Anatomy of the Biphoton State
And Absolute Brightness of Spontaneous Parametric Down-Conversion

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The Biphoton State (from first-order TDPT)

$$|\psi\rangle_{SPDC} \cong \sum_{\substack{\vec{\mu}_p,\vec{\mu}_1,\vec{\mu}_2\\ k_p k_1 k_2}} \sqrt{\frac{2\hbar\omega_p\omega_1\omega_2 d_{eff}^2}{\epsilon_0 n_p^2 n_1^2 n_2^2 L_z^3}} \underbrace{\left(\int dxdy\, g^*_{\vec{\mu}_p}(x,y) g_{\vec{\mu}_1}(x,y) g_{\vec{\mu}_2}(x,y)\right)}_{\substack{\Phi_{xy}\\ \text{the mode overlap}}} \underbrace{\left(\int dz\, \bar{\chi}^{(2)}_{eff}(z)\, e^{-i\Delta k_z z}\right)}_{\substack{\Phi_z\\ \text{phase matching}}} \underbrace{\left(\int_0^T dt'\, e^{i\Delta\omega t'}\right)}_{\text{Conservation of Energy}} \underbrace{\alpha_{k_p\vec{\mu}_p}}_{\text{Pump Amplitude}} \hat{a}^\dagger_{\vec{\mu}_1,k_1}\hat{a}^\dagger_{\vec{\mu}_2,k_2}|0,0\rangle$$

First-order perturbation theory is accurate for CW pump powers in the mW to W scale, where absolute conversion probability is at best $\approx\ 10^{-8}$

Pump Photon Flux: $\sum_{\vec{\mu}_p,k_p}\left|\alpha_{k_p\vec{\mu}_p}\right|^2 = \frac{P}{\hbar\omega_p}T_{DC} = \frac{P}{\hbar\omega_p}\left(\frac{n_p L_z}{c}\right)$

Mode Overlap

$$\Phi_{xy} \equiv \int dxdy\, g^*_{\vec{\mu}_p}(x,y) g_{\vec{\mu}_1}(x,y) g_{\vec{\mu}_2}(x,y)$$

- $g_{\vec{\mu}_p}(x,y)$ is the transverse spatial mode of order $\vec{\mu}_p$
  - We are free to chose the basis of transverse spatial modes, with Laguerre-Gaussian and Hermite-Gaussian modes being the most popular.
- $g_{\vec{\mu}_p}(x,y)$ is normalized as a quantum wavefunction whose magnitude square sums to unity
- For a given subset of Laguerre-Gaussian modes, $\vec{\mu}_p$ corresponds to Orbital Angular Momentum (OAM) eigenvalues, and $g_{\vec{\mu}_p}(x,y)$ to OAM modes.
- For all Gaussian modes with standard deviations σ_p and σ_1 for pump and signal/idler respectively:

$$\Phi_{xy} = \sqrt{\frac{2}{\pi\sigma_p^2}}\left(\frac{\sigma_p^2}{\sigma_1^2 + 2\,\sigma_p^2}\right)$$

Energy Conservation

$$\left|\int_0^T dt'\, e^{i\Delta\omega t'}\right|^2 \xrightarrow[T\to large]{} 2\pi T\delta(\Delta\omega)$$

- Accurate for times longer than pump coherence time (uncertainty of $\Delta\omega$ smaller than pump bandwidth)
- Width of $\Delta\omega$ is much smaller than phase-matching bandwidth of SPDC light, giving rise to strong frequency correlations
- Non-energy-conserving transitions are possible, but probability is vanishingly small after integration over rapid oscillations of $\Delta\omega$.

Phase Matching

$$\Phi_z \equiv \int dz\ \bar{\chi}^{(2)}(z)\, e^{-i\Delta k_z z} \propto \mathcal{F}\left[\bar{\chi}^{(2)}(z)\right]$$

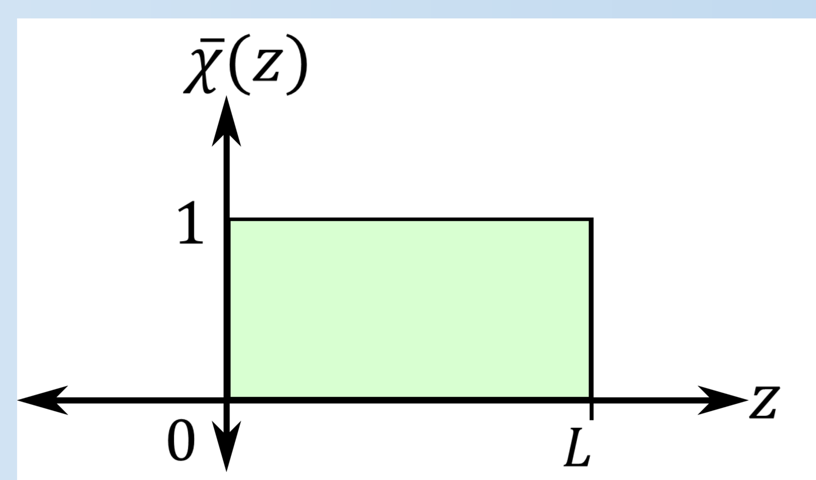
- Can be combined with Φ_{xy} to describe SPDC in variable width waveguides and tightly focused pump beams.
- Is a function contributing to likelihood of SPDC due to momentum conservation, whose enforcement mechanism is distinct from energy conservation.
  - Momentum: $\frac{p}{\hbar} = k = \frac{n(\omega)\omega}{c}$
  - Momentum only conserved at certain combinations of frequencies due to variation of $n(\omega)$ (i.e., dispersion).
  - Non-momentum-conserving transitions are possible, but improbable due to washing out when integrating over a rapidly oscillating phase (as in energy conservation)

Bulk Crystal

- When changing the optical properties and orientation of the crystal is enough to achieve phase matching (e.g., by using different polarizations/birefringence or anomalous dispersion):

$$|\Phi_z|^2 = Sinc^2\left(\frac{\Delta k_z\, L_z}{2}\right)$$

$$\Delta k_z = k_1 + k_2 - k_p$$

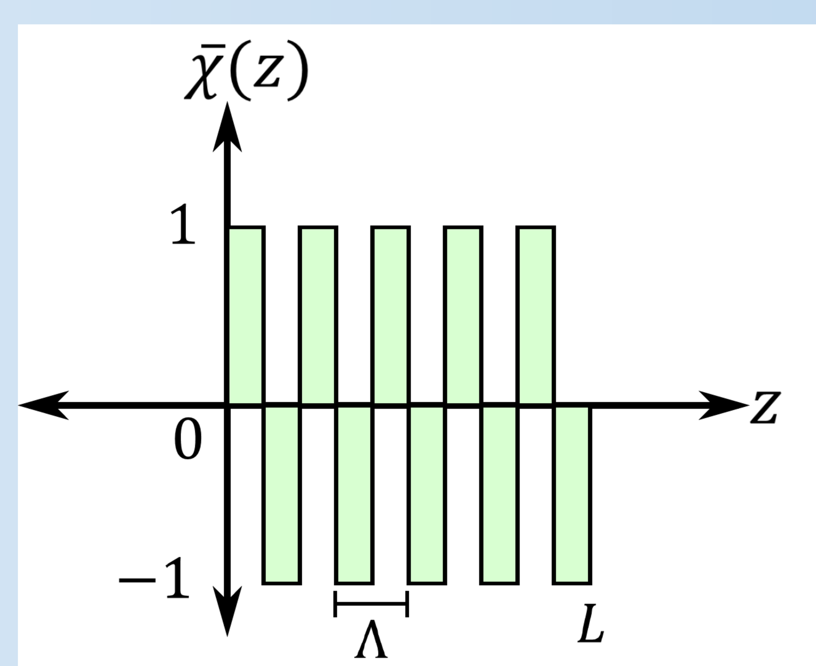


Periodic Poled Crystal

- When the dispersion is incompatible with SPDC at desired wavelengths, you can periodically flip the crystal axis orientation creating an effective nonlinear grating that imparts its own momentum to balance out the offset:

$$|\Phi_z|^2 \to \frac{4}{n^2\pi^2}|\Phi_z|^2$$

$$\Delta k_z \to k_1 + k_2 - k_p - \frac{2\pi n}{\Lambda}$$



Absolute Brightness

$$R_{\vec{\mu}_1,\vec{\mu}_2} = \frac{1}{\pi\epsilon_0 c^3}\left(\frac{n_{g1}n_{g2}}{n_p n_1^2 n_2^2}\right) P\, d_{eff}^2\, |\Phi_{xy}|^2 \int d\omega_1 d\omega_2\ \ \omega_1\omega_2|\Phi_z|^2\delta(\Delta\omega)$$

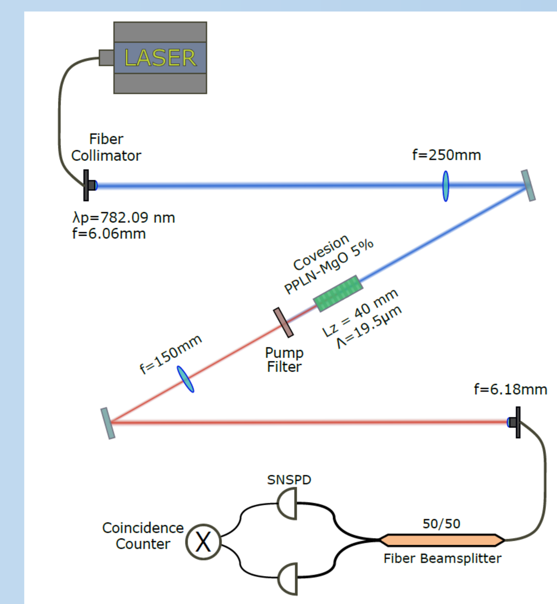
Where in the continuum limit:

$$\sum_{k_{1z}k_{2z}} \longrightarrow \left(\frac{L_z}{2\pi}\right)^2 \int dk_{1z}dk_{2z} \longrightarrow \left(\frac{L_z}{2\pi}\right)^2\left(\frac{n_{g1}n_{g2}}{c^2}\right)\int d\omega_1 d\omega_2$$

and

$$k(\omega_0 + \delta) = k(\omega_0) + \frac{n_{g0}}{c}\delta + \frac{1}{2}\kappa_0\,\delta^2 + \ldots$$

Type-0/I, SM



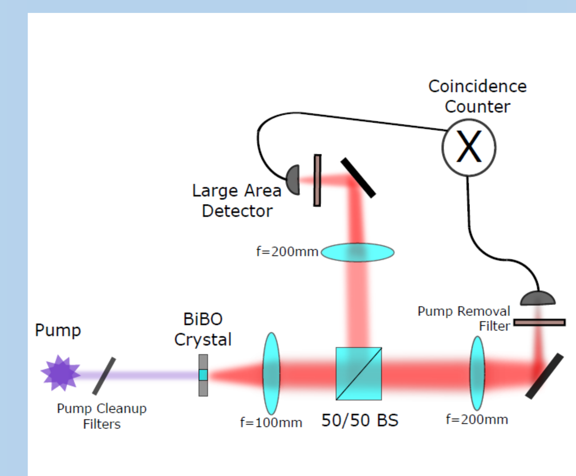
Experiment uses 782nm OBIS laser coupled into Single-mode fiber, followed by collimating and focusing optics to create an ideal Gaussian pump beam in the crystal center with focal spot diameter of about 250 microns. Similar optics are used on the exit side of the experiment to mode-match the down-converted light for good coupling into the exit single-mode fiber. Theoretical formula does not include first-order quasi-phase matching factor of $\frac{4}{\pi^2}$ needed to predict the brightness in periodically poled crystals.

$$R_{th} = \sqrt{\frac{2}{\pi^3}}\frac{2}{3\epsilon_0 c^3}\frac{n_{g1}n_{g2}}{n_p n_1^2 n_2^2}\frac{\left(d_{eff}\right)^2\omega_p^2}{\sqrt{\kappa}}\left|\frac{\sigma_p^2}{\sigma_1^2 + 2\,\sigma_p^2}\right|^2\frac{P}{\sigma_p^2}L_z^{3/2}$$

$$R_{th} \approx (94.86 \pm 10.89)\times 10^6\ s^{-1}\,mW^{-1}$$

$$R_{exp} \approx (95.63 \pm 2.71)\times 10^6\ s^{-1}\,mW^{-1}$$

Type-0/I, MM



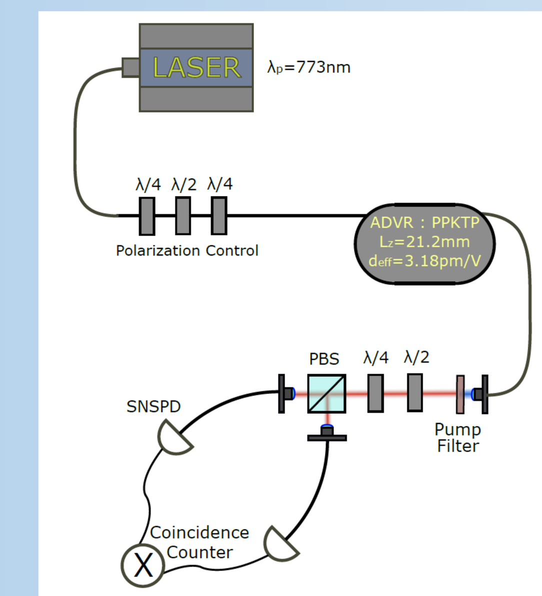
Experiment uses 405nm OBIS laser incident on BiBO nonlinear crystals. The down-converted light is first collimated, then split by a 50/50 beamsplitter, and collected using focusing optics and Large area Single-Photon detectors to capture all of the SPDC light instead of a single spatial mode. The fitting constant ϕ was estimated theoretically to be about 0.335, and comes from fitting the peaks of the full predicted biphoton wavefunction to the wavefunction obtained for degenerate SPDC. A more accurate rate may be calculated numerically, but the qualitative dependence would be lost.

$$R_{th} = \frac{32\sqrt{2\pi^3}}{27\epsilon_0 c}\frac{n_{g1}n_{g2}}{n_1^2 n_2^2}\frac{\left(d_{eff}\right)^2}{\lambda_p^3\sqrt{\kappa}}\frac{P}{\phi}L_z^{1/2}$$

$$R_{th} \approx (53.87 \pm 10.87)\times 10^6\ s^{-1}\,mW^{-1}$$

$$R_{exp} \approx (64.68 \pm 1.69)\times 10^6\ s^{-1}\,mW^{-1}$$

Type-II, SM



Experiment uses 773nm NewFocus laser coupled into PPKTP waveguide manufactured by diffusing dopant into the periodically poled medium to create a waveguide channel. The asymmetric distribution of dopant yields an oblong non-Gaussian spatial mode, which we approximated as Gaussian. Using the exact eigenmodes of the waveguide would greatly increase the accuracy of the theoretical description, and its corresponding agreement to the experiment.

$$R_{th} = \frac{1}{\pi\epsilon_0 c^2}\frac{n_{g1}n_{g2}}{n_p n_1^2 n_2^2}\frac{\left(d_{eff}\right)^2\omega_p^2}{\Delta n_g}\left|\frac{\sigma_p^2}{\sigma_1^2 + 2\,\sigma_p^2}\right|^2\frac{P}{\sigma_p^2}L_z$$

$$R_{th} \approx (23.58 \pm 5.60)\times 10^6\ s^{-1}\,mW^{-1}$$

$$R_{exp} \approx (35.5 \pm 0.8)\times 10^6\ s^{-1}\,mW^{-1}$$

SPDC Bandwidth (FWHM (ω))

Type-I	$\approx \frac{3.33653}{\sqrt{L_z\kappa}}$	$\kappa \equiv \frac{d^2k}{d\omega^2}\Big\vert_{\frac{\omega_p}{2}}$
Type-I No GVD	$\approx \frac{4.80794}{(L_z\Upsilon)^{1/4}}$	$\Upsilon \equiv \frac{d^4k}{d\omega^4}\Big\vert_{\frac{\omega_p}{2}}$
Type-II	$\approx \frac{5.56624}{L_z\,\vert\Delta n_g\vert}$	$\frac{n_g}{c} \equiv \frac{dk}{d\omega}\Big\vert_{\omega}$

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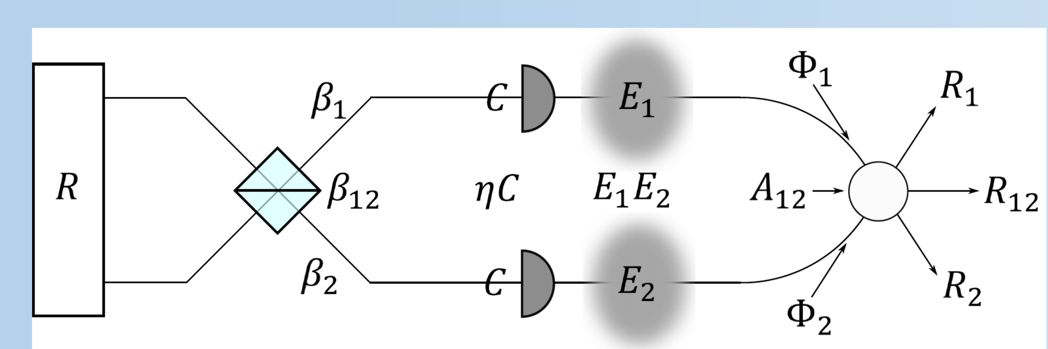
Extracting the Raw generation rate from lossy experiments

Singles rate: $R_1 = R\cdot E_1 C\,\beta_1 + \Phi_1$

Idlers rate: $R_2 = R\cdot E_2 C\,\beta_2 + \Phi_2$

Coinc. rate: $R_{12} = R\cdot E_1E_2\eta C\,\beta_{12} + A_{12}$



With a Non-polarizing Beamsplitter:

$$R = \frac{(R_1 - \Phi_1)(R_2 - \Phi_2)}{(R_{12} - A_{12})}\left(\frac{\beta_{12}}{\beta_1\beta_2}\right)\frac{\eta}{C}$$