Characterizing Entanglement from Correlations: The Multi-partite case

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Outline

- What is quantum entanglement?
- What makes 3-party entanglement special?
- Measuring entanglement from correlations
- Measures of three party entanglement
  - Residual vs resource measures
- Our method of quantifying three-party entanglement from correlations
- The entanglement-correlation connection

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The Nature of Entanglement

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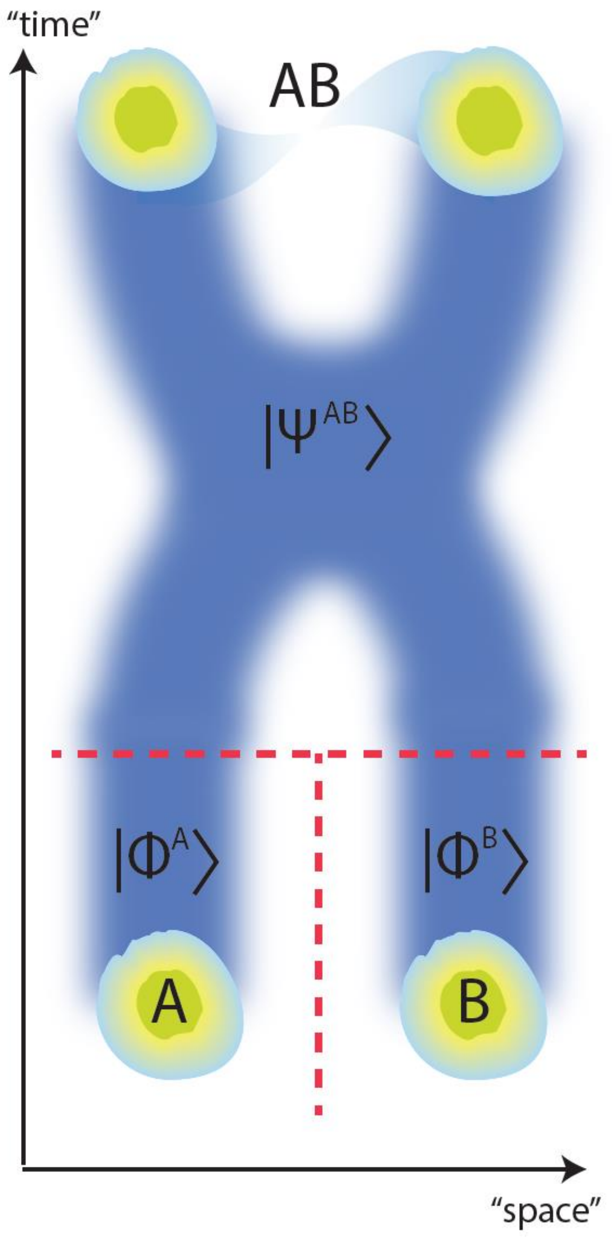
What is entanglement?

(It's more the rule than the exception)

- What isn't entangled?
  - Independent particles
    - $|\psi_{ABC}\rangle = |\psi_A\rangle \otimes |\psi_B\rangle \otimes |\psi_C\rangle$
  - Classically correlated states
    - $\hat{\rho}_{ABC} = \sum_i p_i \left(\hat{\rho}_{Ai} \otimes \hat{\rho}_{Bi} \otimes \hat{\rho}_{Ci}\right)$

  Everything not separable like this is entangled
- Where does entanglement come from?
  - Interactions between parties
  - Generation of entangled particles
- What is entanglement good for:
  - The speedup in quantum computation
  - The secrecy of correlations
  - Enhanced Measurement



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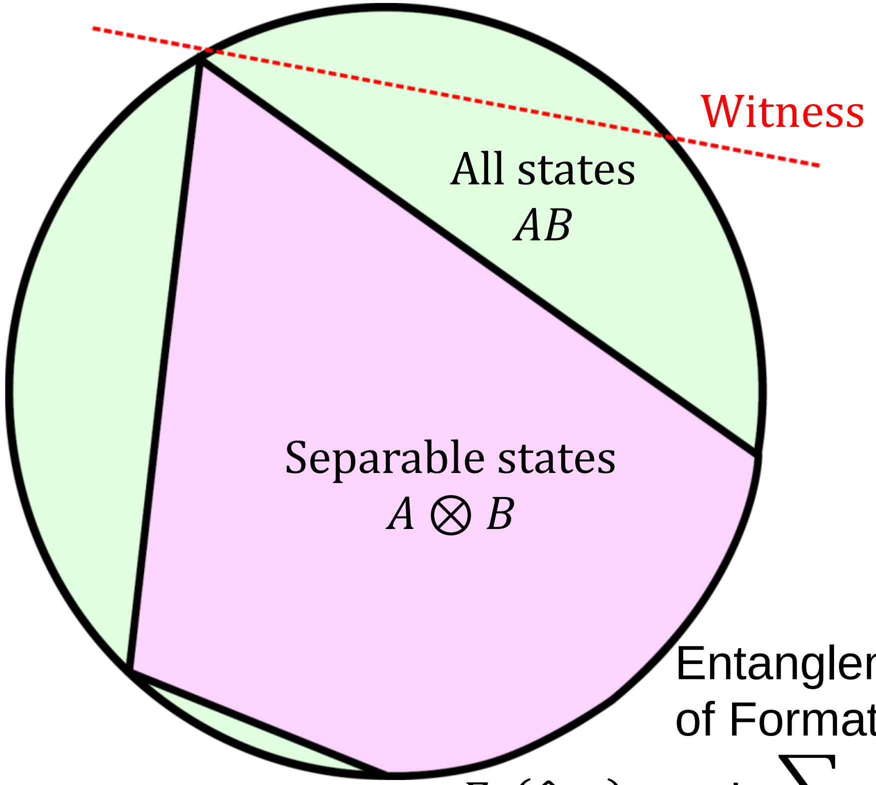
Entanglement between two parties

Entanglement is not *just* a yes or no question

Some states are more entangled than others

- Entanglement measures:
  - Geometric:
    - Degree of divergence/distance from separable states
  - Resource-based:
    - Unit of two-party entanglement is the **ebit**, entangled bit or two-qubit Bell state
    - How many ebits do you need to make a state $|\psi\rangle$ along with local operations and classical communication (LOCC)?
      - How many ebits can you distill out of $|\psi\rangle$?
- Entanglement witnesses:
  - Things that all separable states do
    - E.g., obey Bell inequalities



Entanglement of Formation: E_F

$$E_F(\hat{\rho}_{AB}) \equiv \min_{|\psi_i\rangle} \sum_i p_i\, S_i(A)$$

$$S_i(A) = -Tr[\hat{\rho}_{Ai} \log_2 \hat{\rho}_{Ai}]$$

$$\hat{\rho}_{AB} = \sum_i p_i\, |\psi_{ABi}\rangle\langle\psi_{ABi}|$$

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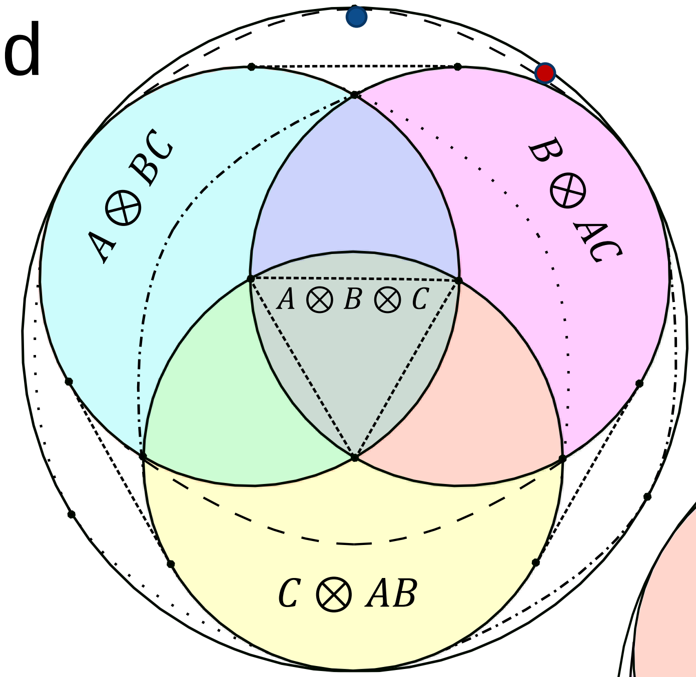
Entanglement between three or more parties

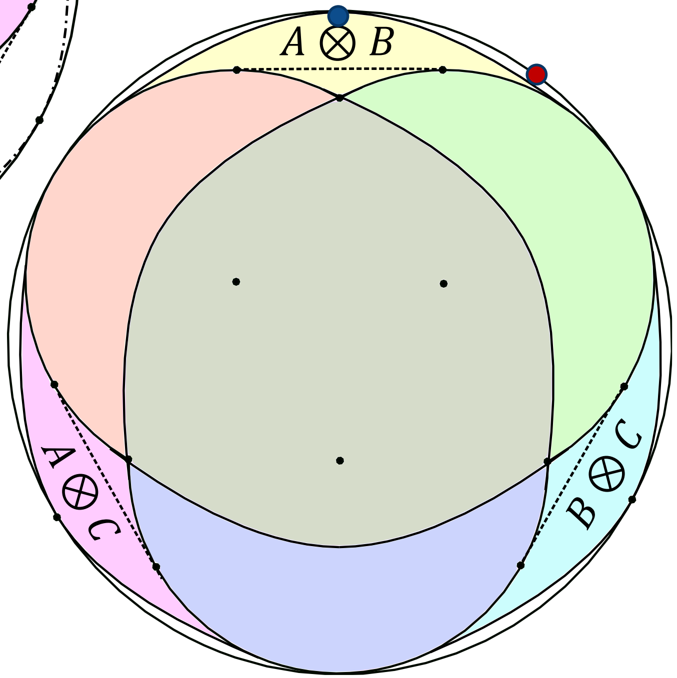
- Some states are more entangled than others

And..

- Some states are entangled differently than others
  - Example:
    - $|GHZ\rangle = \frac{1}{\sqrt{2}}(|0,0,0\rangle + |1,1,1\rangle)$
    - $|W\rangle = \frac{1}{\sqrt{3}}(|0,0,1\rangle + |0,1,0\rangle + |1,0,0\rangle)$
- Multiple forms of separability mean multiple forms of entanglement:



Three-party classes



Two-party classes

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Entanglement between three or more parties

Example: Classes of three-party states

- Fully separable:

$$\hat{\rho}_{ABC} = \hat{\rho}_{A\otimes B\otimes C} = \sum_i p_i \left(\hat{\rho}_{Ai} \otimes \hat{\rho}_{Bi} \otimes \hat{\rho}_{Ci}\right)$$

- Bi-separable (3 possible ways):

$$\hat{\rho}_{ABC} = \hat{\rho}_{A\otimes BC} = \sum_i p_i \left(\hat{\rho}_{Ai} \otimes \hat{\rho}_{BCi}\right) \neq \hat{\rho}_{A\otimes B\otimes C}$$

- Fully Inseparable:

$$\hat{\rho}_{ABC} \neq \hat{\rho}_{A\otimes BC} \neq \hat{\rho}_{B\otimes AC} \neq \hat{\rho}_{C\otimes AB} \neq \hat{\rho}_{A\otimes B\otimes C}$$

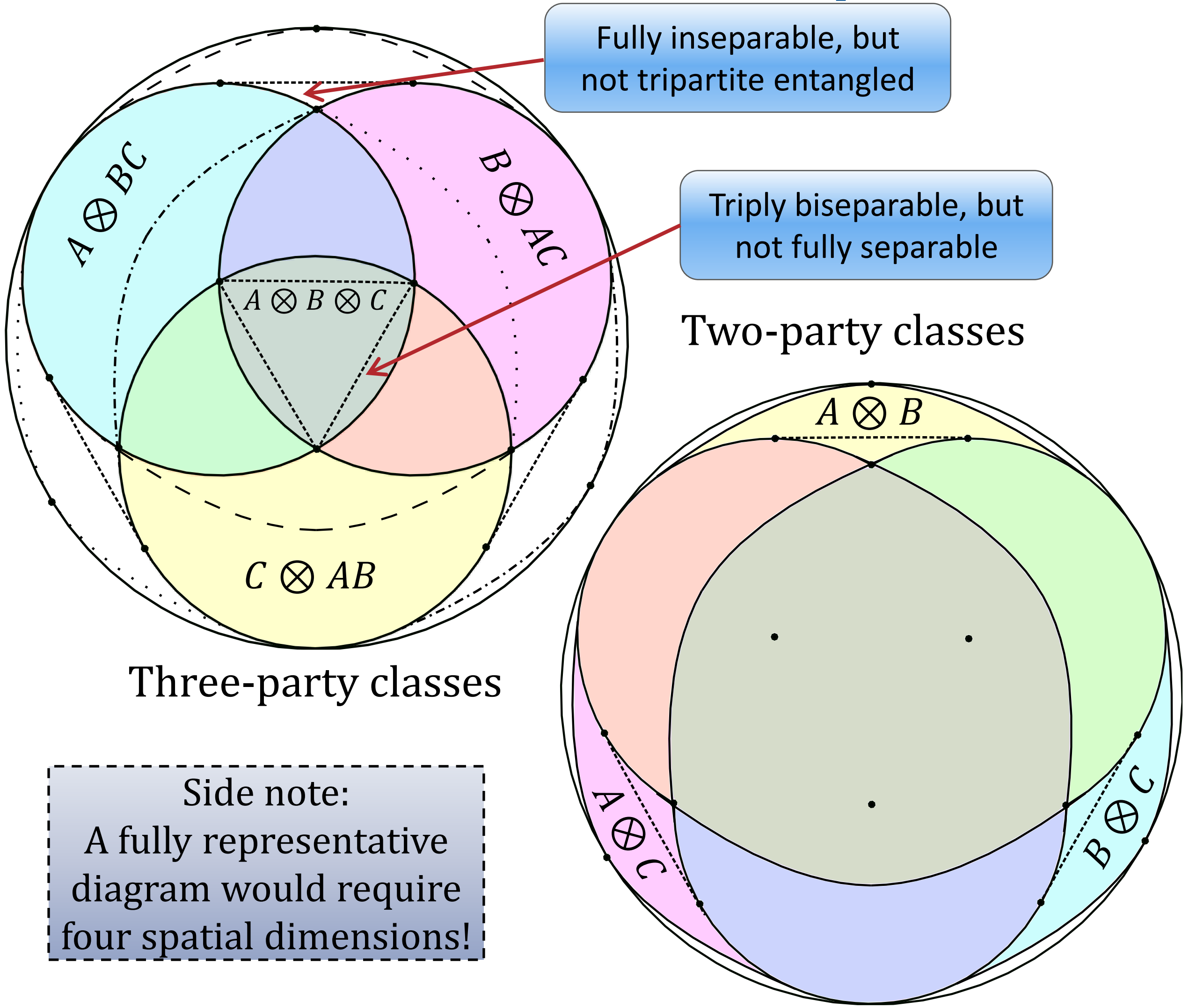
- Genuine tripartite-entangled

$$\hat{\rho}_{ABC} \neq p\,\hat{\rho}_{A\otimes BC} + q\,\hat{\rho}_{B\otimes AC} + r\,\hat{\rho}_{C\otimes AB} + s\,\hat{\rho}_{A\otimes B\otimes C}$$

$$: (p, q, r, s) \geq 0$$

$$p + q + r + s = 1$$

(So $\hat{\rho}_{ABC}$ is genuinely tripartite entangled iff it can't be derived from any combination of biseparable states



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Measuring Entanglement

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How do we measure entanglement?

Problems with the direct approach:

- Full state tomography is completely impractical at high-dimension / many qubits
  - An N-qubit system needs $4^N - 1$ measurements to completely determine the state:
    - A two-qubit system requires 15 measurements
    - A 12-qubit system requires over 1.6×10^7 measurements
    - A 52-qubit system requires over 2×10^{31} measurements
      - Note: 2×10^{31} is larger than the number of silicon atoms in the world's largest supercomputer!
      - Also: 2×10^{31} (32-bit) floating point numbers is over 80 trillion exabytes

- Computing entanglement measures is NP-hard
  - Completely intractable at high-dimension
    - This happens well before tomography becomes impractical

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Measuring Entanglement with Correlations

Solutions:

- Quantitative Entanglement witnesses:
  - Witnesses a minimum nonzero amount of entanglement

Bounding entanglement through correlations:

- Two-party entanglement from entropic uncertainty:

$$H(Q) = -\sum_i P(q_i)\log(p(q_i))$$

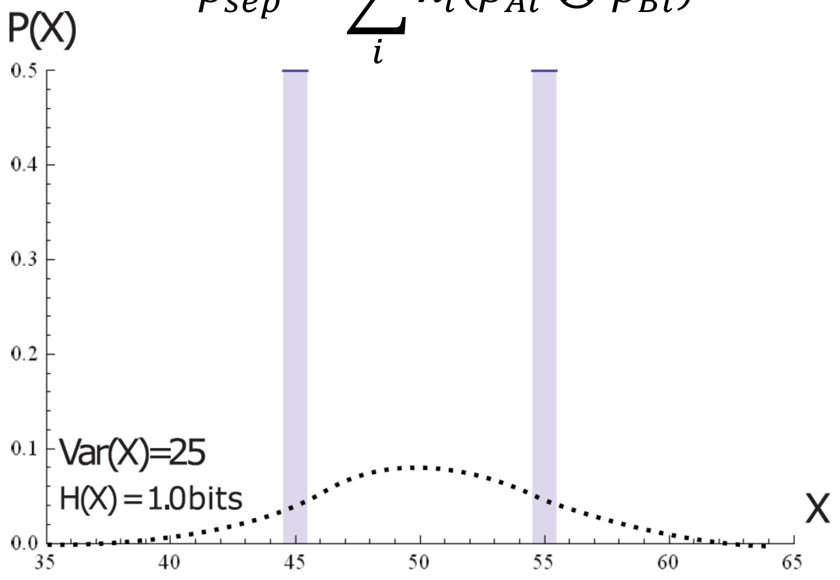
$$H(Q_A|Q_B) = H(Q_A,Q_B) - H(Q_B)$$

$$\Omega \equiv \min_{i,j}\left(\frac{1}{|\langle q_i|r_j\rangle|^2}\right) \in [0, N_A]$$

$$H(Q_A) + H(R_A) \geq \log(\Omega)$$

$$H(Q_A|\lambda) + H(R_A|\lambda) \geq \log(\Omega)$$

$$\hat{\rho}_{sep} = \sum_i \lambda_i(\hat{\rho}_{Ai} \otimes \hat{\rho}_{Bi})$$



Separable states: $H(Q_A|Q_B) + H(R_A|R_B) \geq \log(\Omega)$

All states: $H(Q_A|Q_B) + H(R_A|R_B) \geq 0$

Two-party correlations can be arbitrarily strong

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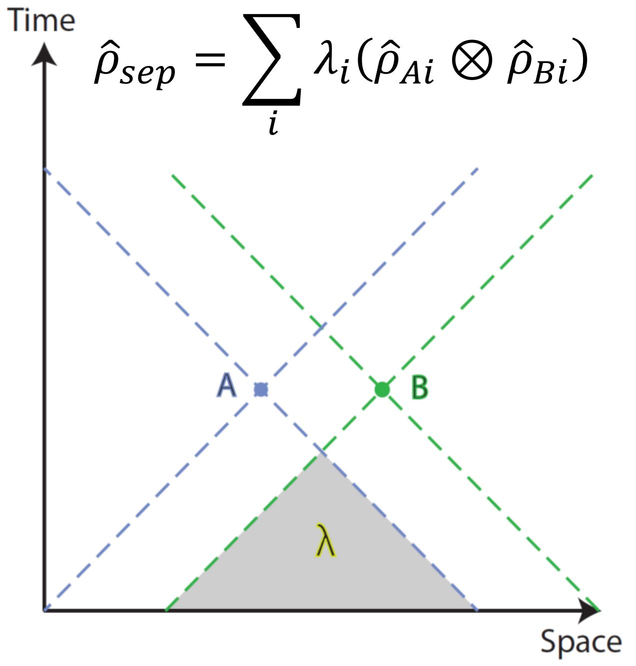

Separable states: $H(Q_A|Q_B) + H(R_A|R_B) \geq \log(\Omega)$

All states: $H(Q_A|Q_B) + H(R_A|R_B) \geq 0$

Two-party correlations can be arbitrarily strong

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Measuring Entanglement with Correlations

Solutions:

- Quantitative Entanglement witnesses:
  - Witnesses a minimum nonzero amount of entanglement

$$h(x) = -\int dx \rho(x) \log(\rho(x))$$

Bounding entanglement through correlations:

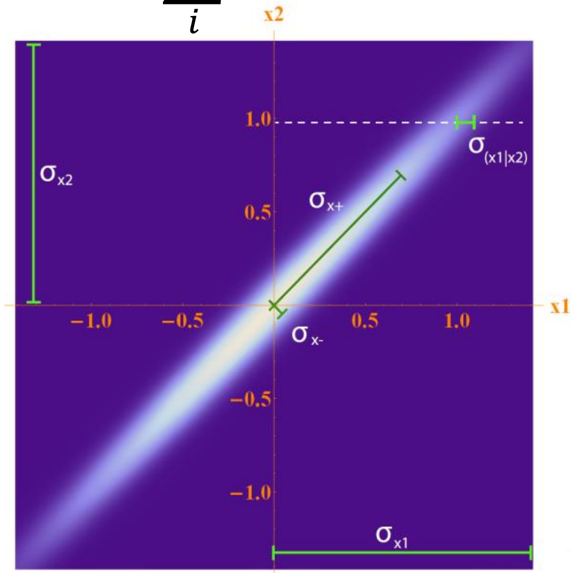
$$h(x_A|x_B) = h(x_A, x_B) - h(x_B)$$

- Two-party entanglement from entropic uncertainty:

$$h(x_A) + h(k_A) \geq \log(\pi e)$$

$$h(x_A|\lambda) + h(k_A|\lambda) \geq \log(\pi e)$$

$$\hat{\rho}_{sep} = \sum_i \lambda_i (\hat{\rho}_{Ai} \otimes \hat{\rho}_{Bi})$$



Separable states: $h(x_A|x_B) + h(k_A|k_B) \geq \log(\pi e)$

All states: $h(x_A|x_B) + h(k_A|k_B) > -\infty$

Two-party correlations can be arbitrarily strong

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Measuring Entanglement with Correlations

Solutions:

- Quantitative Entanglement witnesses:
  - Witnesses a minimum nonzero amount of entanglement

Bounding entanglement through correlations:

- Two-party entanglement from entropic uncertainty:

$$H(Q_A|Q_B) + H(R_A|R_B) \geq \log(\Omega) + S(A|B)$$

$$h(x_A|x_B) + h(k_A|k_B) \geq \log(2\pi) + S(A|B)$$

$$\{E_F, E_{RE}, E_{SQ}\} \geq \max\{0, -S(A|B), -S(B|A)\}$$

$$E_D \geq \max\left\{0, \frac{-S(A|B) - S(B|A)}{2}\right\}$$

$$H(Q) = -\sum_i P(q_i)\log\big(p(q_i)\big)$$

$$H(Q_A|Q_B) = H(Q_A, Q_B) - H(Q_B)$$

$$S(A) = -\mathrm{Tr}[\hat{\rho}_A \log(\hat{\rho}_A)]$$

$$S(A|B) = S(AB) - S(B)$$

$$\hat{\rho}_{sep} = \sum_i \lambda_i(\hat{\rho}_{Ai} \otimes \hat{\rho}_{Bi})$$

$$\Omega \equiv \min_{i,j}\left(\frac{1}{|\langle q_i|r_j\rangle|^2}\right) \in [0, N_A]$$

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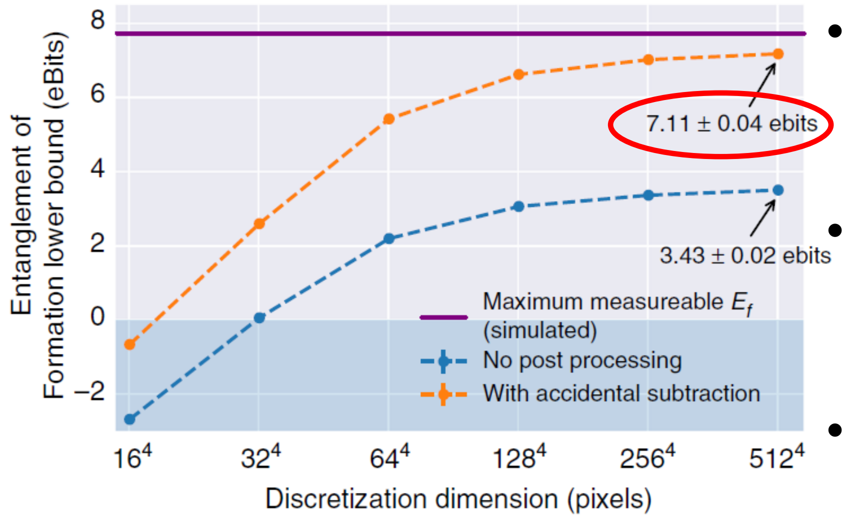
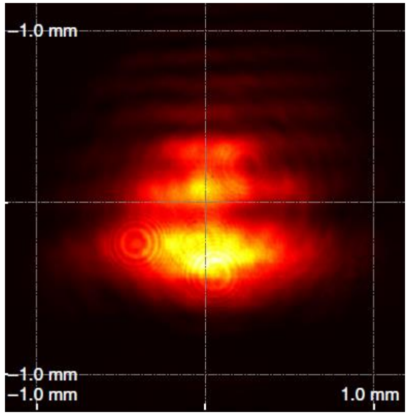
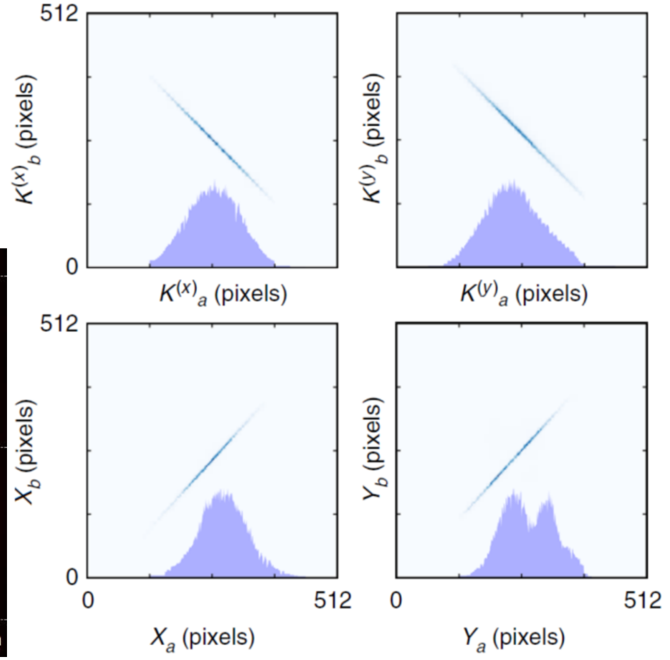
Measuring Entanglement with Correlations

This can be a very successful approach! (currently Record-setting)



- Measured position and momentum correlations of entangled down-converted photon pairs
- Course graining never over-estimates bound
- Needed only 6456 measurements of 68-billion-dimensional state space





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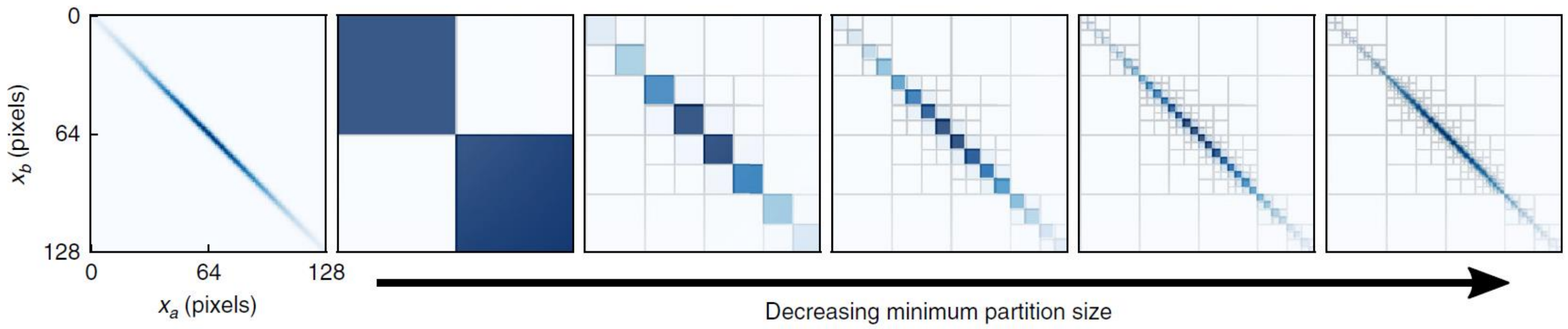
Quantifying entanglement in a 68-billion-dimensional quantum state space

James Schneeloch[1,5], Christopher C. Tison[1,2,3], Michael L. Fanto[1,4], Paul M. Alsing[1] & Gregory A. Howland[1,4,5]



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From two parties to three

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Measures of Three-Party Entanglement

Standard Approach: Residual entanglement

- The entanglement left over when you take away all two-party entanglement

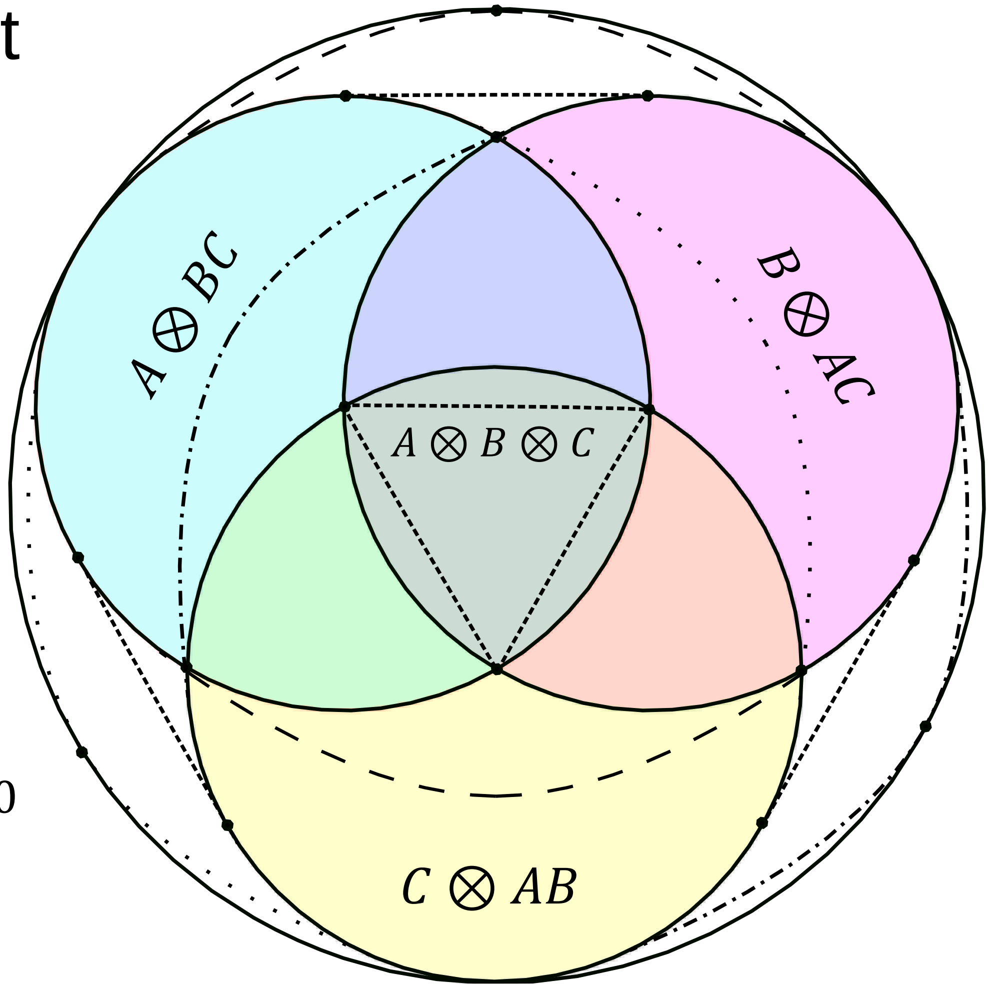
(the three-tangle)

$$\tau_{ABC} \equiv E(A{:}BC) - E(A{:}B) - E(A{:}C)$$

The underlying idea:

- Any entangled three-party state without any two-party entanglement, must be tri-partite entangled.
  - Demonstrably true for pure states.
    - Pure state: $S(ABC) = 0\ldots$
    - No 2-party entanglement: $\{S(A|B), S(B|C), S(C|A)\} \geq 0$
    - Together shows: $S(A) = S(B) = S(C)$
    - Giving result either $|\psi\rangle_{ABC} = |\mu\rangle_A \otimes |\nu\rangle_B \otimes |\phi\rangle_C$

      or $\{S(A|BC), S(B|AC), S(C|AB)\} < 0$



Three-party classes

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Measures of Three-Party Entanglement

Issues with Residual entanglement

(the three-tangle)

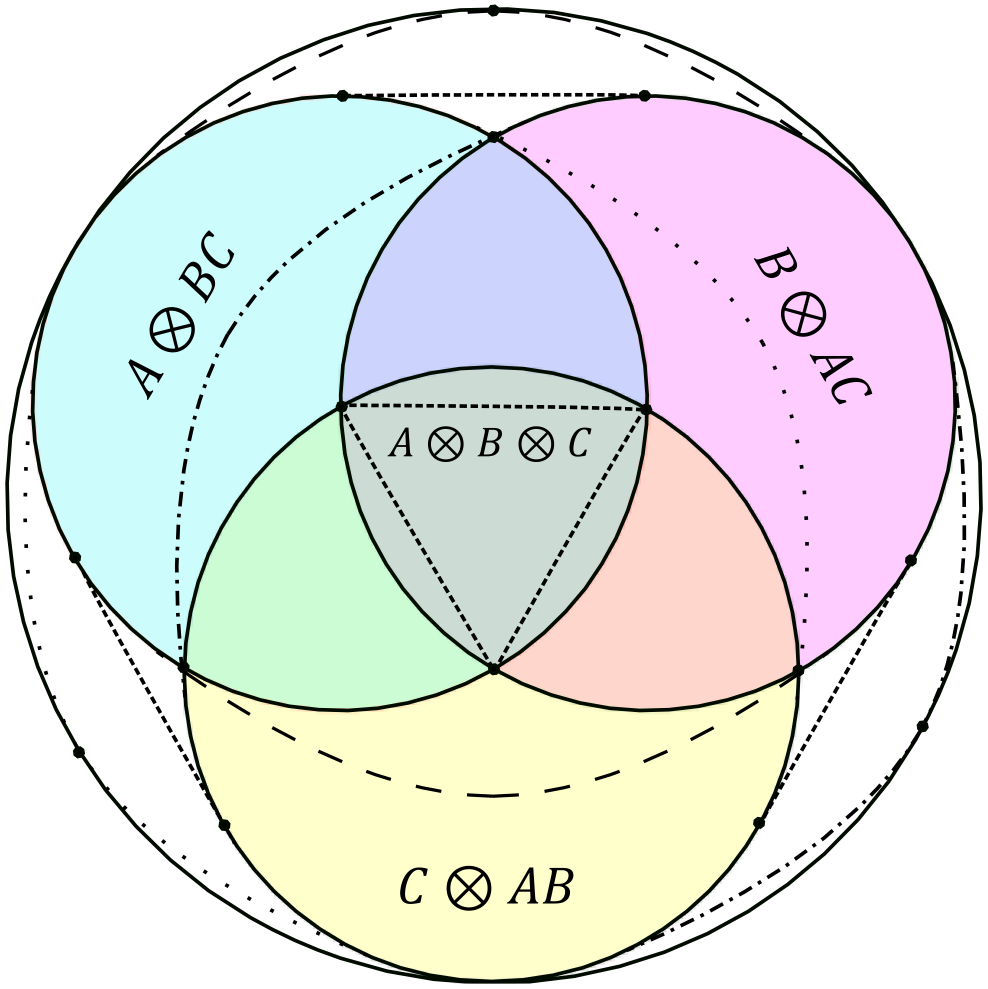
$$\tau_{ABC} \equiv E(A\!:\!BC) - E(A\!:\!B) - E(A\!:\!C)$$

The underlying idea:

- Any entangled three-party state without any two-party entanglement, must be tri-partite entangled.
  - True **only** for pure states.
    - Counter-example mixed state:

$$\hat{\rho}_{ABC} = \frac{1}{3}\left(|\Phi^+\rangle\langle\Phi^+|_{AB} \otimes |0\rangle\langle 0|_C + |\Phi^+\rangle\langle\Phi^+|_{AC} \otimes |0\rangle\langle 0|_B + |\Phi^+\rangle\langle\Phi^+|_{BC} \otimes |0\rangle\langle 0|_A\right)$$

      - Biseparable (by construction)
      - No two-party entanglement
      - Is entangled (across all three bipartitions)
  - Only valid for some entanglement measures
  - Not additive : $\tau_{ABC}(\hat{\rho} \otimes \hat{\sigma}) \neq \tau_{ABC}(\hat{\rho}) + \tau_{ABC}(\hat{\sigma})$
  - Zero for some tripartite-entangled states (e.g., $|W\rangle\langle W|$)



Three-party classes

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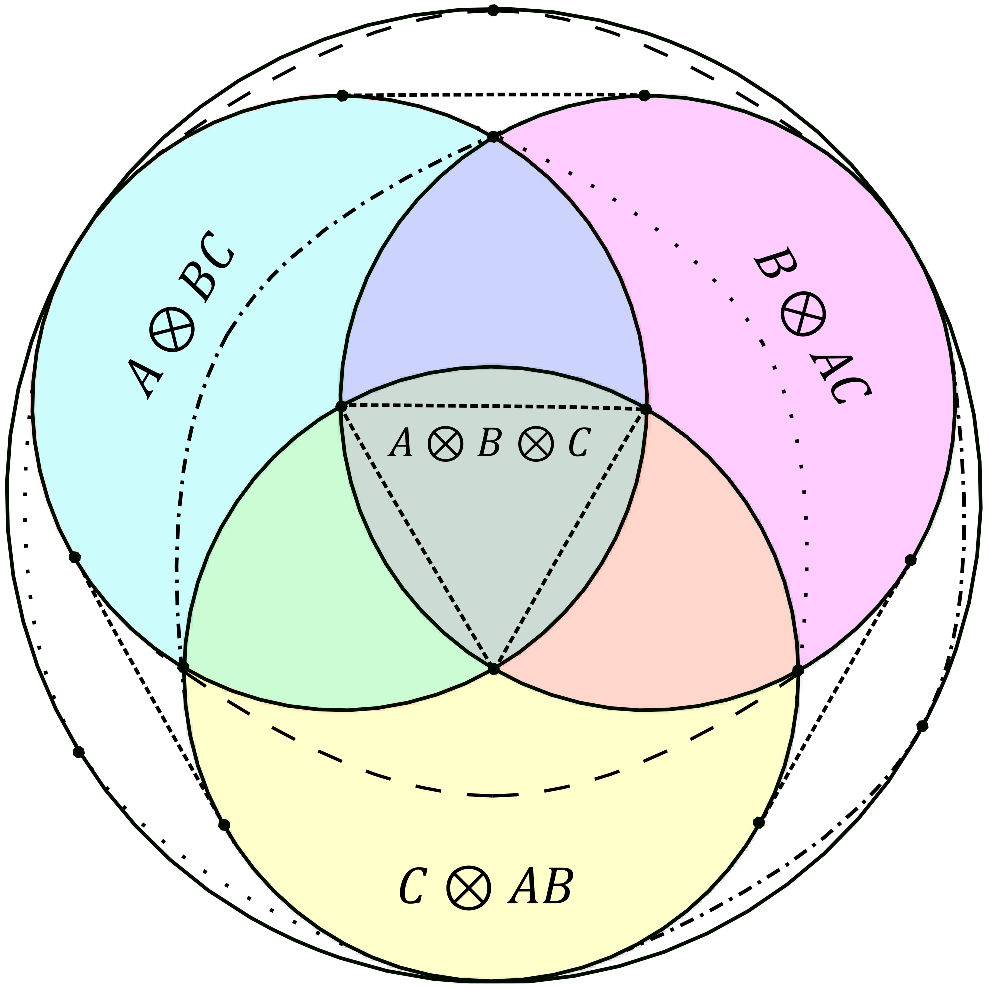
Measures of Three-Party Entanglement

The Tripartite entanglement of formation

$$E_{3F}(\hat{\rho}_{ABC}) \equiv \min_{|\psi_i\rangle} \sum_i p_i \min\{S_i(A), S_i(B), S_i(C)\}$$

- Zero iff $\hat{\rho}_{ABC}$ is biseparable
- Monotonically decreases under LOCC
- Additive on tensor products of pure states
  - Can compare tripartite entanglement in a few high-dimensional systems to that in many low dimensional systems
  - New unit of multi-partite entanglement: the three-party **gebit**, or 3-qubit GHZ state
- Straightforward to compute for pure states
- Straightforward to bound for mixed states



Three-party classes

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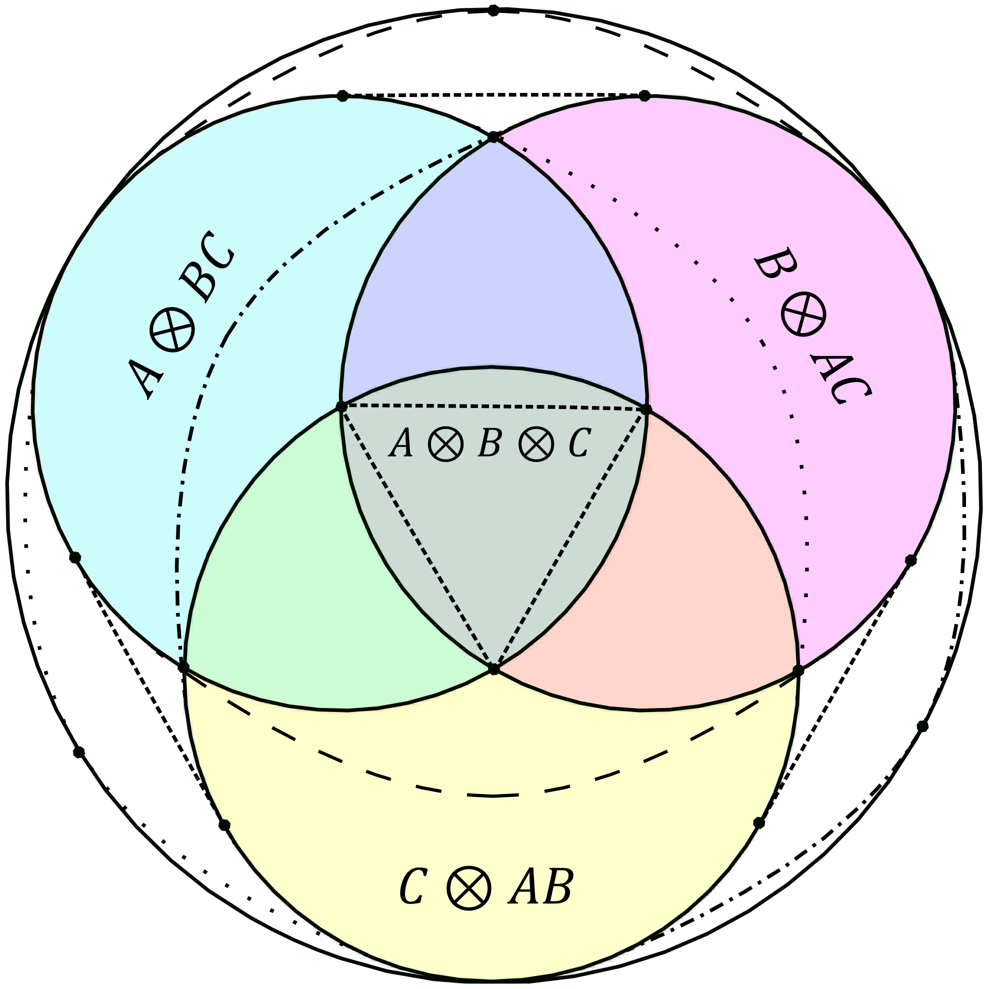
Measures of Three-Party Entanglement

The Tripartite entanglement of formation

$$E_{3F}(\hat{\rho}_{ABC}) \equiv \min_{|\psi_i\rangle} \sum_i p_i \min\{S_i(A), S_i(B), S_i(C)\}$$

Fundamental issues with gebits:

- There's no known set of entangled states that can synthesize all three-party states with LOCC
- The underlying protocol needs rigor
  - Are we saying we need this many gebits along with other resources to make the state?
  - What about distillability?



Three-party classes

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Bounding Three-Party Entanglement

The Tripartite entanglement of formation

$$E_{3F}(\hat{\rho}_{ABC}) \equiv \min_{|\psi_i\rangle} \sum_i p_i \min\{S_i(A), S_i(B), S_i(C)\}$$

- Easy to bound for pure states:

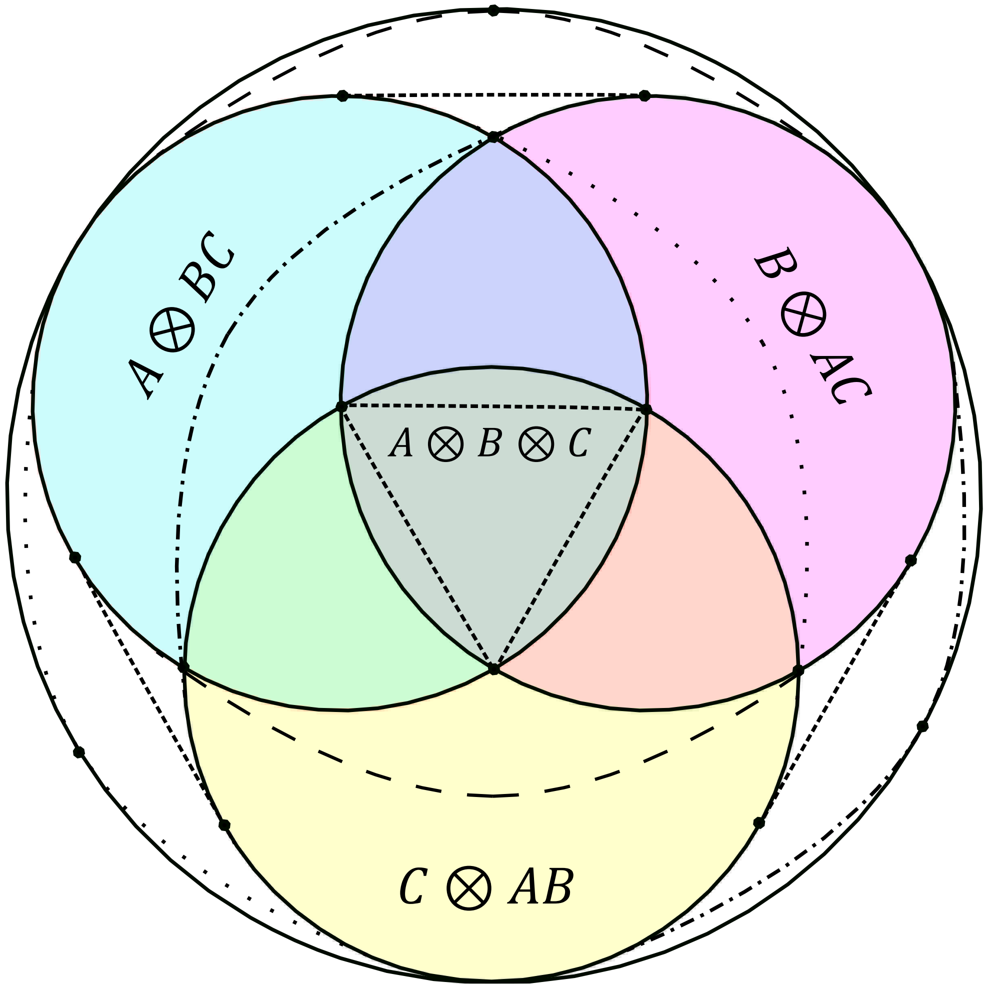
$$E_{3F}(|\psi\rangle_{ABC}) = \min\{-S(A|BC), -S(B|AC), -S(C|AB)\}$$

$$H(Q_A|Q_B, Q_C) + H(R_A|R_B, R_C) \geq \log(\Omega) + S(A|BC)$$

- Challenging, but possible to bound for mixed states:

$$E_{3F} \geq -S(A|BC) - S(B|AC) - S(C|AB) - 2\log(D_{max})$$

- Inequality is tight



Three-party classes

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How successful is our strategy?

GHZ-Werner state:

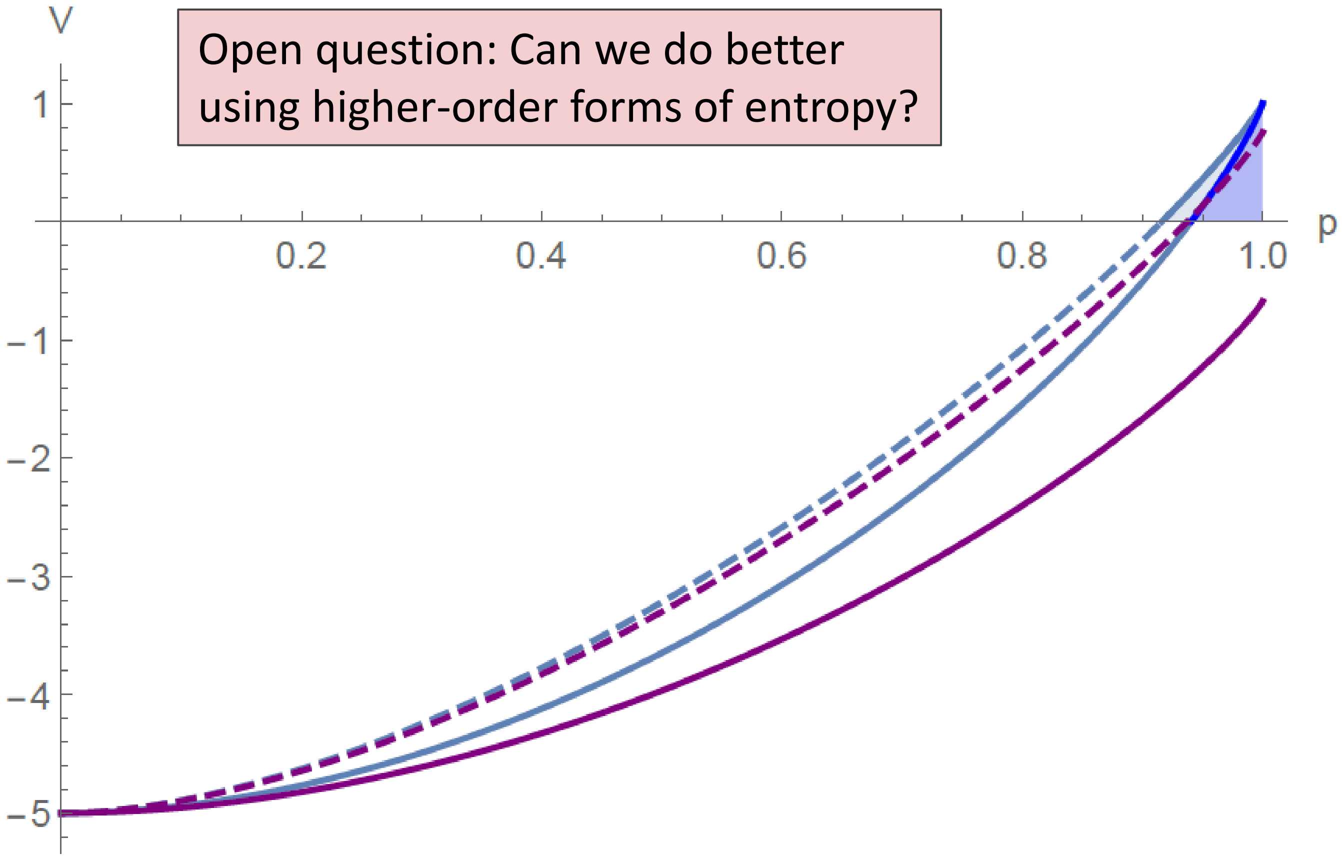
$$\hat{\rho}_{GHZW} = p\,|GHZ\rangle\langle GHZ| + (1-p)\hat{\rho}_{MM}$$

- Test: Measure spin correlations in σ_x and σ_z.
  - Successful witness for $p > 0.9406$
- Test: Directly calculating quantum entropies
  - Successful witness for $p > 0.9161$
- Quantifies all entanglement as $p \rightarrow 1$

W-Werner state:

$$\hat{\rho}_{WW} = p\,|W\rangle\langle W| + (1-p)\hat{\rho}_{MM}$$

- Test: Measure spin correlations in σ_x and σ_z.
  - No witnessing accomplished
- Test: Directly calculating quantum entropies
  - Successful witness for $p > 0.9374$
- Quantifies most entanglement as $p \rightarrow 1$



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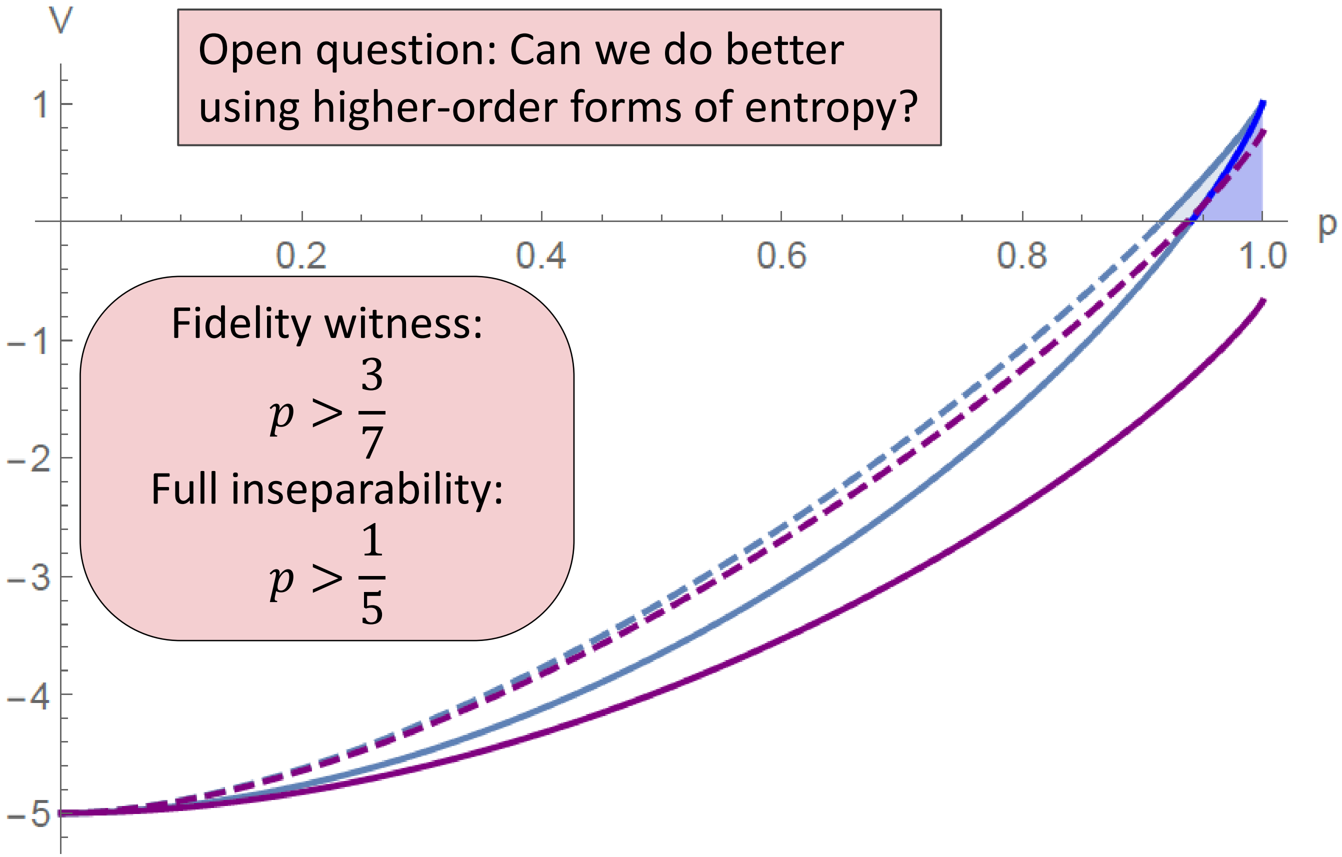
- Test: Measure spin correlations in σ_x and σ_z.
  - Successful witness for $p > 0.9406$
- Test: Directly calculating quantum entropies
  - Successful witness for $p > 0.9161$
- Quantifies all entanglement as $p \to 1$

W-Werner state:

$$\hat{\rho}_{WW} = p\,|W\rangle\langle W| + (1-p)\hat{\rho}_{MM}$$

- Test: Measure spin correlations in σ_x and σ_z.
  - No witnessing accomplished
- Test: Directly calculating quantum entropies
  - Successful witness for $p > 0.9374$
- Quantifies most entanglement as $p \to 1$



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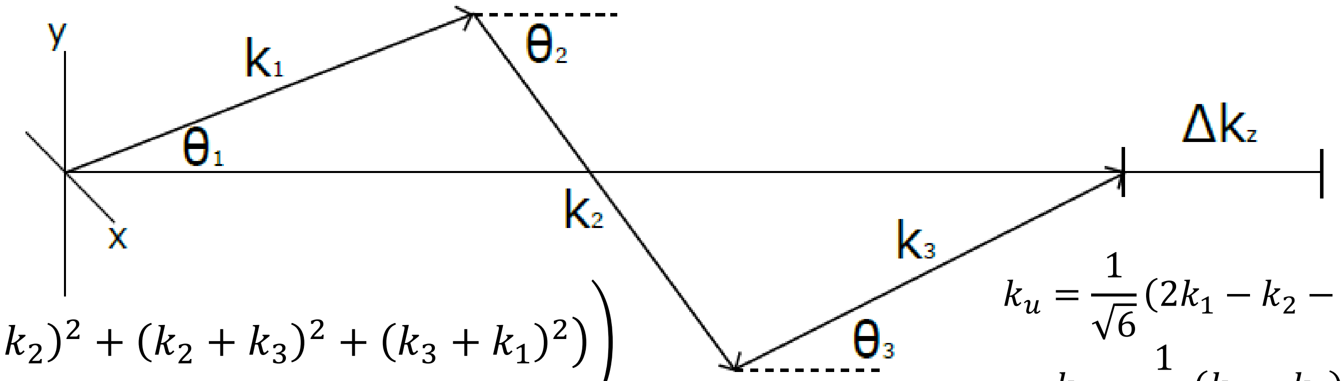
How successful is our strategy? (Examples)

- Tri-photon pure state wavefunction from third-order SPDC



$$a = \frac{3L_z\lambda_p}{8\pi n_p}$$

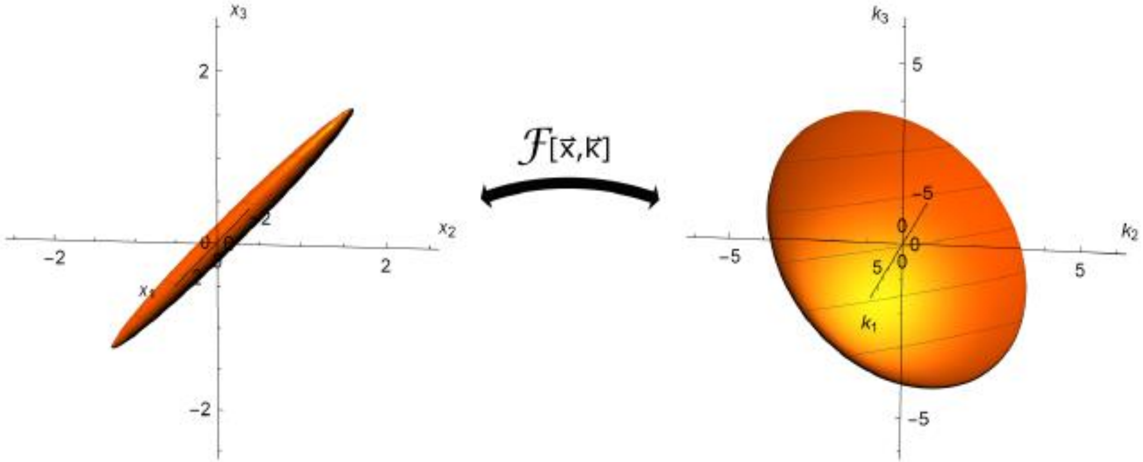
$$\psi(k_1,k_2,k_3) = N\psi_p(k_1+k_2+k_3)Sinc\left(\frac{3L_z}{4k_p}\left((k_1+k_2)^2+(k_2+k_3)^2+(k_3+k_1)^2\right)\right)$$

$$\psi(k_1,k_2,k_3) \approx N\ Exp[-\frac{8a}{9}(k_u^2+k_v^2)\ -\left(\sigma_p^2+\frac{32a}{9}\right)k_w^2]$$

$$k_u = \frac{1}{\sqrt{6}}(2k_1-k_2-k_3)$$

$$k_v = \frac{1}{\sqrt{2}}(k_2-k_3)$$

$$k_w = \frac{1}{\sqrt{3}}(k_1+k_2+k_3)$$



For pure states of arbitrary dimension:

$$E_{3F}(ABC) \geq \min\{-S(A|BC), -S(B|CA), -S(C|AB)\}$$

Classical entropies (measured experimentally) can be used to bound quantum entropies:

$$h(x_a|x_b,x_c) + h(k_a|k_b,k_c) \geq \log(2\pi) + S(A|BC)$$

$$H(X_A|X_B,X_C) + H(K_A|K_B,K_C) \geq \log\left(\frac{2\pi}{\Delta x \Delta k}\right) + S(A|BC)$$

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How successful is our strategy? (Examples)

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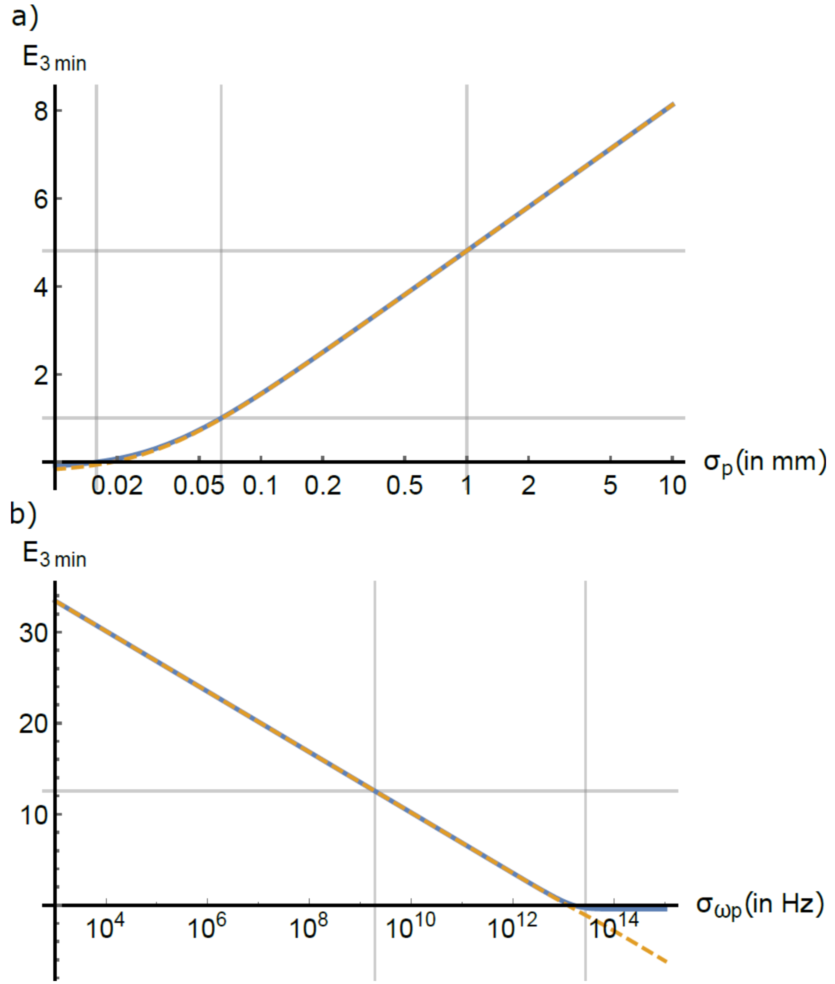
$$a = \frac{3L_z\lambda_p}{8\pi n_p}$$

$$\psi(k_1, k_2, k_3) \approx N\ Exp[-\frac{8a}{9}(k_u^2 + k_v^2) - \left(\sigma_p^2 + \frac{32a}{9}\right)k_w^2]$$

Example: AlN crystal
$L_z = 10\text{mm}$,
$\lambda_p = 325\text{nm}$,
$\sigma_{\omega_p} = 1.9\text{GHz}$
$\sigma_p = 1.0\text{mm}$
Degenerate collinear triplets at 975nm.



In position-momentum:

- Minimum E_{3F} is: 4.808 3-party gebits in one spatial degree of freedom
  - That's more tripartite entanglement than can be supported on a 14-qubit state space
  - With both transverse degrees, this doubles!

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How successful is our strategy? (Examples)

- Tri-photon pure state wavefunction from third-order SPDC

$$a = \frac{3L_z\lambda_p}{8\pi n_p}$$

$$\psi(k_1,k_2,k_3) \approx N\,Exp[-\frac{8a}{9}(k_u^2+k_v^2) - \left(\sigma_p^2+\frac{32a}{9}\right)k_w^2]$$

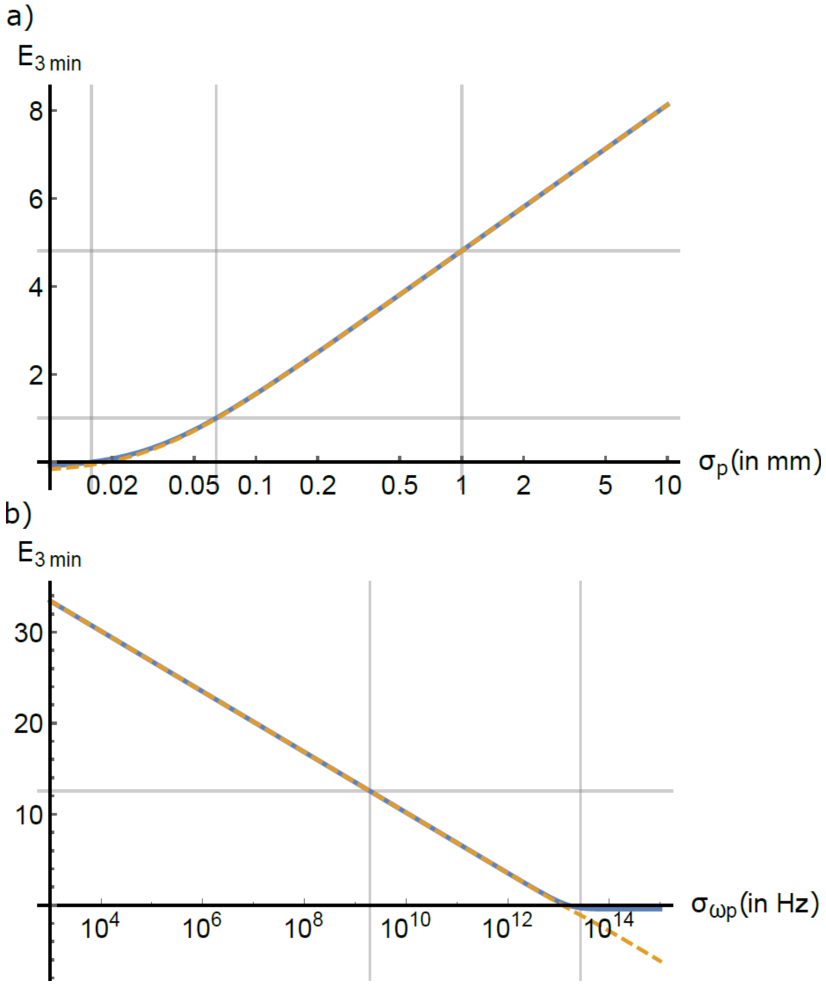
Example: AlN crystal
$L_z = 10\text{mm}$,
$\lambda_p = 325\text{nm}$,
$\sigma_{\omega_p} = 1.9\text{GHz}$
$\sigma_p = 1.0\text{mm}$
Degenerate collinear triplets at 975nm.

In Energy-time:

- Minimum E_{3F} is: 12.54 3-party gebits
  - More tripartite entanglement than can be supported on a 37-qubit state space!
  - Note: $2^{37} \approx 137.$ billion.

In position-momentum:

- Minimum E_{3F} is: 4.808 3-party gebits in one spatial degree of freedom
  - That's more tripartite entanglement than can be supported on a 14-qubit state space
  - With both transverse degrees, this doubles!



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From three parties to N

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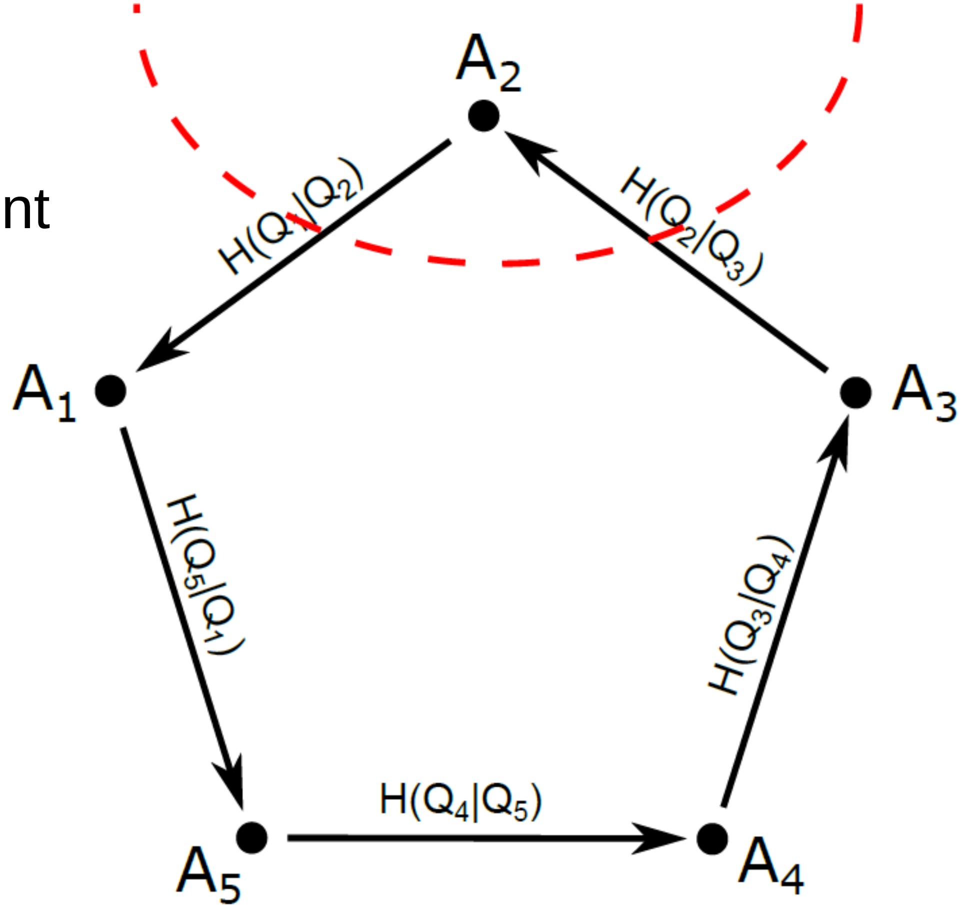
Witnesses for N Parties

Cyclic entropic correlation:

$$\sum_{i=1}^{N}\left(H(Q_i|Q_{i+1}) + H\left(R_i\middle|R_{i+1},\ldots,R_{i+(N-1)}\right)\right) \geq 2\log(\Omega)$$

- Violation witnesses genuine N-partite entanglement



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Witnesses for N Parties

Cyclic entropic correlation:

$$\sum_{i=1}^{N}\left(H(Q_i|Q_{i+1}) + H\left(R_i\middle|R_{i+1},\ldots,R_{i+(N-1)}\right)\right) \geq 2\log(\Omega)$$

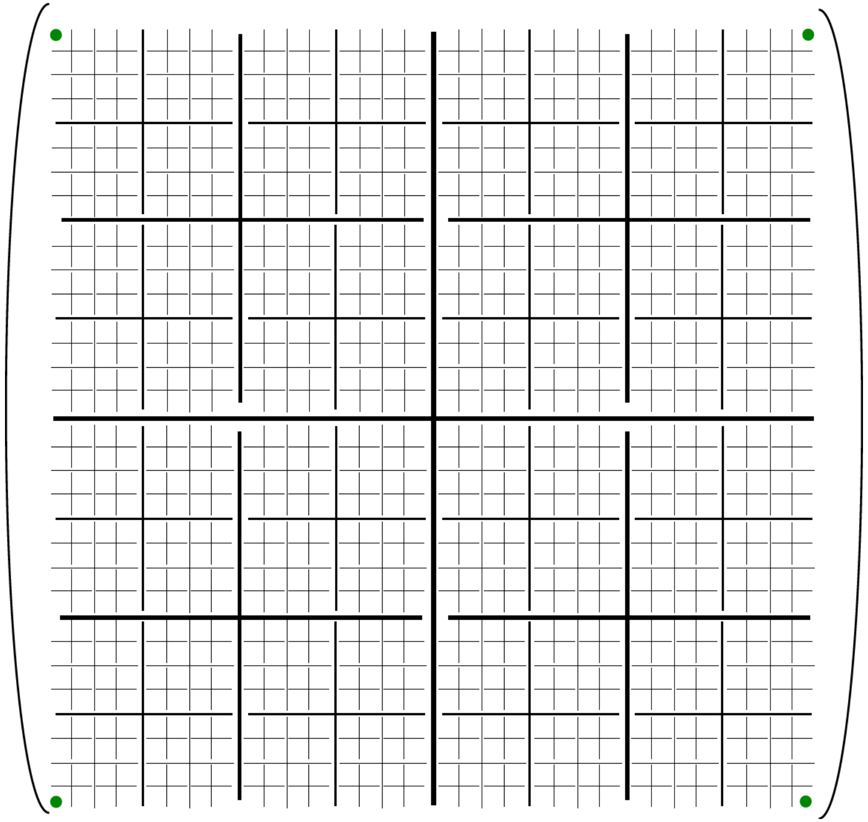
- Violation witnesses genuine N-partite entanglement

Four-corners method:

$$\mathcal{B} \geq \left|\langle 0^{\otimes N}|\hat{\rho}|0^{\otimes N}\rangle\right| + \left|\langle 0^{\otimes N}|\hat{\rho}|1^{\otimes N}\rangle\right| + \left|\langle 1^{\otimes N}|\hat{\rho}|0^{\otimes N}\rangle\right| + \left|\langle 1^{\otimes N}|\hat{\rho}|1^{\otimes N}\rangle\right| - 1$$

$$E_{NF}(\hat{\rho}) \geq -\log_2\left(1 - \frac{\mathcal{B}^2}{2}\right)$$

- Works for GHZ-Werner state for $p > \frac{3}{7}$



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The entanglement-correlation connection

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Quantum Uncertainty limits N-partite correlations

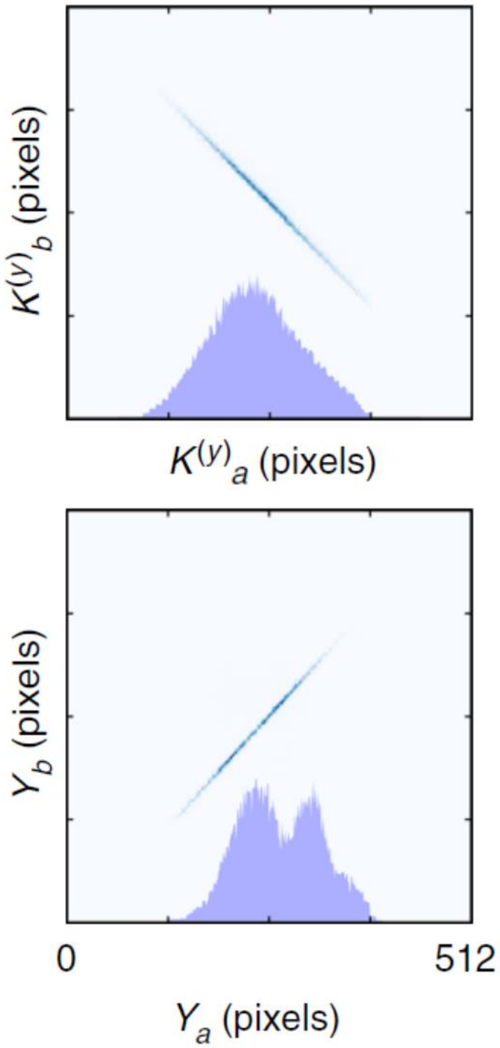
Two parties:

- The entropic uncertainty principle between maximally uncertain $\hat{Q}$ and $\hat{R}$ (and maximum entanglement):

$$H(Q_A|Q_B) + H(R_A|R_B) \geq 0$$

$$h(x_A|x_B) + h(k_A|k_B) > -\infty$$

(No upper limit to correlations between two parties)



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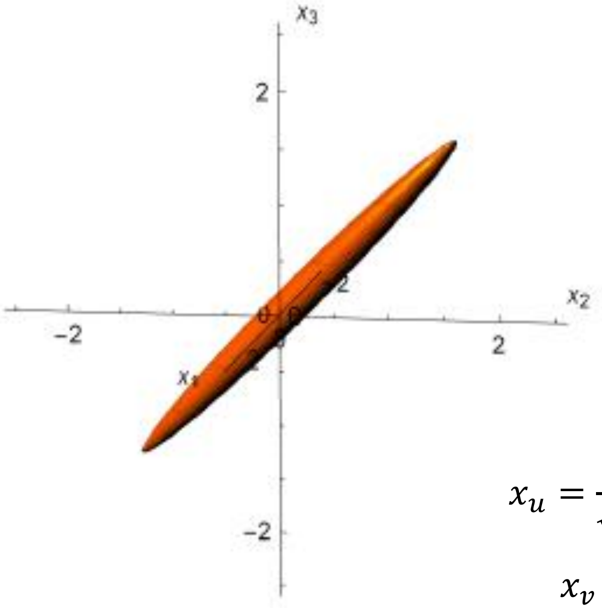
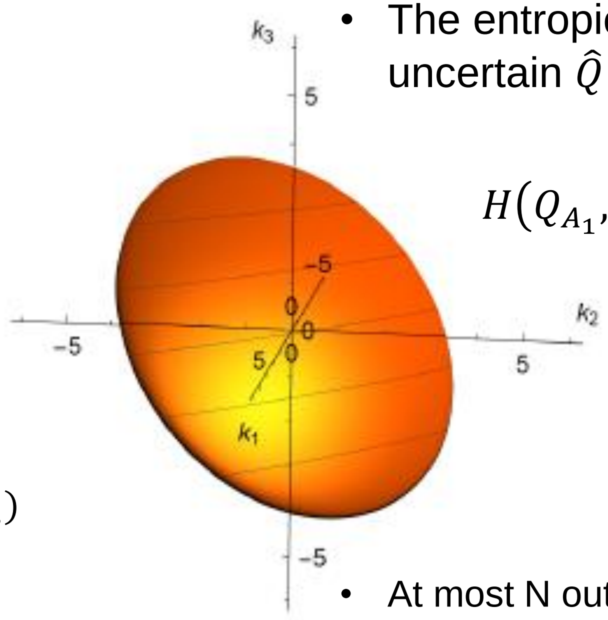
Quantum Uncertainty limits N-partite correlations





$$x_u = \frac{1}{\sqrt{6}}(2x_1 - x_2 - x_3)$$
$$x_v = \frac{1}{\sqrt{2}}(x_2 - x_3)$$
$$x_w = \frac{1}{\sqrt{3}}(x_1 + x_2 + x_3)$$

Three or more parties:

- The entropic uncertainty principle between maximally uncertain $\hat{Q}$ and $\hat{R}$ (and maximum entanglement):

$$H\left(Q_{A_1}, Q_{A_2} \middle| Q_B\right) + H\left(R_{A_1}, R_{A_2} \middle| R_B\right) \geq \log(D)$$

$$h(x_u) + h(k_u) \geq \log(\pi e)$$
$$h(x_v) + h(k_v) \geq \log(\pi e)$$
$$h(x_w) + h(k_w) \geq \log(\pi e)$$

- At most N out of N pairs of conjugate observables can be determined through correlation
  - So with perfect correlations in Q (one determines the rest)…
  - …the best-case correlations in R are where $N-1$ determines last

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Conclusion

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Concluding points

- Entanglement is only defined from separability
  - Multiple forms of separability → Multiple forms of entanglement
- Entanglement can be efficiently quantified through correlations
- There are resource-based measures of multi-partite entanglement that can also be quantified by correlations
- The relationship between entanglement and correlation is different for more parties

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Thanks for listening!

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Contingency slide: Four Corners Method

See PRA 86, 022319 (2012)

- The minimum quantum entropy over all bipartite splits measures multipartite entanglement:

$$\text{Bound } \mathcal{B} \leq E_M(\hat{\rho}) = \min_{|\psi\rangle} \sum_i p_i \min_{\gamma} \sqrt{S_L\left(\hat{\rho}_{\gamma i}\right)}$$

$S_R(\hat{\rho}) = -\log_2\left(1 - \frac{S_L(\hat{\rho})}{2}\right)$ is a concave-up function of $\sqrt{S_L(\hat{\rho})}$, so…

$$-\log_2\left(1 - \frac{\mathcal{B}^2}{2}\right) \leq \min_{|\psi\rangle} \sum_i p_i \min_{\gamma} S_R\left(\hat{\rho}_{\gamma i}\right)$$

$$\min_{|\psi\rangle} \sum_i p_i \min_{\gamma} S_R\left(\hat{\rho}_{\gamma i}\right) \leq \min_{|\psi\rangle} \sum_i p_i \min_{\gamma} S\left(\hat{\rho}_{\gamma i}\right) = E_{NF}(\hat{\rho})$$

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Contingency slide 2: Four Corners Method

See PRA 86, 022319 (2012)

- Example bound (PRA 83, 062325 (2011)):

$$\mathcal{B} = 2\left|\langle 0^{\otimes N}|\hat{\rho}|1^{\otimes N}\rangle\right| - \sum_{q=1}^{2^N-2}\sqrt{\langle q|\hat{\rho}|q\rangle\langle 2^N-1-q|\hat{\rho}|2^N-1-q\rangle}$$

What is $|q\rangle$?

Example: 6 qubits, and q=17

In binary, $17 \rightarrow 10001$

and $|17\rangle \rightarrow |0,1,0,0,0,1\rangle$

Example state: GHZ-Werner of N-qubits

$$\hat{\rho}_{GW} = p|GHZ\rangle\langle GHZ| + (1-p)\hat{\rho}_{MM}$$

$\mathcal{B} > 0$ for $p > \frac{2^{N-1}-1}{2^N-1}$ e.g., $> \frac{3}{7}$ for 3-qubit $\hat{\rho}_{GHZW}$

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Contingency slide 3: Multipartite negativity of GHZ-Werner state

The N-partite Negativity

$$\mathcal{N}_N(\hat{\rho}) \equiv \min_{|\psi_i\rangle} \sum_i p_i \min_{\alpha} \left\{ \mathcal{N}_i \left(\hat{\rho}_{\alpha|\bar{\alpha}} \right) \right\}$$

N-qubit GHZ-Werner state:

$$\hat{\rho}_{GHZW} = p\,|GHZ\rangle\langle GHZ| + (1-p)\hat{\rho}_{MM}$$

- Fully separable under partial trace
- Set of eigenvalues of partial transpose is constant over all possible partial transposes:

$$\vec{\lambda} = \left(\frac{1-p}{2^N}, \ldots, \frac{1-p}{2^N}, \frac{1+(2^N-1)p}{2^N} \right)$$

$$\vec{\lambda}_{PT} = \left(\frac{1-p}{2^N}, \ldots, \frac{1-p}{2^N}, \frac{1-(2^{N-1}+1)p}{2^N}, \frac{1+(2^{N-1}-1)p}{2^N} \right)$$

- Fully inseparable for $p > \frac{1}{1+2^{N-1}}$

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