequality?
  - Definitely maybe!



Conclusions and Future Questions

- **Classical** information theory remains a fertile ground for new research in **quantum** information

- How does the measurement–disturbance tradeoff work for other pairs (or groups) of observables?
- What are the ultimate applications of (free-space) position-momentum entanglement?
  - Position-momentum quantum cryptography?
- What can be done with multi-partite position-momentum entanglement that can't be done with photon pairs?

Thanks for listening!

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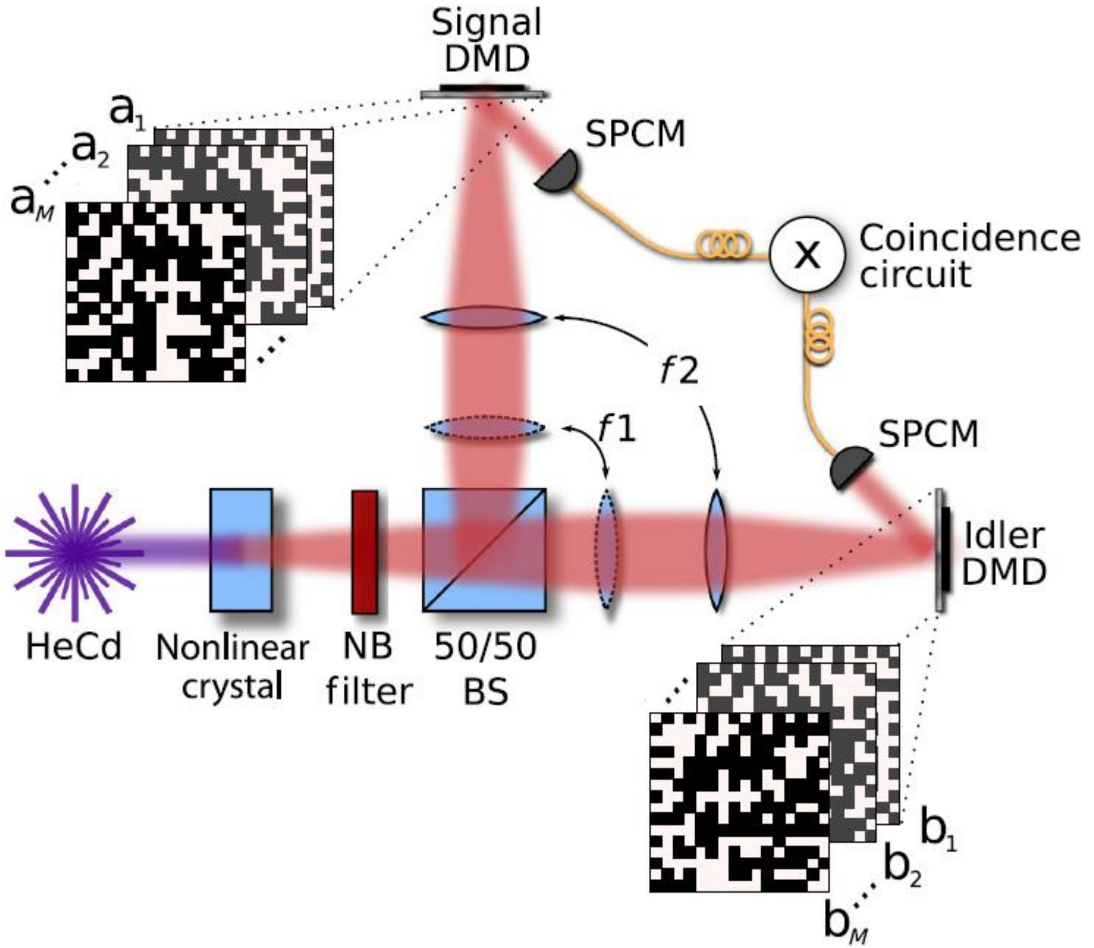
Contingency Slide: Experimental setup

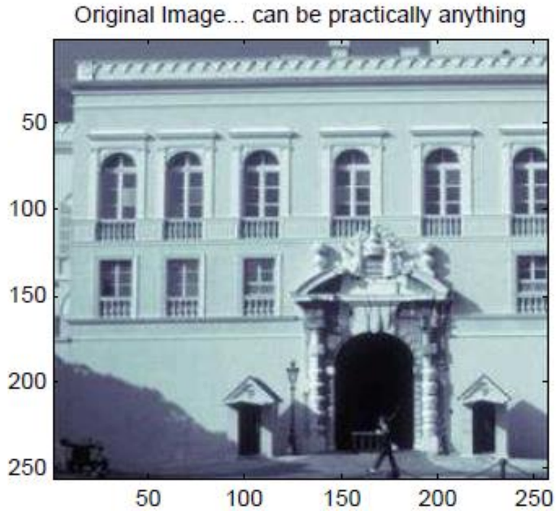
- Experimental setup for measuring position and momentum joint distributions with compressive sensing
- Use random patterns in signal and idler arms
- Flux through patterns gives correlation between pattern and signal
  - E.g. a pattern resembling the signal will let a lot of the signal through
- With a lot of these correlations, we can reconstruct the signal by brute force
- We can do much better using compressive sensing algorithms.

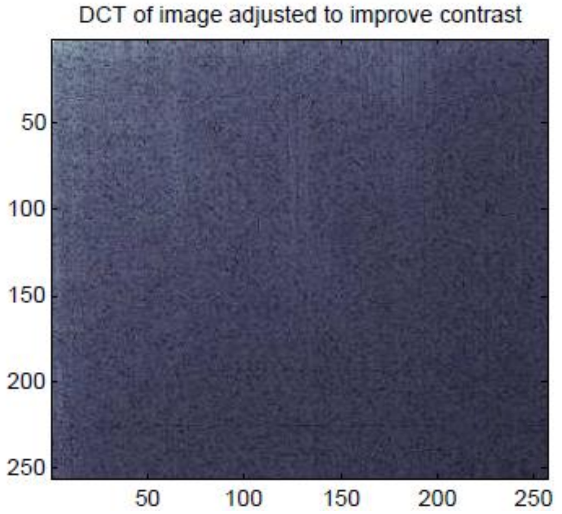


experimental diagram from PRX 3 011013 (2013)

Contingency Slide: Compressive Sensing in a Nutshell

- Many signals are compressible in some sparse basis
  - i.e, a basis where the signal has only a few significant components.
- We could sense in this sparse basis efficiently if we knew where the significant components were
- Using random measurements unbiased with the sparse basis, you can get lots of info about all significant components of the signal with each measurement.
- The original signal is (ideally) the unique solution to an optimization problem.



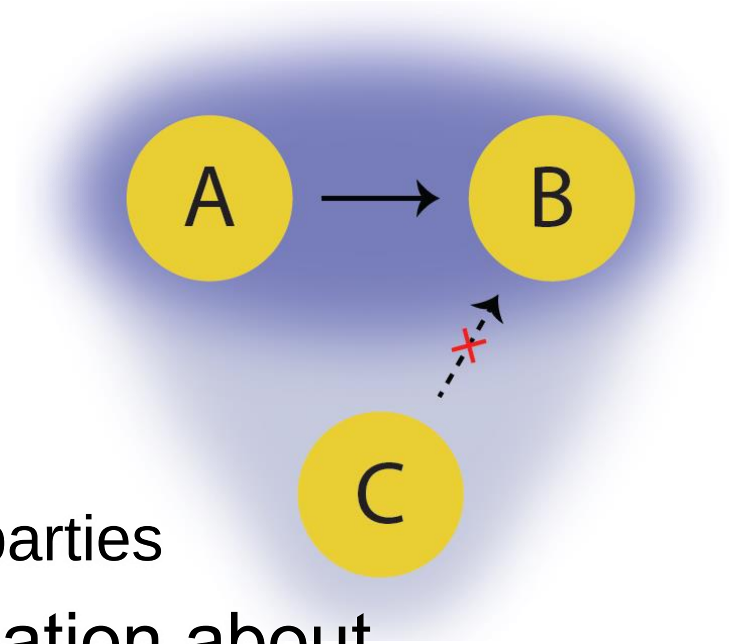


$$\vec{y} = \boldsymbol{A}\vec{x} + \vec{\phi}$$

$$\vec{x} = \min_{x_0}\left(\frac{1}{2}\|\vec{y} - \boldsymbol{A}\vec{x}_o\|_2^2 + \tau\|\vec{x}_o\|_1^1\right)$$

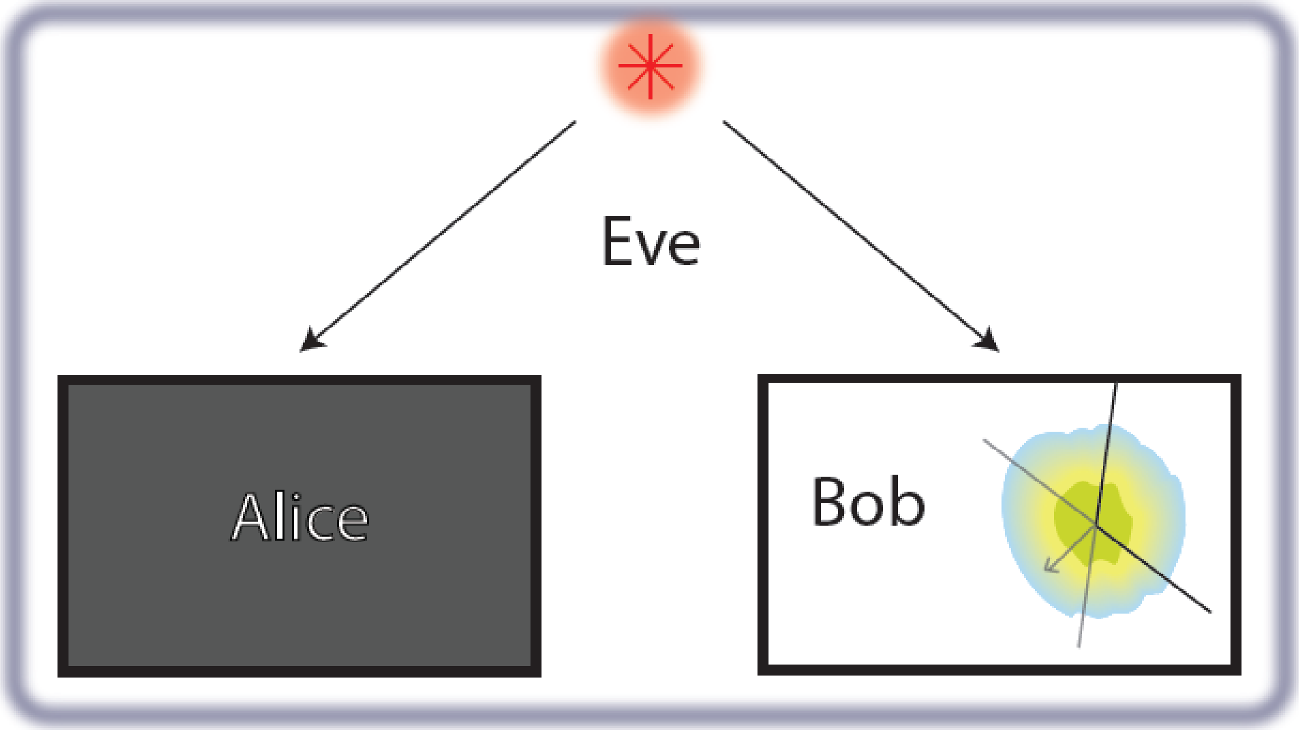
Okay, so what are these Demonstrations good for?

- Quantum metaphysics is fine, but…
- EPR-steering correlations are *Monogamous*!
  - i.e., the more correlated systems A and B are… …the less any third system can be correlated with either of them!
  - Also, no system can be steered both by two independent parties
- Limiting a third party's correlations, limits their information about (A and B)'s correlations.
  - Useful in *one-sided device-independent* quantum key distribution!



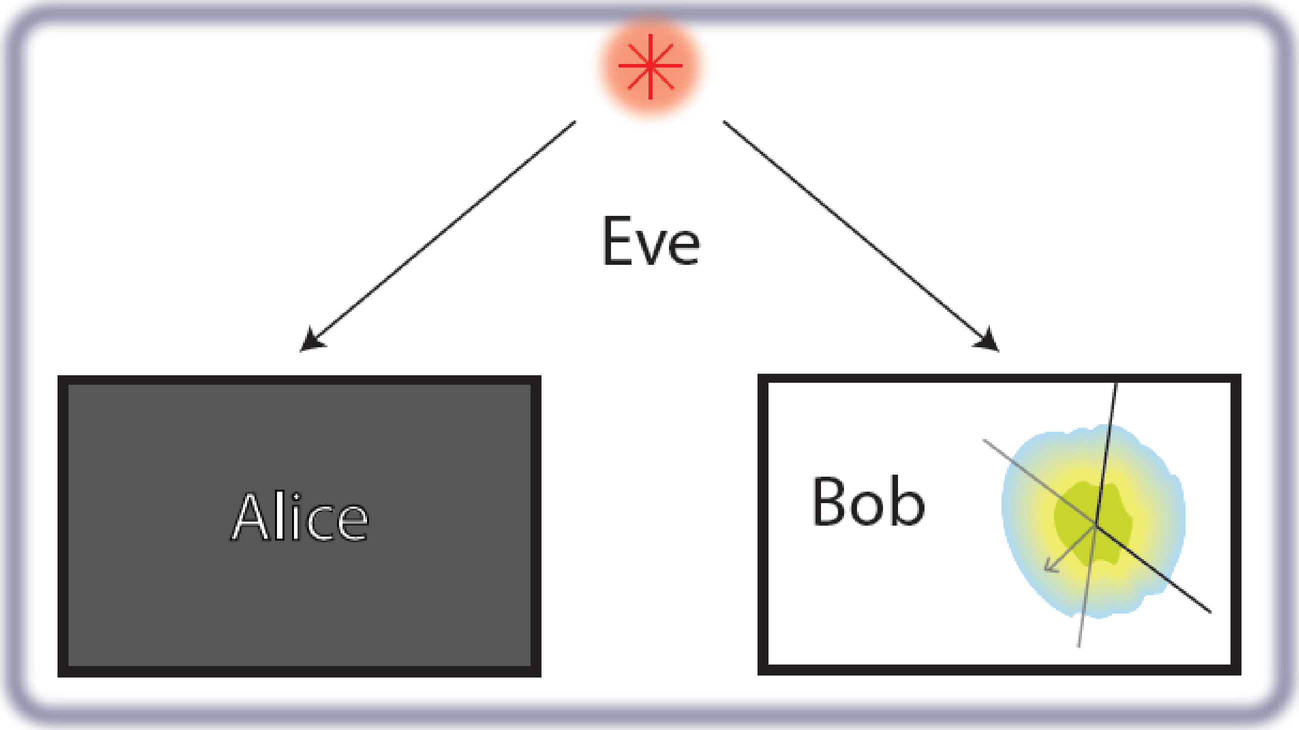
One-sided device-*independent* Quantum Key Distribution?

- One-sided:
  Alice's device is an untrusted black box with settings and outputs.
- Bob's device is trusted.
- If Alice doesn't trust her measurement device, but Bob trusts his…
- What about Eve?



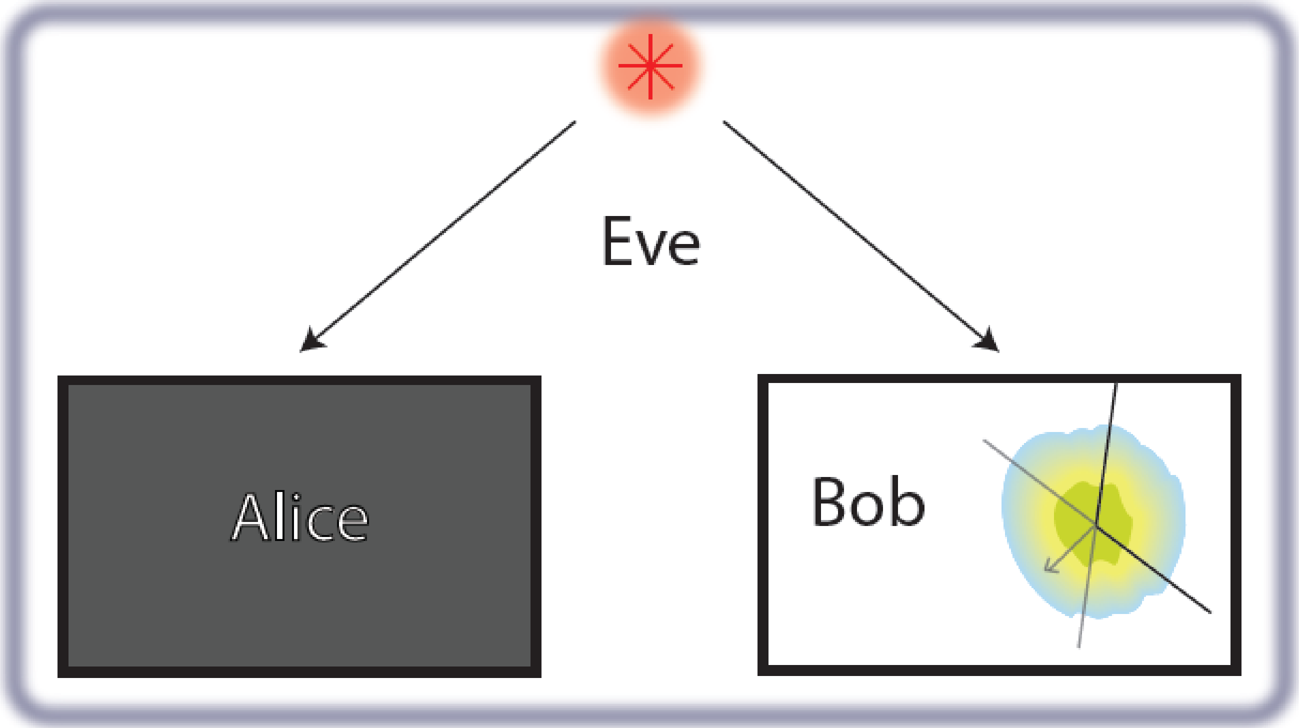
One-sided device-*independent* Quantum Key Distribution?

- What about Eve?
  - Eve sends particles to Alice and Bob
  - Bob measures in a random basis
  - Bob dictates Alice's measurement setting
  - Alice measures in chosen setting.
- What does Eve control?
  - Eve knows the state sent to Alice and Bob
  - Eve knows Alice's measurement settings
  - Eve can force Alice's measurement device to display any outcome
  - Eve ***does not know*** Alice's measurement outcome otherwise



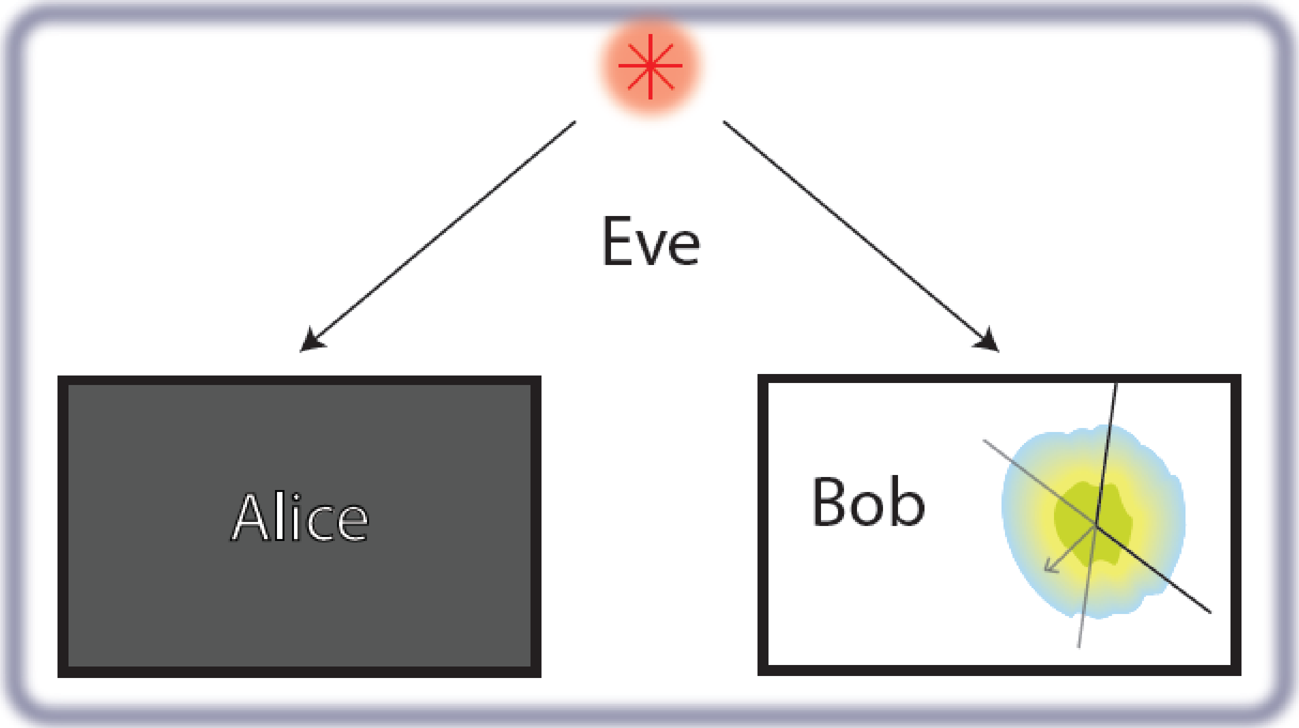
One-sided device-*independent* Quantum Key Distribution?

- What about Eve?
- What can Eve do?
- The best Eve could do:
  - send Bob a prepared ensemble of photons
  - puppeteer Alice’s device to display expected outcomes



One-sided device-*independent* Quantum Key Distribution?

- What about Eve?
- What can Eve do?
- The best Eve could do:
  - send Bob a prepared ensemble of photons
  - puppeteer Alice's device to display expected outcomes
- But then, Bob's receiving local hidden states
- No steering inequality can be violated:
  - Eve could've sent a photon well defined in position, but Alice is told to measure in momentum.



Extra bit: CV Fano inequality

- Assuming $\langle W \rangle$ is finite:

$$H(W) \leq \frac{h_2(\mu)}{\mu}$$

- $\boldsymbol{\mu} \equiv P(W = 0)$, the probability that the outcomes of X^A and X^B are within the experimental window.

  Why?

  - Maximum entropy decreases with increasing maximum probability
  - $\boldsymbol{\mu} \leq$ maximum probability