Air Force Research Laboratory, Rome, NY
April 14, 2016

The Biphoton Birth Zone in Spontaneous Parametric Down-Conversion

Foundations and Applications

James Schneeloch

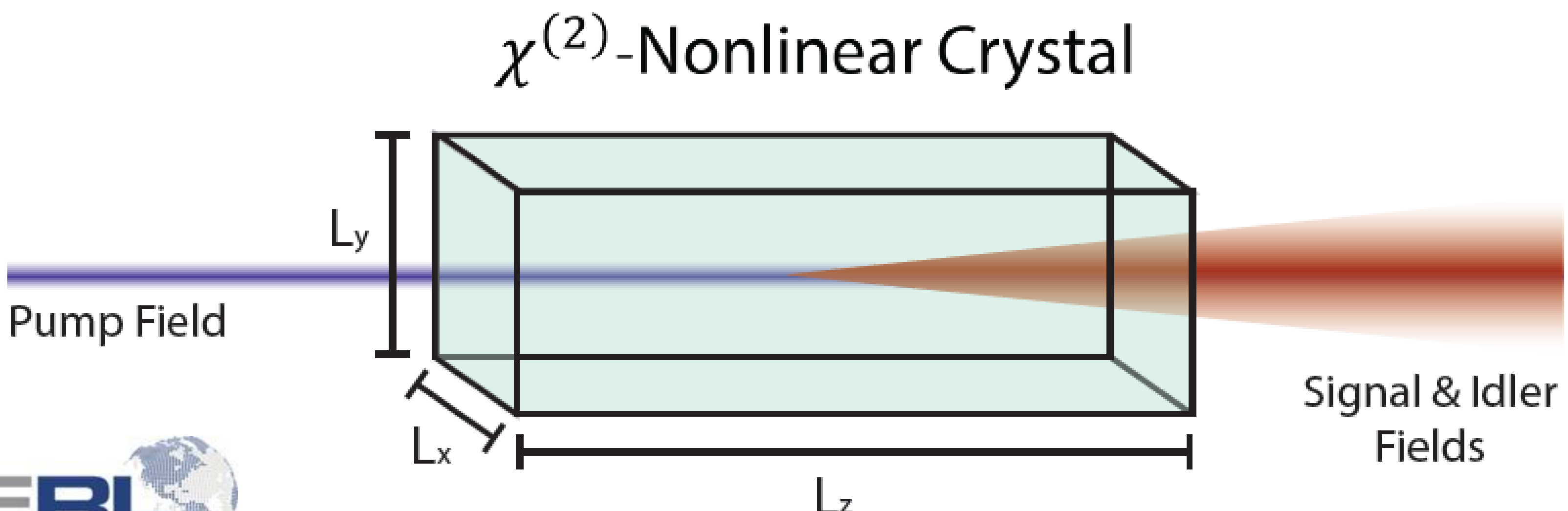
NRC Postdoctoral Fellow: Alsing Research Group

Outline

- Important concepts in SPDC
- Theoretical treatment of correlations
- Experimental methods
- EPR-steering with these correlations
- Application: Monogamy of EPR correlations

What is **SPDC**?

- **S**pontaneous: There's no seeding or stimulation
- **P**arametric: Optical parameters temporarily altered by pump beam
  - The final quantum state of the medium is left unchanged, though
- **D**own-**C**onversion: Pump photons are down-converted into correlated signal-idler photon pairs.
  - The energies and frequencies of the signal and idler photons are below that of the pump photon.

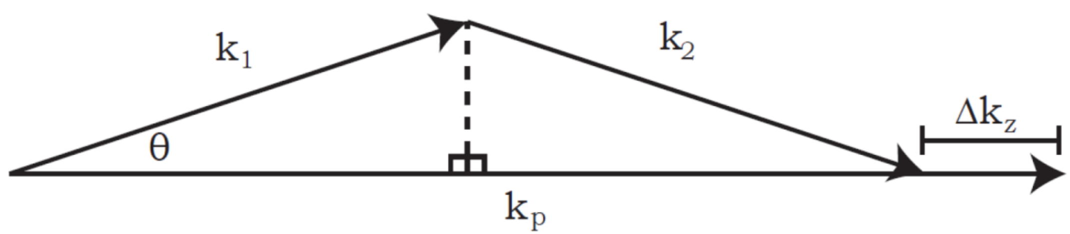


What Correlations are there?

Optical Parametric processes Conserve…

Energy: $\omega_{pump} = \omega_1 + \omega_2 \quad : \quad E = \hbar\omega$

Momentum: $\vec{k}_{pump} = \vec{k}_1 + \vec{k}_2 \quad : \quad \vec{\mathrm{p}} = \hbar\vec{k}$



If ω_{pump} and $\vec{\mathrm{k}}_{pump} \approx$ constant…

$\Rightarrow$ Energy and momentum correlations!

The Biphoton Birth Zone

- The region where the signal-idler photons are likely to be found given where the destroyed pump photon was:

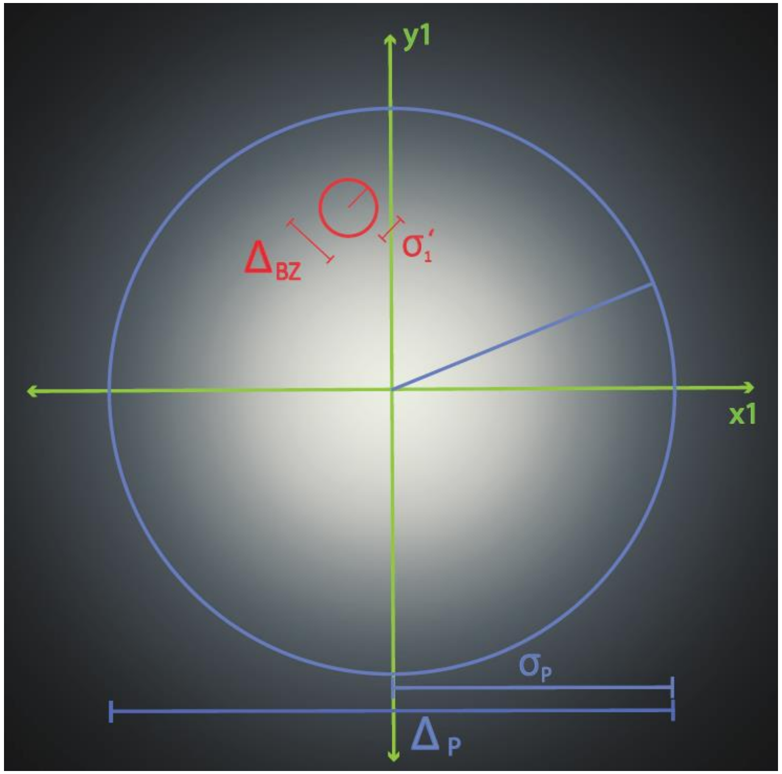
$$\Delta_{BZ} \equiv 2\sigma_{\left(x_1 \middle| \frac{x_1+x_2}{2}\right)}$$

$$\Delta_{pump} = 2\sigma_{\left(\frac{x_1+x_2}{2}\right)}$$

- The Birth zone number:

$$N \equiv \left(\frac{\Delta_{pump}}{\Delta_{BZ}}\right)^d$$

(a measure of the strength of these correlations)

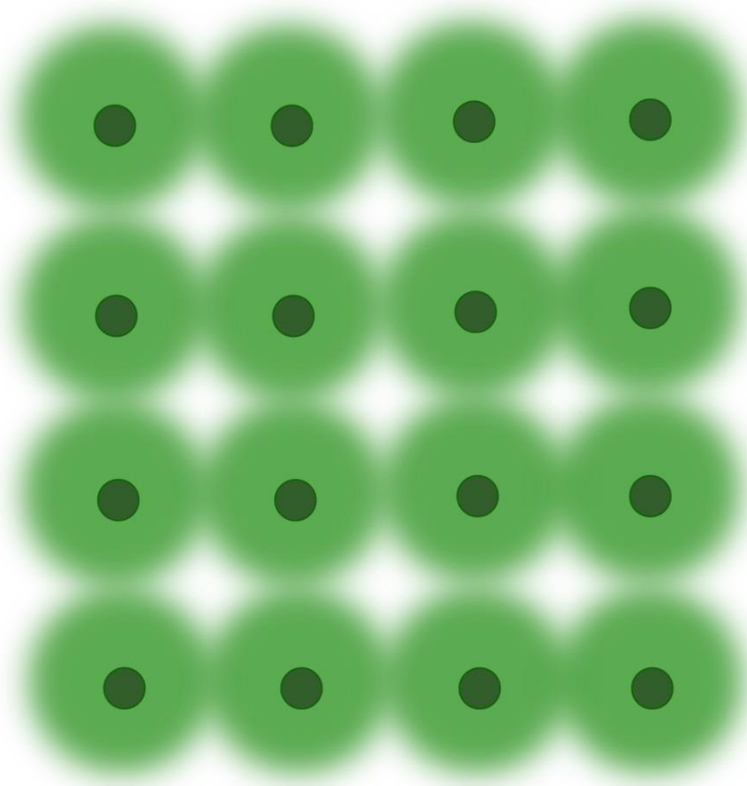


AFRL

How is SPDC even possible?

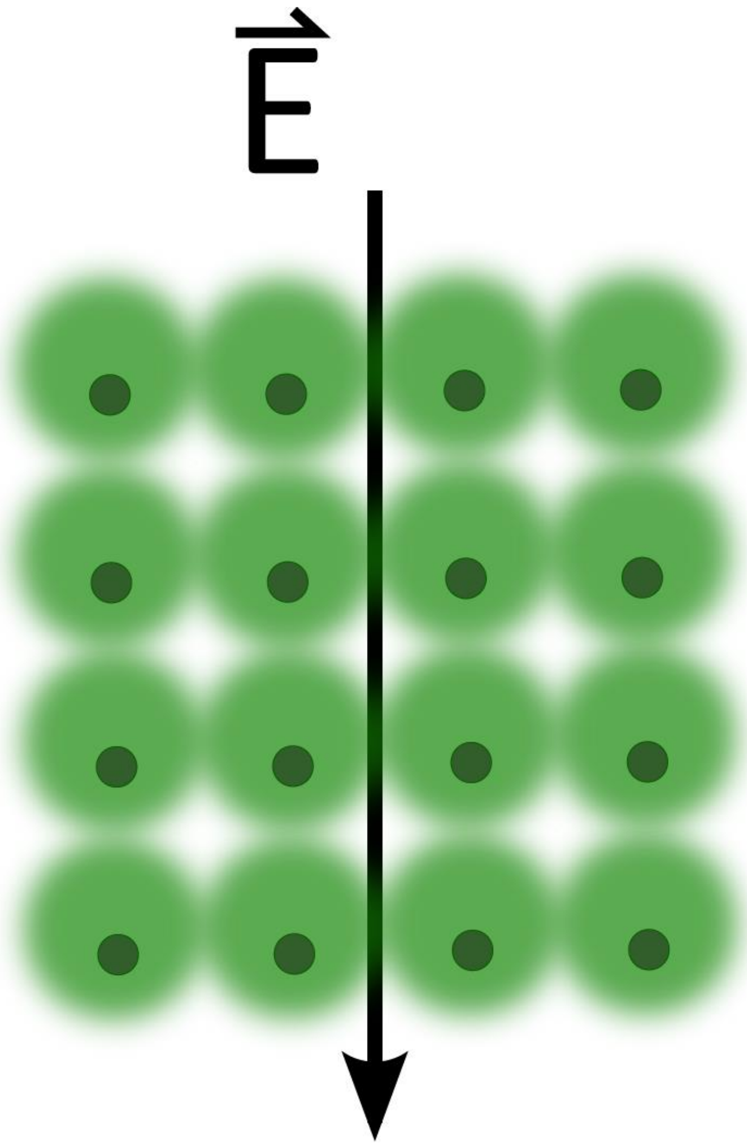
- Standard (linear) optics:

What does light do to matter?



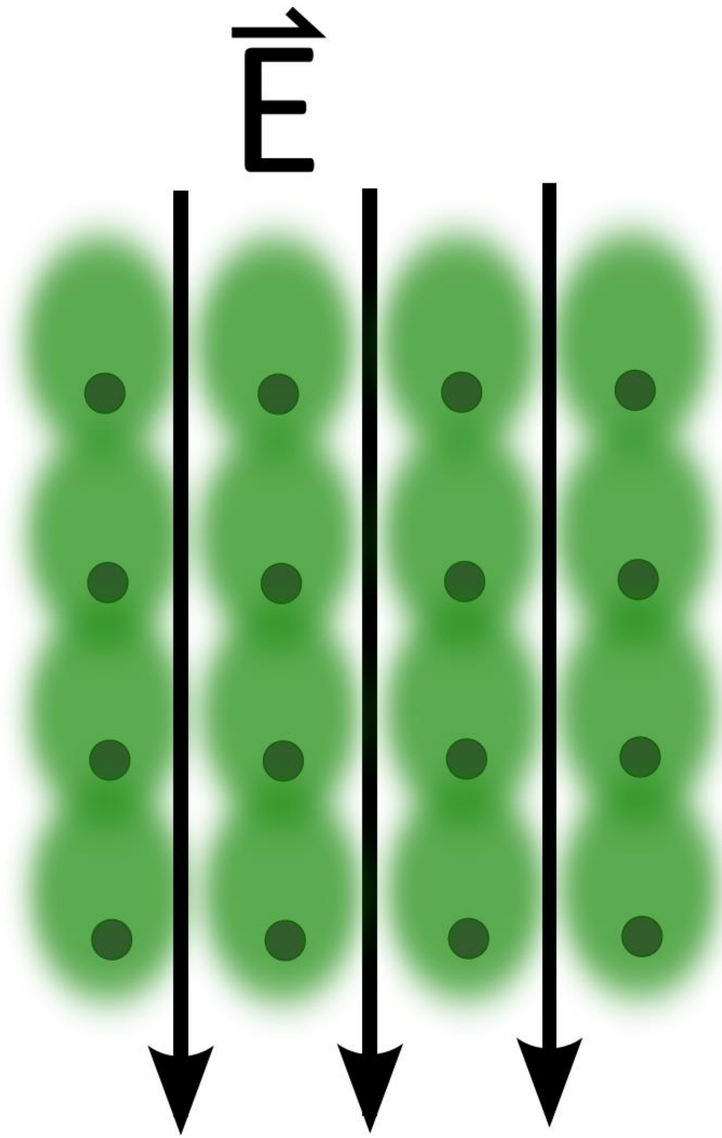
How is SPDC even possible?

- Standard (linear) optics:
  - EM waves slightly disturb bound electrons in a solid
    - They recoil back and forth like springs.
  - Steady state response is proportional to input electric field.



How is SPDC even possible?

- Standard (linear) optics:
  - EM waves slightly disturb bound electrons in a solid
    - They recoil back and forth like springs.
  - Steady state response is proportional to input electric field.
- Nonlinear optics:
  - Sufficiently intense light pushes and pulls electrons hard/far enough that they can’t be treated so simply.
  - Steady state response is no longer proportional:
    - You can have different frequency components in the output
  - Examples:
    - Optical Rectification
    - Second harmonic generation



AFRL

The Birth Zone Width from Quantum Optics

In four (easier-said-than-done) steps:

1. Quantize the electromagnetic field

$$\left[\hat{A}, \hat{B}\right] = i\hbar\{A, B\}$$

The Birth Zone Width from Quantum Optics

In four (easier-said-than-done) steps:

1. Quantize the electromagnetic field

2. Find the Hamiltonian for the SPDC process

$$H_{EM} = \frac{1}{2} \int d^3\boldsymbol{r}\,(\vec{\boldsymbol{D}} \cdot \vec{\boldsymbol{E}} + \vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{H}})$$

AFRL

The Birth Zone Width from Quantum Optics

In four (easier-said-than-done) steps:

1. Quantize the electromagnetic field

2. Find the Hamiltonian for the SPDC process

3. Use the Schrödinger equation to get the biphoton state $|\Psi_{SPDC}\rangle$.
   – The spatially varying part is the biphoton wavefunction

$$i\hbar \frac{d}{dt}|\psi\rangle = H_{EM}|\psi\rangle$$

The Birth Zone Width from Quantum Optics

In four (easier-said-than-done) steps:

1. Quantize the electromagnetic field
2. Find the Hamiltonian for the SPDC process
3. Use the Schrödinger equation to get the biphoton state $|\Psi_{SPDC}\rangle$.
   - The spatially varying part is the biphoton wavefunction
4. Calculate the Birth Zone Width from the biphoton wavefunction.

The Birth Zone Width from Quantum Optics

Assumptions:

- The pump beam is well-collimated and narrowband
- The pump beam is bright enough to be treated classically
  - But not so bright as to damage the material or to include multi-photon effects

The Birth Zone Width from Quantum Optics

Assumptions:

- The pump beam is well-collimated and narrowband
- The pump beam is bright enough to be treated classically
  - But not so bright as to damage the material or to include multi-photon effects
- The crystal is wider than the beam (and a lot wider than an optical wavelength)
- Optical constants are uniform throughout the crystal.
- The crystal is AR-coated (so we needn't consider multiple internal reflections)

The Birth Zone Width from Quantum Optics

Assumptions:

- The pump beam is well-collimated and narrowband
- The pump beam is bright enough to be treated classically
  - But not so bright as to damage the material or to include multi-photon effects
- The crystal is wider than the beam (and a lot wider than an optical wavelength)
- Optical constants are uniform throughout the crystal.
- The crystal is AR-coated (so we needn't consider multiple internal reflections)
- We use frequency filtering to look at the part of the downconverted spectrum where $\omega_1 \approx \omega_2 \approx \frac{\omega_{pump}}{2}$

The Birth Zone Width from Quantum Optics

The Quantum Biphoton State from SPDC

$$|\Psi_{\text{SPDC}}\rangle \approx C_0|0_1,0_2\rangle$$
$$+C_1 d_{eff}\sqrt{I_{pump}T^2}\iint d^3k_1 d^3k_2\ \mathbf{\Phi}(\vec{k}_1,\vec{k}_2)\ \hat{a}^\dagger(\vec{k}_1)\hat{a}^\dagger(\vec{k}_2)\ |0_1,0_2\rangle$$

The Biphoton Wavefunction

$$\mathbf{\Phi}(\vec{k}_1,\vec{k}_2) = \mathcal{N}\ Sinc\left(\frac{\Delta k_z L_z}{2}\right)v(k_{1x}+k_{2x},k_{1y}+k_{2y})$$

$$:\Delta k_z = k_{1z}+k_{2z}-k_{pump\ z}$$

What can we learn about rate R of produced photon pairs?

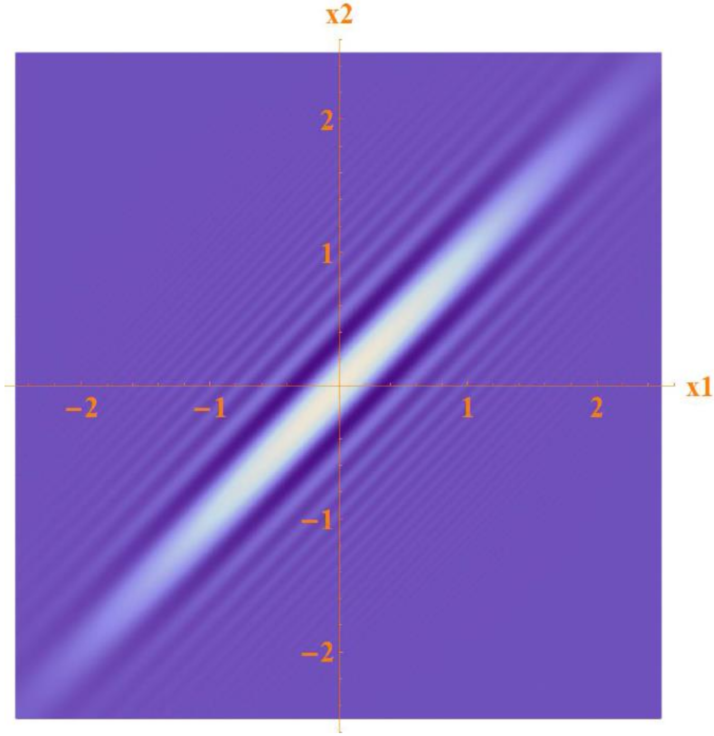
- $R \propto d_{eff}^2$ (high nonlinearities are especially important)
- $R \propto I_{pump}$ (pump photons in $\rightarrow$ output pairs out)
- $R \propto L_z^2$ (The amplitude is a sum over the paths in the crystal)

The Birth Zone Width from Quantum Optics

The Biphoton Wavefunction (in 1D)

- With a Gaussian Pump Beam profile:

$$\Phi(k_{1x}, k_{2x}) = \mathcal{N}\ Sinc\left(\frac{L_z \lambda_{pump}}{8\pi}(k_{1x} - k_{2x})^2\right) e^{-\sigma_{pump}^2 (k_{1x}+k_{2x})^2}$$



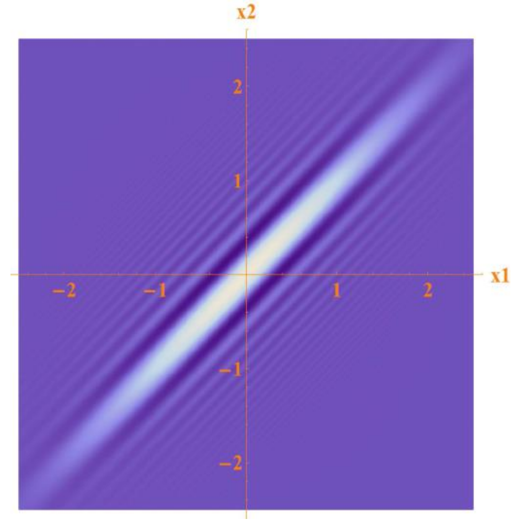
- We can calculate the Transverse Correlation Width $\sigma_{(x_1 - x_2)}$

$$\sigma_{(x_1 - x_2)} = \sqrt{\frac{9 L_z \lambda_{pump}}{10\pi}} = \Delta_{BZ}$$

(agrees with experimental data too!)

The Double-Gaussian Approximation

- The exact biphoton wavefunction is hard to work with...

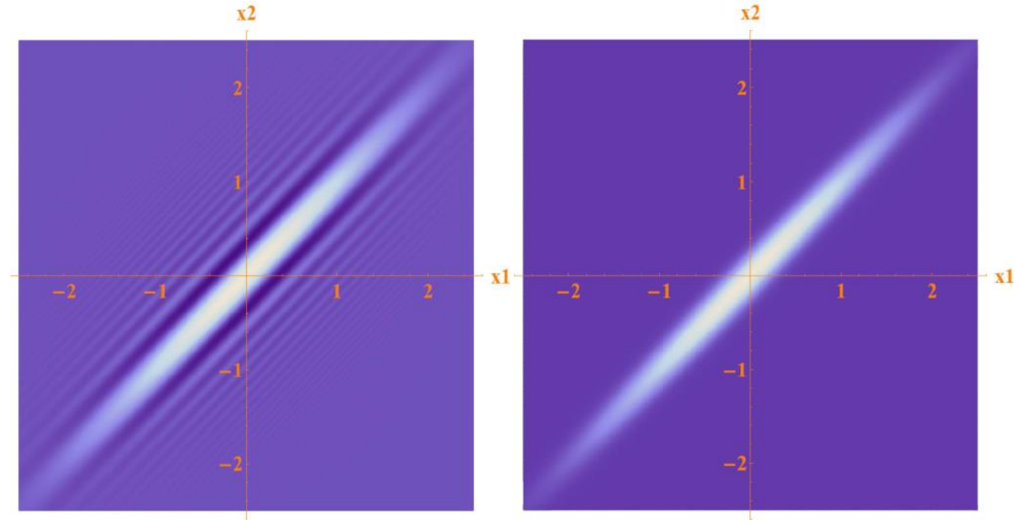


$$\psi(x_1, x_2) = \mathcal{N}\left[(x_1 - x_2)\sqrt{\pi}\left(\mathcal{S}\left(\frac{x_1 - x_2}{2\sqrt{\pi a}}\right) - \mathcal{C}\left(\frac{x_1 - x_2}{2\sqrt{\pi a}}\right)\right) + 2\sqrt{a}\left(\cos\left(\frac{(x_1 - x_2)^2}{8a}\right) + \sin\left(\frac{(x_1 - x_2)^2}{8a}\right)\right)\right] e^{-\frac{(x_1+x_2)^2}{16\sigma_{pump}^2}}$$

$$\mathcal{S}(x) = \int_0^x \sin\left(\frac{\pi}{2}t^2\right)dt \qquad \mathcal{C}(x) = \int_0^x \cos\left(\frac{\pi}{2}t^2\right)dt \qquad a \equiv \frac{L_z \lambda_{pump}}{4\pi}$$

The Double-Gaussian Approximation

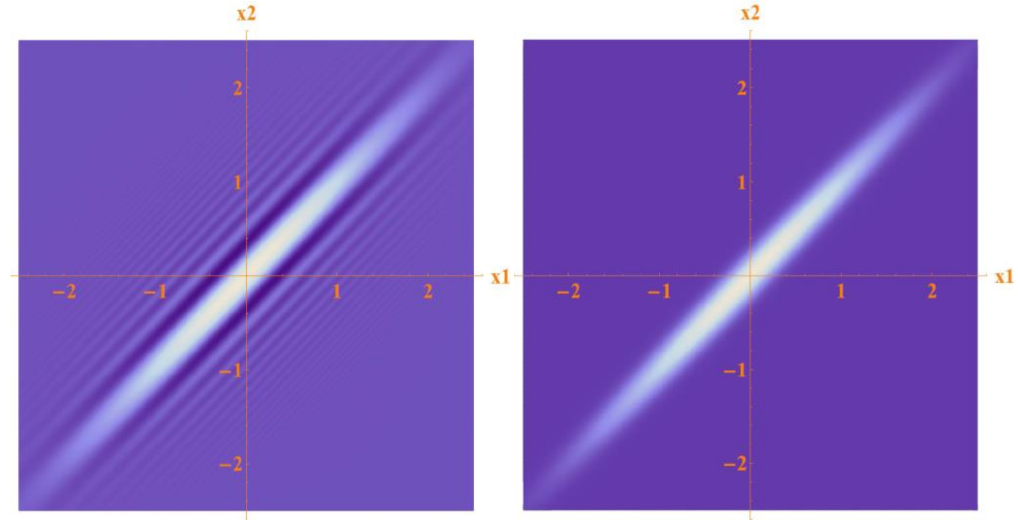
- The exact biphoton wavefunction is hard to work with.
- Gaussian wavefunctions are well studied and have many useful properties.
  - Useful Fourier-Transform properties
    - They saturate uncertainty relations
  - Statistical properties like mutual information are easy to calculate
  - You can actually find *quantum* entropies even though it's infinite-dimensional



$$\psi^{DG}(x_1, x_2) = \frac{1}{\sqrt{2\pi\sigma_{pump}\Delta_{PM}}} e^{-\frac{(x_1+x_2)^2}{16\sigma_{pump}^2}} e^{-\frac{(x_1-x_2)^2}{4\Delta_{PM}^2}}$$

The Double-Gaussian Approximation

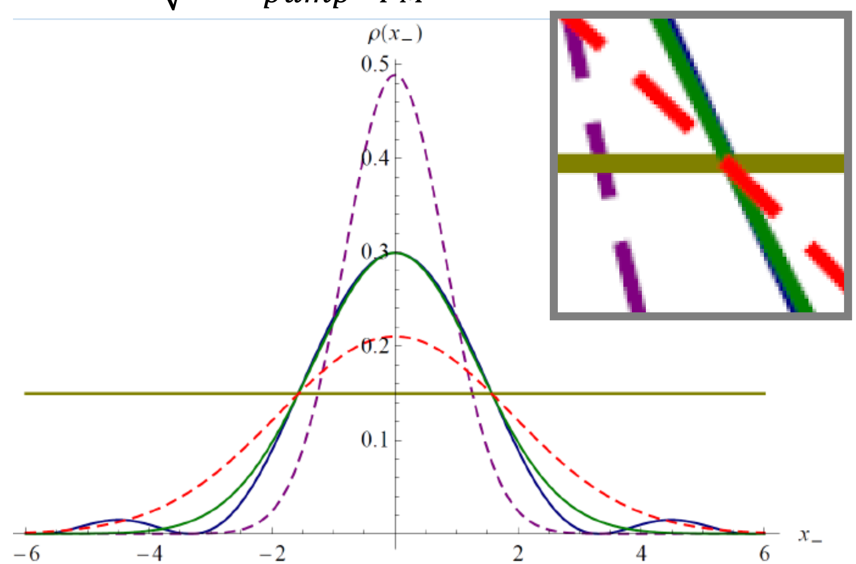
- The center of the biphoton wavefunction is well-approximated with a double-Gaussian by matching peaks:
- (the low-level oscillating wings are not)
- Experiments may neglect wings due to noise floor.



$$\psi^{DG}(x_1,x_2) = \frac{1}{\sqrt{2\pi\sigma_{pump}\Delta_{PM}}} e^{-\frac{(x_1+x_2)^2}{16\sigma_{pump}^2}} e^{-\frac{(x_1-x_2)^2}{4\Delta_{PM}^2}}$$

$$\Delta_{PM} = \sqrt{\frac{4\, L_z \lambda_{pump}}{9\pi}}$$

(fits Full Width at 48.2% of Max)



How strong are these Correlations?

$$\sigma_{(x_1-x_2)} = \sqrt{\frac{9L_z\lambda_{pump}}{10\pi}} = \Delta_{BZ} \gtrsim \sigma_{(x_1|x_2)}$$

- Example:
  - BiBO crystal for Type-I SPDC
  - λ_{pump}=775nm
  - L_z =3mm
  - $\sigma_{pump} = 0.5\text{mm}$

  $\Longrightarrow \Delta_{BZ}$ =25.8μm

  $\Longrightarrow$ (Birth Zone Number) $N = (38.75)^2 \approx 1502$

  $\Rightarrow$ (Mutual Information) $h(\vec{x}_1 : \vec{x}_2) \approx 9.55$ bits

  $\Longrightarrow$ (Entanglement of Formation): $\mathcal{E}(\hat{\rho}) \approx 9.4$ ebits

  $\Rightarrow$ (Pearson R-value) $R \approx 0.9987$

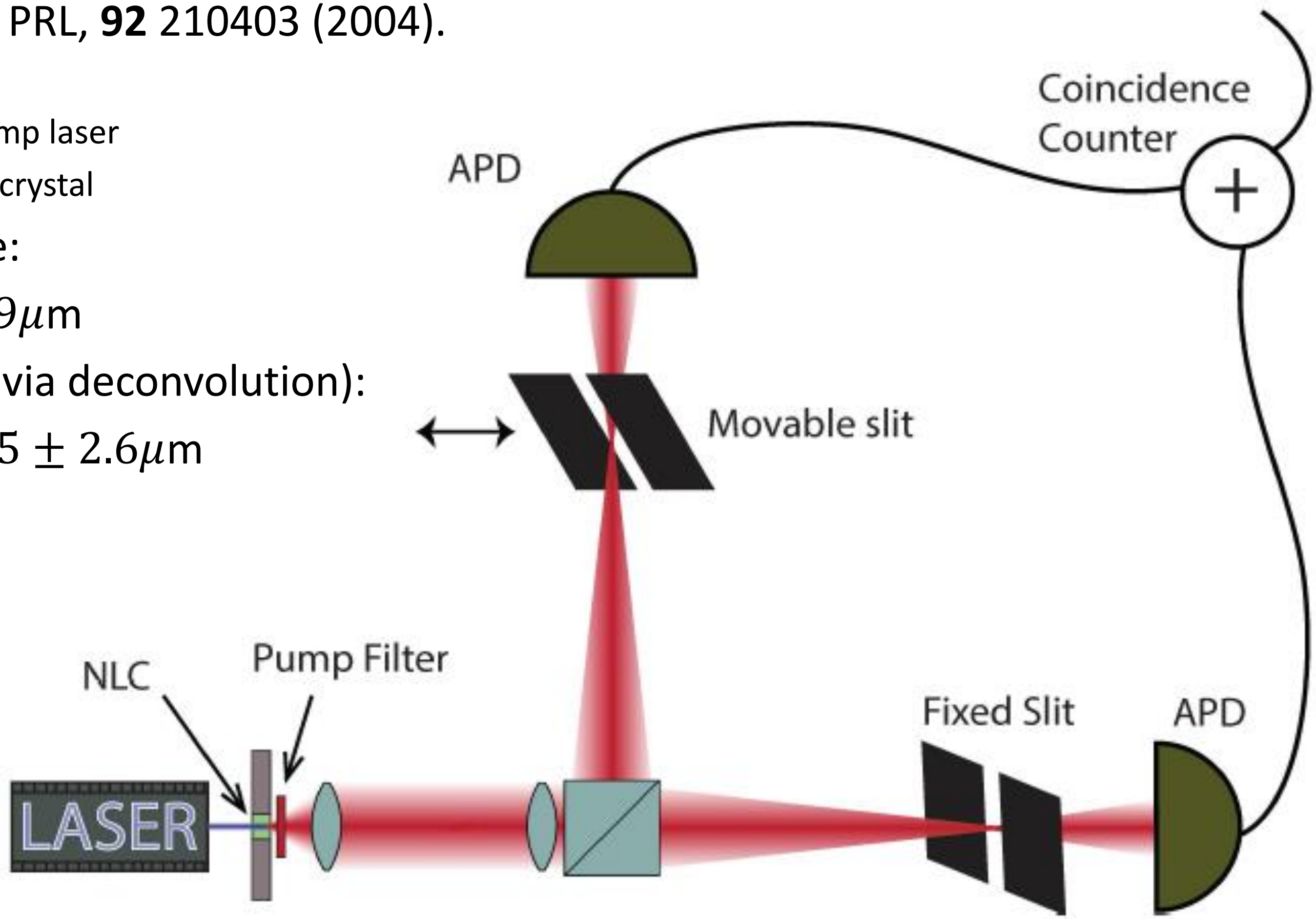
AFRL

How to measure these correlations?

- To measure just Δ_{BZ} …
  - Consider the following, based on Howell *et* al: PRL, **92** 210403 (2004).
    - 40μm slits
    - 390nm pump laser
    - 2mm BBO crystal
  - Our estimate:

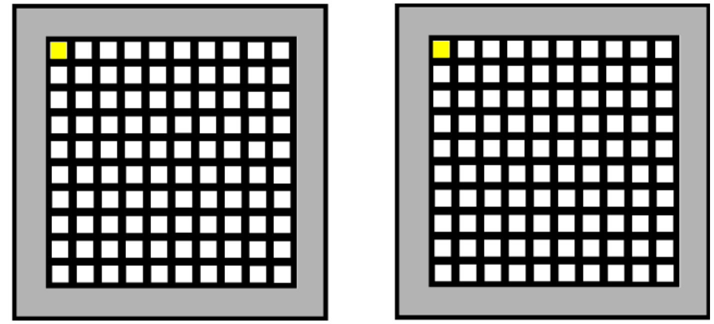
    $\Delta_{BZ} \approx 14.9\mu\text{m}$
  - Their result (via deconvolution):

    $\Delta_{BZ} \approx 13.5 \pm 2.6\mu\text{m}$



How to measure *all* the correlations?

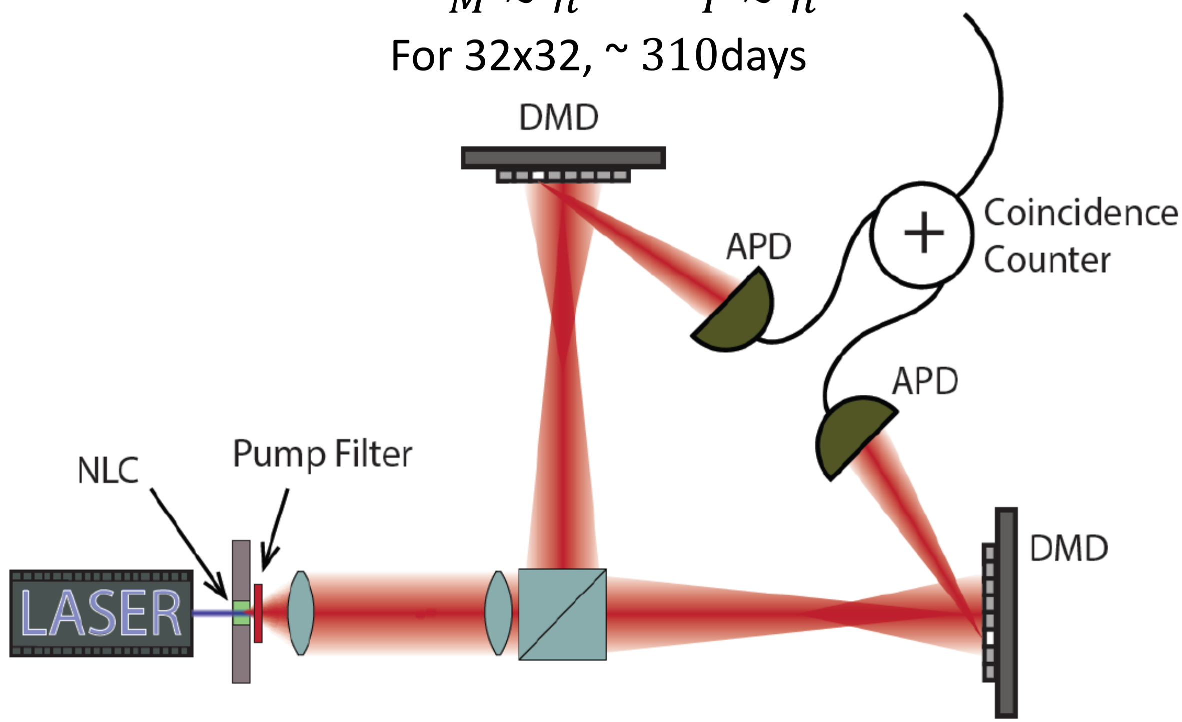
Hard way: Measure the coincidences coming from every pair of pixels in transverse position and momentum planes.



(Brute force)

$M \sim n^4 \qquad T \sim n^6$

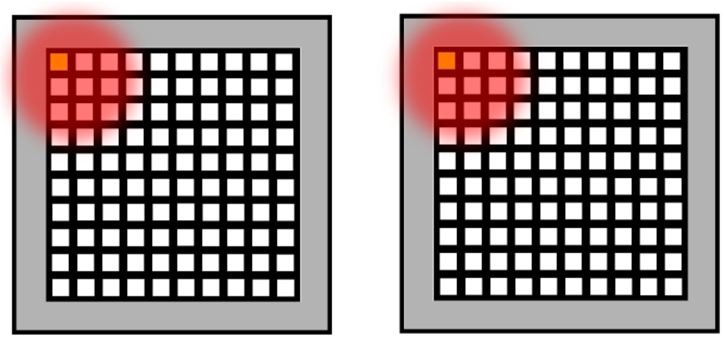
For 32x32, ~ 310days



How to measure *all* the correlations?

Hard way: Measure the coincidences coming from every pair of pixels in transverse position and momentum planes.

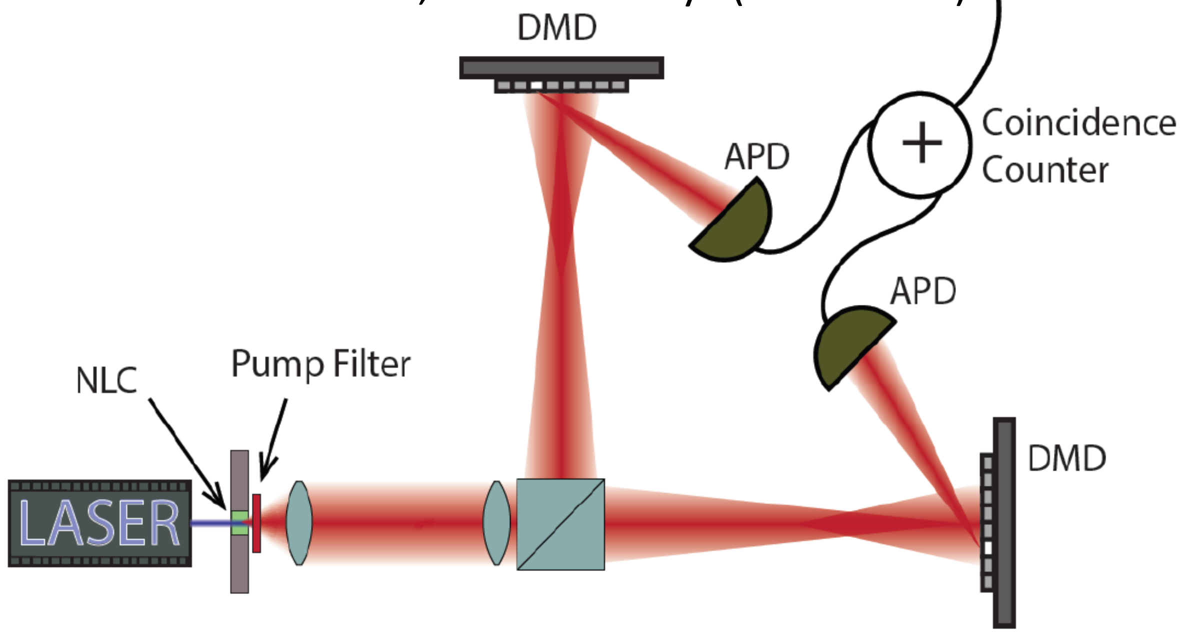
Medium way: Measure coincidences coming from pairs expected to be correlated, and local neighborhood.



(Nearest K Neighbors)

$M \sim k \times n^2 \qquad T \sim k\, n^4$

For 32x32, ~ 14 -15 days (for $k \sim 49$)

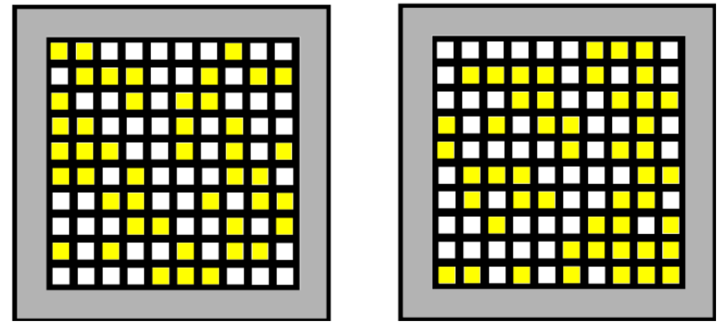


How to measure *all* the correlations?

Hard way: Measure the coincidences coming from every pair of pixels in transverse position and momentum planes.

Medium way: Measure coincidences coming from pairs expected to be correlated, and local neighborhood.

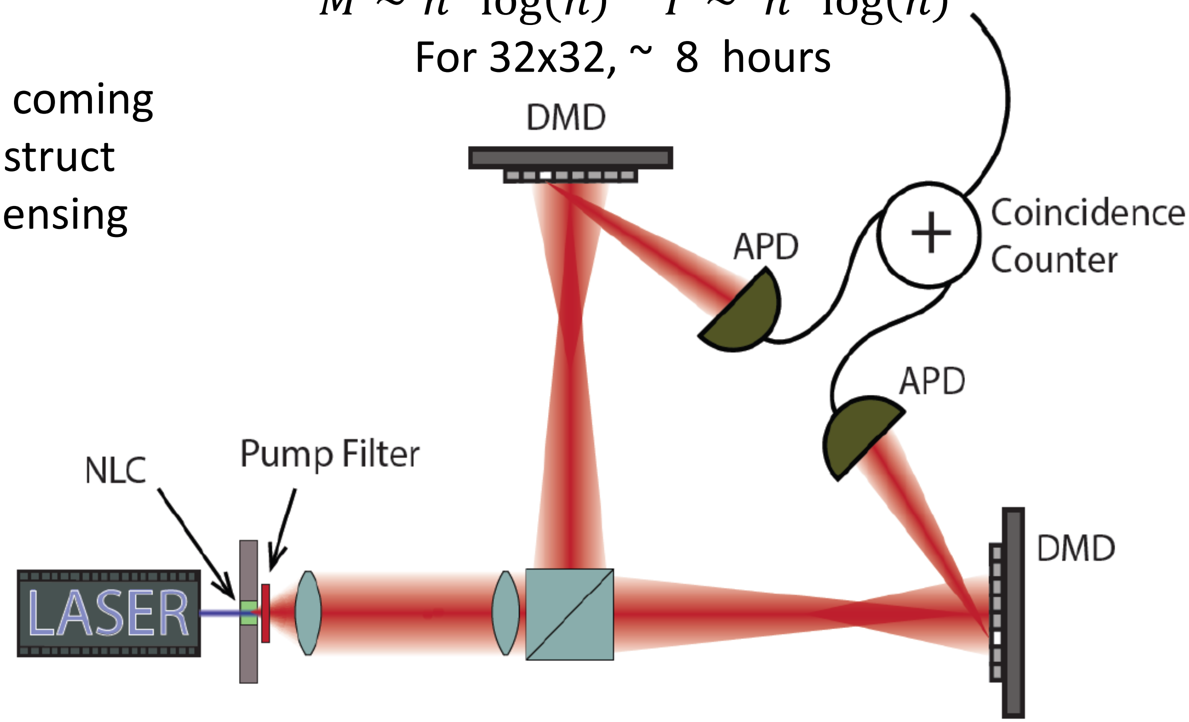
Easy way: Measure coincidences coming from random patterns and reconstruct whole thing using Compressive Sensing Tomography.



(Random Projections)

$M \sim n^2 \log(n) \quad T \sim n^4 \log(n)$

For 32x32, ~ 8 hours



Why measure these correlations?

With strong enough correlations,
In complementary domains,
you can prove there's entanglement
by way of **EPR-steering**!

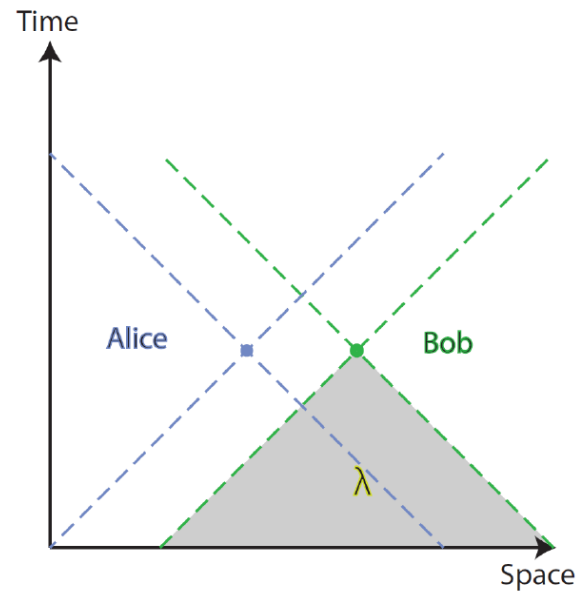
$$\Delta(x_A) \cdot \Delta(k_A) \geq \frac{1}{2}$$

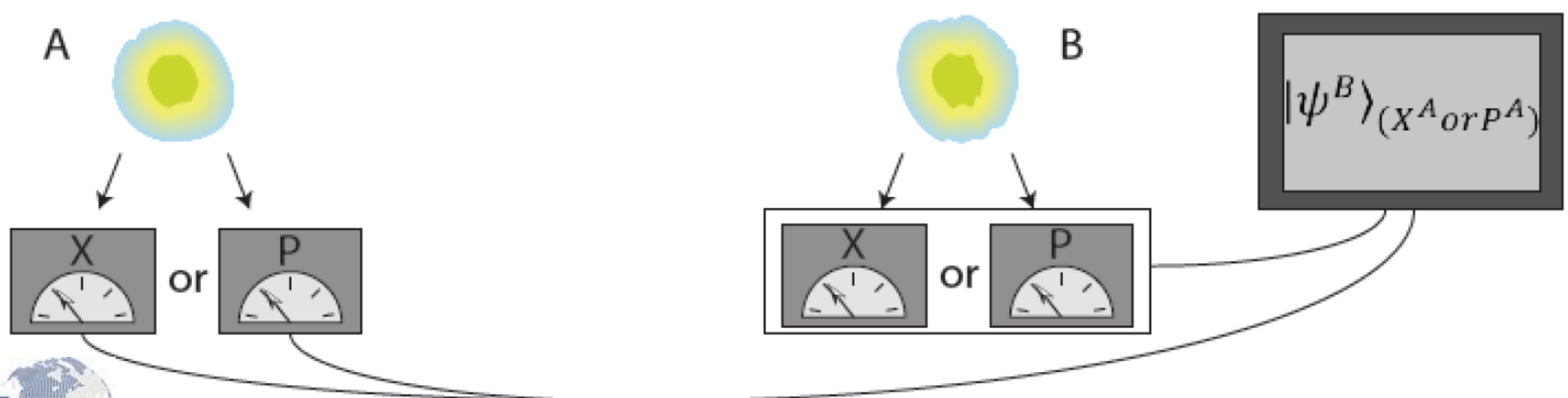
But…

$$\Delta(x_A|x_B) \cdot \Delta(k_A|k_B) \geq 0$$

EPR-steering: Essential Concepts

- **The situation**: Alice and Bob share a separated pair of particles A and B entangled in (e.g.,) position and momentum
- **Locality**: The effect of measurement cannot travel faster than light
- **Completeness**: Quantum Mechanics gives a complete description of reality. The uncertainty principle is an absolute fundamental limit.
- **The steering**: Alice's choice of measurement controls the ensemble of possible states Bob measures.

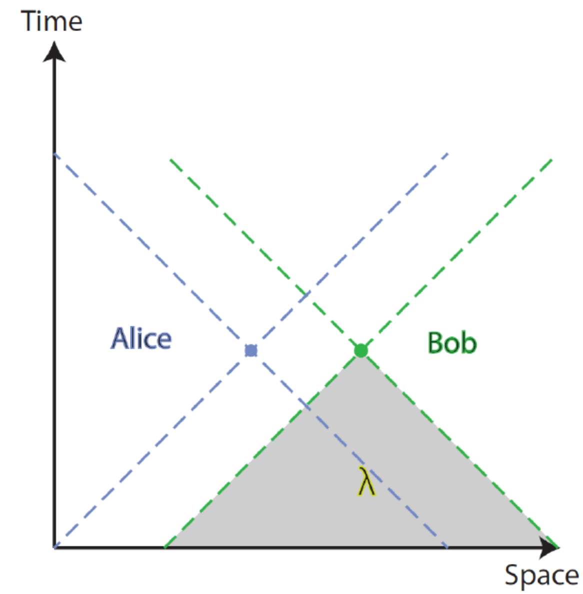




From the EPR-paradox to EPR-steering

- **Locality**:
  Everything about particle B is in information λ in B's past light cone.
  - Conditioning on Alice's results cannot reduce the uncertainty more than conditioning on all of λ

$$\Delta(x_B|x_A) \geq \Delta(x_B|\lambda)$$



- **Completeness**:
  The uncertainty principle still holds, even when conditioning on all this information λ.

$$\Delta(x_B|\lambda) \cdot \Delta(k_B|\lambda) \geq \frac{1}{2}$$

- Putting Locality and completeness together:
  We get the first EPR-steering inequality (Reid, 1989)

$$\Delta(x_B|x_A) \cdot \Delta(k_B|k_A) \geq \frac{1}{2}$$

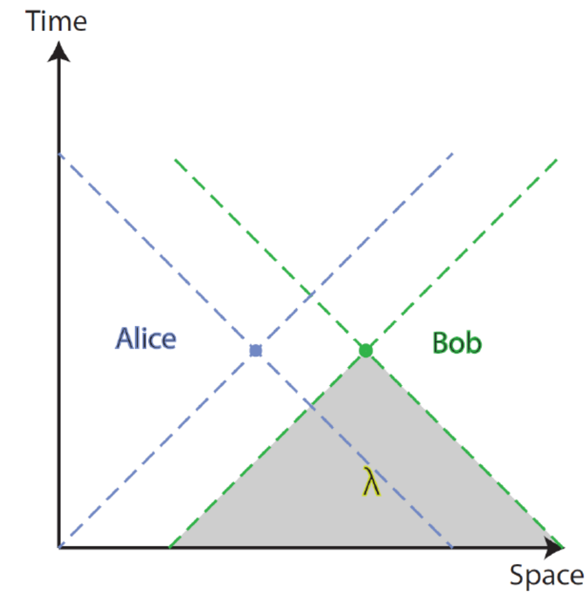
- But in general, we know that:

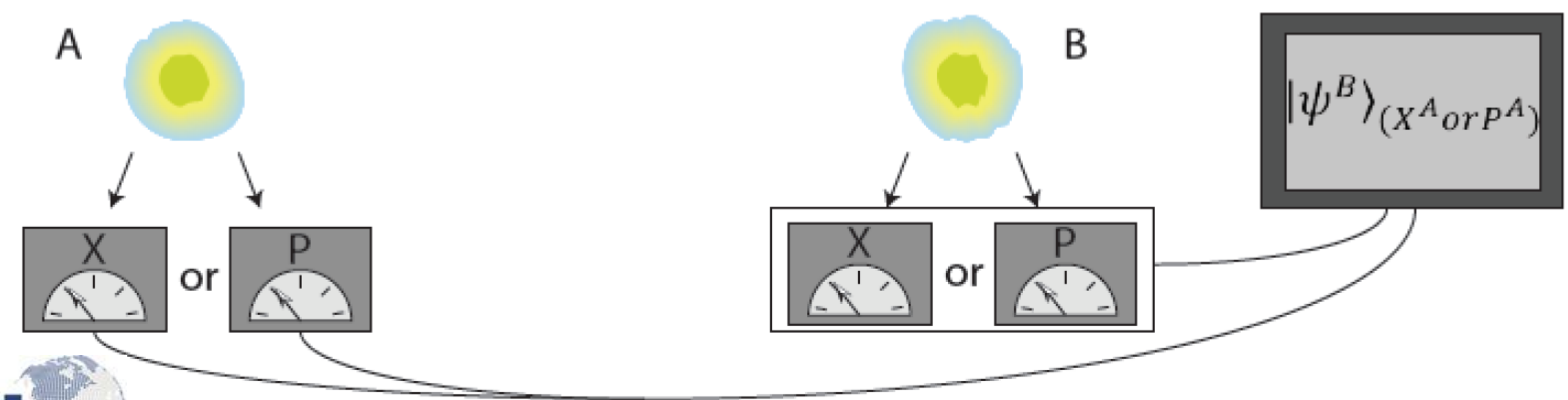
$$\Delta(x_B|x_A) \cdot \Delta(k_B|k_A) \geq 0$$

The EPR Paradox:
Locality and Completeness are mutually exclusive.

The Flavors of EPR-steering

- **Easy**: Show the joint entangled state is EPR-steerable
- **Medium**: Demonstrate EPR-steering correlations (i.e., the EPR paradox)
  - Useful in more robust Quantum Key Distribution
- **Hard**: "EPR-steer" something
  - Actually do the tomography on Bob's systems to show that Bob's state conditioned on Alice's measurement result is under her control





Proving EPR-steering (Theoretically)

- Lots of correlations can be explained classically
  - Particles A and B could be classically correlated to begin with
  - Alice or Bob could have an untrusted measurement device (a “black box”)

$$h(x) \equiv -\int dx\rho(x)\log\big(\rho(x)\big)$$

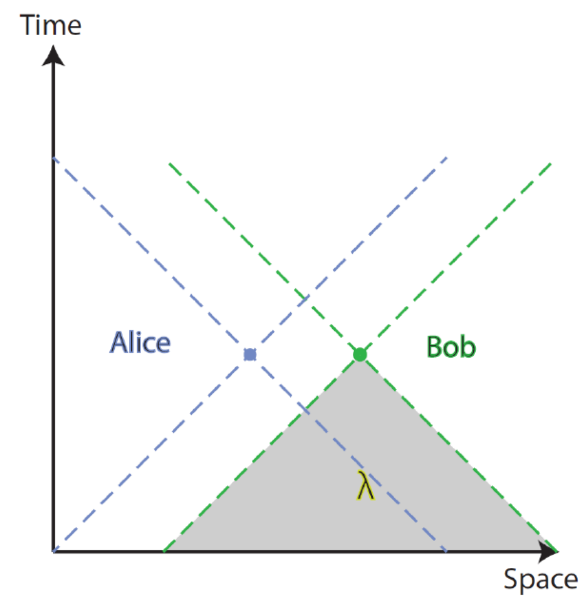
$$h(x_B|x_A) \geq \int d\lambda\, \rho(\lambda) h(x_B|\lambda)$$
$$h(k_B|k_A) \geq \int d\lambda\, \rho(\lambda) h(k_B|\lambda)$$

$$h(x_B|\lambda) + h(k_B|\lambda) \geq \log(\pi e)$$

- But if Bob was not receiving halves of entangled pairs…
  - Then there’s a limit to how well Alice can predict Bob’s measurement results.

$$h(x_B|x_A) + h(k_B|k_A) \geq \log(\pi e)$$

(Walborn et al., 2011)



AFRL

Proving EPR-steering (Experimentally)

$$h(x_B|x_A) + h(k_B|k_A) \geq \log(\pi e)$$

– This relies on knowing continuous probability densities

$$h(x) \equiv -\int dx \rho(x) \log(\rho(x))$$

$$H(X) \equiv -\sum_i P(X_i) \log(P(X_i))$$

But…

- Discrete approximation never decreases entropy!*

$$H(X_B|X_A) + \log(\Delta x_B) \geq h(x_B|x_A)$$

- So violating the discrete inequality…

$$H(X_B|X_A) + H(K_B|K_A) \geq \log\left(\frac{\pi e}{\Delta x_B \Delta k_B}\right)$$

- ..witnesses position-momentum EPR-steering!

*See PRL **110,** 130407 (2013) for full treatment

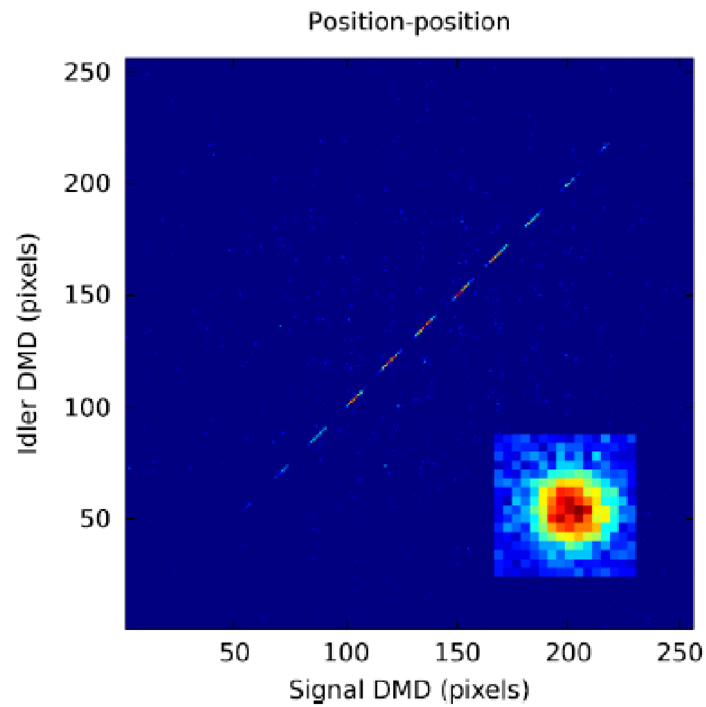


Figure from:
Phys. Rev. X, 3, 011013 (2013).

Application: Monogamy of EPR-steering Correlations

- A and B's (sufficiently) high correlations can guarantee low correlations with any third party
  - (good for security against eavesdroppers)
- Steering deficit: $\delta_{A\to B}$

$$\delta_{A\to B} \equiv H(X_B|X_A) + H(K_B|K_A) - \log\left(\frac{\pi e}{\Delta x_B \Delta k_B}\right)$$

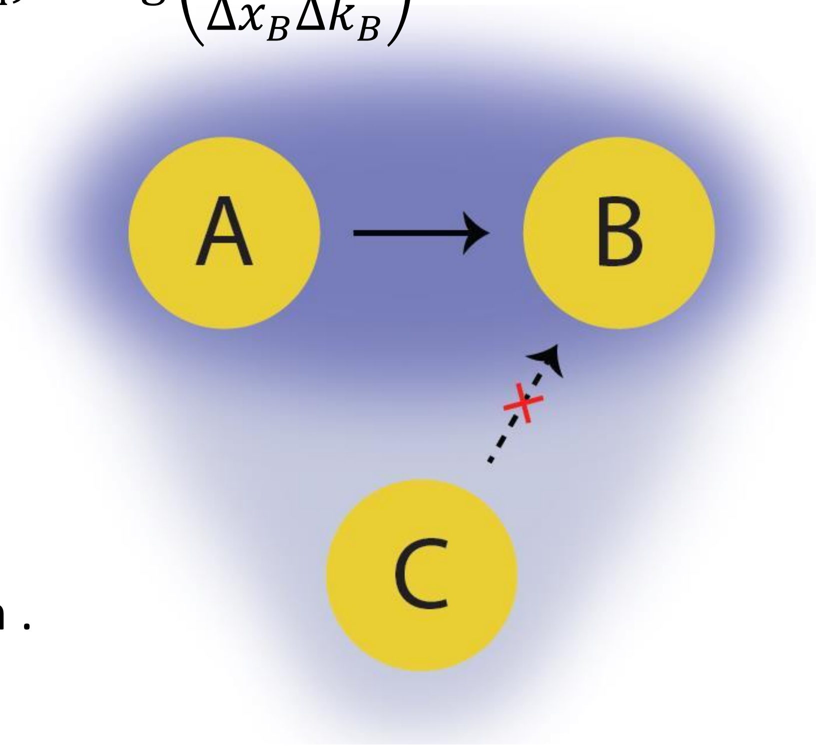
- Monogamy inequality:

$$\delta_{A\to B} + \delta_{C\to B} \geq 0$$

(made by combining two uncertainty relations)

- Security:

Alice and Bob can use their
higher correlations to distill a
secret key for private communication .



Thanks for Listening!

HRG

AFOSR
AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

AFRL

CCQO

DARPA DSO

The National Academies of
SCIENCES · ENGINEERING · MEDICINE

Works Cited

Schneeloch, James and Howell, John C. "Introduction to the Transverse Spatial Correlations in Spontaneous Parametric Down-Conversion through the Biphoton Birth Zone", *J. Opt.* **18**,5 (2016).

Howell, John C., Bennink, Ryan S., Bentley, Sean J., and Boyd, R.W. "Realization of the Einstein-Podolsky-Rosen Paradox using Momentum- and Position Entangled Photons from Spontaneous Parametric Down-Conversion", *Phys. Rev. Lett*. **92**, 21, (2004).

Reid, M. D., "Demonstration of the Einstein-Podolsky-Rosen paradox using nondegenerate parametric amplification", *Phys. Rev. A* **40**, 913 (1989).

Walborn, S.P., Salles, A., Gomes, R. M., Toscano, F., and Souto-Ribeiro, P.H., "Revealing Hidden Einstein-Podolsky-Rosen Nonlocality", *Phys. Rev. Lett*. **106**, 130402 (2011)

Schneeloch, James, Dixon, P. Ben, Howland, Gregory A., Broadbent, Curtis J., and Howell, John C. "Violation of Continuous-Variable Einstein-Podolsky-Rosen Steering with Discrete Measurements", *Phys. Rev. Lett*. **110**, 130407 (2013).

Schneeloch, James, "The Relationship between Discrete and Continuous Entropy in EPR-steering Inequalities", arXiv **1312.2640** (2013).

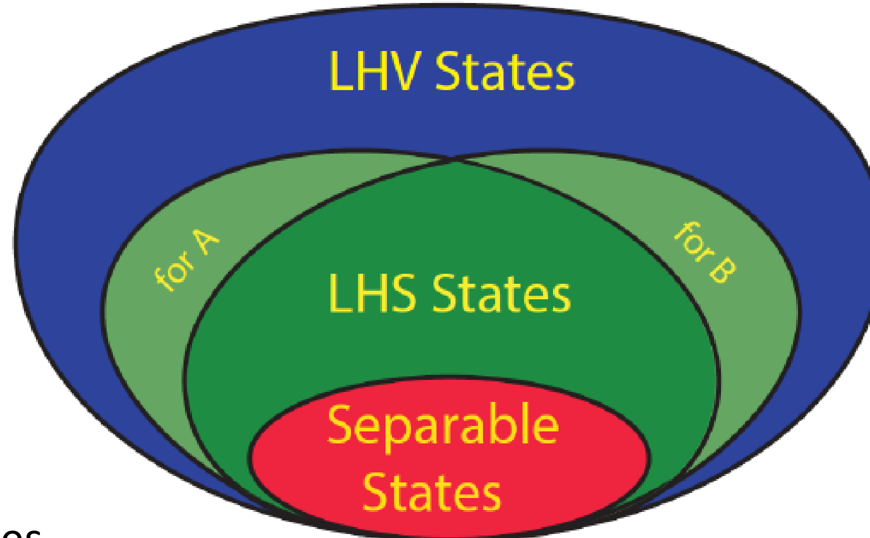
Howland, Gregory A., and Howell, John C. "Efficient High-Dimensional Entanglement Imaging with a Compressive-Sensing Double-Pixel Camera", *Phys. Rev. X* **3**, 011013 (2013).

Entanglement and the Hierarchy of Locality

- Local Hidden Variables (LHV):
  - Information existing in past light cone
  - LHV Models:

$$\rho(x_A, x_B) = \int d\lambda\, \rho(\lambda)\rho(x_A|\lambda)\rho(x_B|\lambda)$$

  - Ruled out by Violating a Bell Inequality



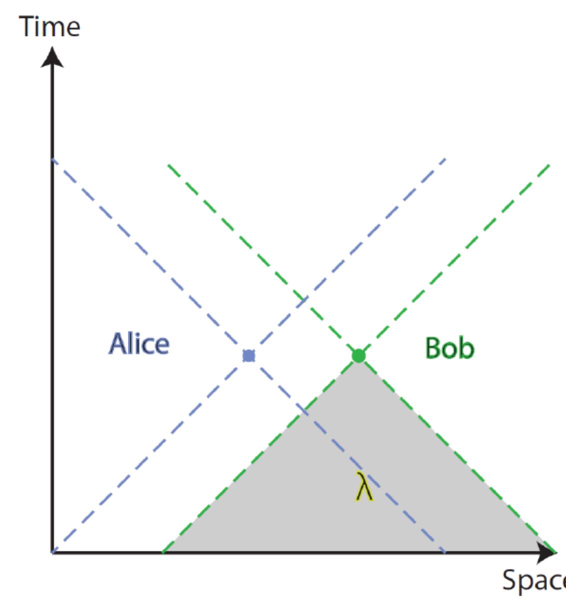
- Local Hidden States (LHS):
  - States determined by local hidden variables
- LHS model (for Bob):

$$\rho(x_A, x_B) = \int d\lambda\, \rho(\lambda)\rho(x_A|\lambda) Tr\left[\hat{\Pi}_X^B\, \hat{\rho}_\lambda^B\right]$$

  - Ruled out by violating an EPR-steering inequality.
- Separable model:

$$\rho(x_A, x_B) = \int d\lambda\, \rho(\lambda) Tr\left[\hat{\Pi}_X^A\, \hat{\rho}_\lambda^A\right] Tr\left[\hat{\Pi}_X^B\, \hat{\rho}_\lambda^B\right]$$

  - Ruled out by any entanglement witness.



AFRL