Witnessing Continuous Variable Einstein-Podolsky-Rosen Steering with Discrete Measurements

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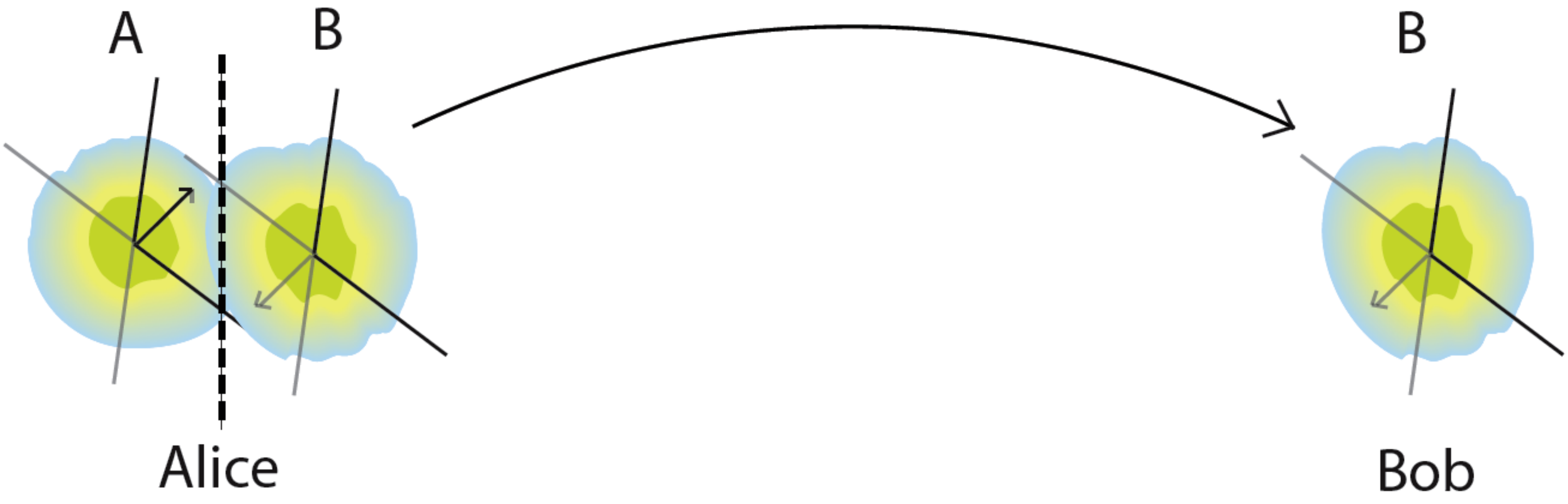
- What our group has done:
  - We've found strong and intuitive inequalities that witness CV entanglement and EPR-steering correlations in the lab.
    - And used them successfully too!
- This is important because:
  - Determining $\hat{\rho}^{AB}$ (by tomography) gets exceptionally hard for high-dimensional systems.
    - Scales as N^4 in needed number of measurements.
  - Sometimes you want to witness more than entanglement.
    - EPR-steering
    - Bell nonlocality

What is EPR-steering?

- It is an intermediate degree of nonlocal correlation.
  - Bell correlations
    - Rules out all LHVs
  - **EPR steering correlations**
    - Rules out some LHV's
    - demonstrates EPR paradox
  - Entanglement correlations
    - Doesn't rule out LHV's
    - Accepted resource in many QI tasks (e.g. superdense coding)
  - Nonclassical correlations
    - Any correlations necessarily destroyed by local measurement .

- Why care about demonstrating EPR-steering?
  - You can verify entanglement **even when one party's measurements are untrusted!**

The situation in EPR-steering



How can Alice prove there's entanglement?

•**If Alice were preparing and sending states to Bob, the measurement correlations could only be so high**.

- Bob could tell Alice to measure $\vec{x}$, even though she sent a state with definite $\vec{k}$.

•An **EPR-steering inequality** gives a bound to these local correlations.

EPR-Steering Inequalities

- (1989) M.D. Reid [1]

  from $\Delta^2(x_B)\Delta^2(k_B) \geq \frac{1}{4}$

  she showed $\Delta^2_{inf}(x_B|x_A)\,\Delta^2_{inf}(k_B|k_A) \geq \frac{1}{4}$

- (2011) S.P. Walborn et. al [2]

  from $h(x_B) + h(k_B) \geq \log(\pi e)$

  they showed $h(x_B|x_A) + h(k_B|k_A) \geq \log(\pi e)$

  (state of the art for CV EPR-steering)

$h(x) = \int dx\, \rho(x) \log\big(\rho(x)\big)$ $h(x_B|x_A) = h(x_A, x_B) - h(x_A)$

Limitations of the state of the art

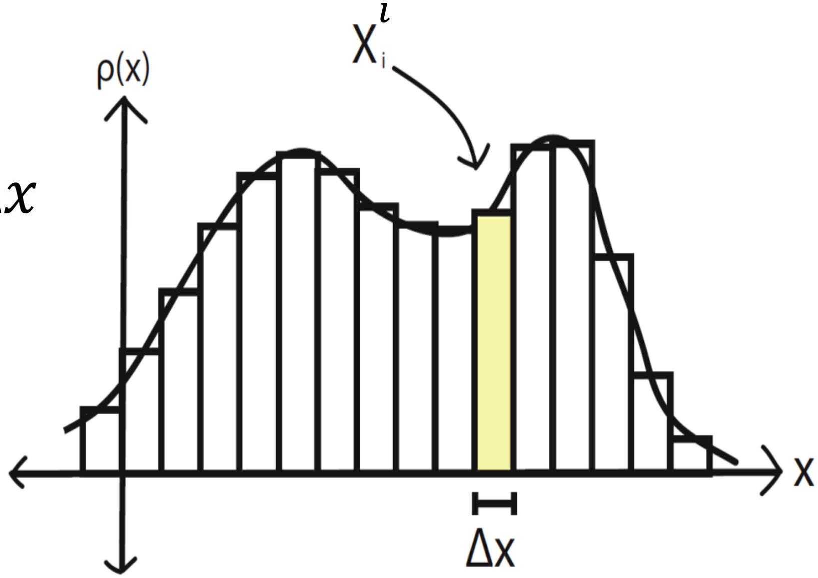
$$h(x_B|x_A) + h(k_B|k_A) \geq \log(\pi e)$$

- Need to know $\rho(x_A, x_B)$ and $\rho(k_A, k_B)$.
- Approximating continuous entropies less useful for experiment
  - No rigorous demonstration

    $$h(x) \approx \log(\Delta x) + H(X)$$

    - x is cut into windows X of size Δx
    - Approximation only good at high resolution
- State of the art is excellent for theoretical investigations

$$H(X) = -\sum_i P(X_i)\log(P(X_i))$$



Our first Inequality

- It is known that..

$$h(x) \leq \log(\Delta x) + H(X)$$

(Bialynicki-Birula 1985) [3]

  - And that..

$$h(x_B|x_A) + h(k_B|k_A) \geq \log(\pi e)$$

(Walborn et. al 2011) [2]

- We proved..

$$h(x|y) \leq \log(\Delta x) + H(X|Y)$$

- Which gave us our first result

$$\boxed{H(X_B|X_A) + H(K_B|K_A) \geq \log\left(\frac{\pi e}{\Delta x_B \Delta k_B}\right)}$$

$$H(X_B|X_A) + H(K_B|K_A) \geq \log\left(\frac{\pi e}{\Delta x_B \Delta k_B}\right)$$

- Relies only on experimental data
  - No approximations necessary
  - Provides rigorous experimental demonstration of entanglement and EPR-steering for CVs.
- Complements existing state of the art
  - Reduces to same inequality for large resolution
  - Witnesses EPR-steering in the same states (for large enough resolution)
- Allows us to formulate second steering inequality based on mutual information

Our Second Inequality

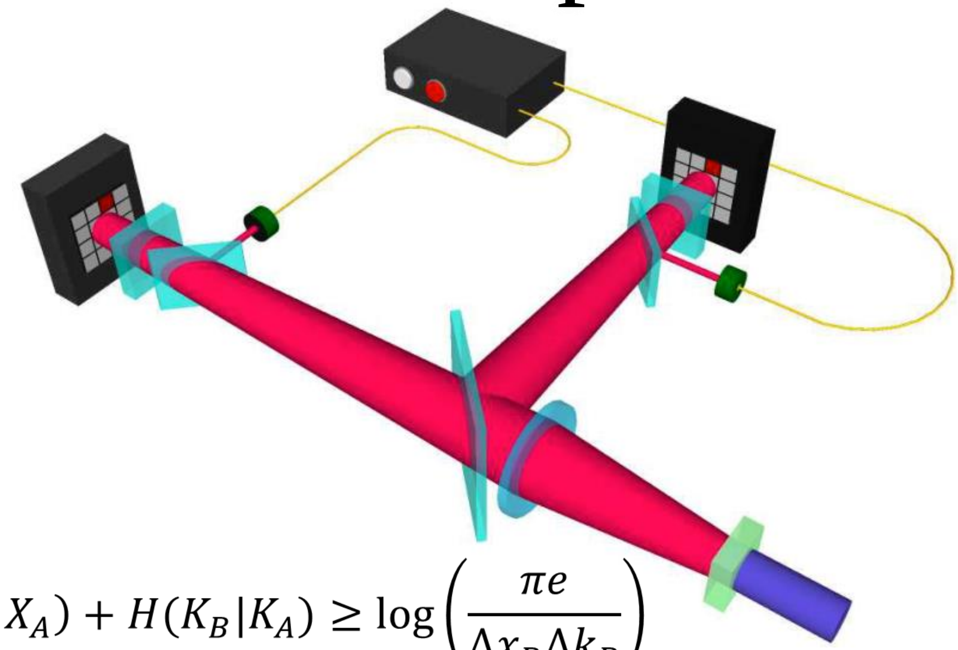
$$I(X_A : X_B) + I(K_A : K_B) \leq \max_{(A,B)} \log\left(\frac{L_x L_k}{\pi e}\right)$$

Why?

$$I(X_A : X_B) = H(X_A) - H(X_A | X_B)$$

- Mutual information is an extremely general measure of correlation, better than covariance and many others.
  - Mutual information captures arbitrary statistical dependence
    - Not just linear dependence.
- Persistence of correlations across conjugate observables is a calling card of entanglement.
- This inequality is symmetric between parties (more restrictive)
  - Violation implies Alice and Bob can trust that they share entanglement even when their mistrust is mutual.

Experimental data

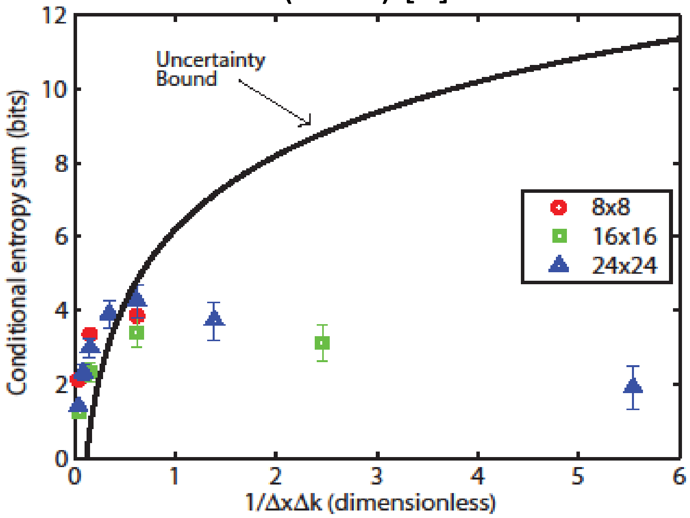


$$H(X_B|X_A) + H(K_B|K_A) \geq \log\left(\frac{\pi e}{\Delta x_B \Delta k_B}\right)$$

- Used down-converted 325 → 650 nm light from BBO nonlinear crystal.
- Measured joint coincident detections to get joint probability distributions in both image and Fourier planes of the crystal.
  - Recorded at different resolutions
- Successful violation of both inequalities at 16x16 and 24x24 resolutions

Results		
First inequality	Nσ min	Nσ max
8x8	3.65	5.9
16x16	8	11.2
24x24	12.3	16.4
Second inequality	Nσ min	Nσ max
8x8	-6.3	-2.8
16x16	3.4	6.96
24x24	6.56	10.7

Experimental diagram and data from PRL 108 142603 (2012) [4]



Asymmetry between parties

- Our first inequality is not symmetric between parties.
  - The bound depends on one party.
  - The conditioning is on the other party.
- In principle, we do not need high resolution in both detectors to demonstrate EPR steering.
  - Bob has a hard lower limit to his detector resolutions
    - Alice does not.

$$H(X_B|X_A) + H(K_B|K_A) \geq \log\left(\frac{\pi e}{\Delta x_B \Delta k_B}\right)$$

Conclusion/Related work

- We have two new inequalities
  - They witness entanglement/EPR-steering in more systems than conditional variances.
  - They only need experimental data
  - We've really used them in the lab! Successfully!
- Related work

  General case for Entropic EPR-steering inequalities:

  - "EPR-steering inequalities from entropic uncertainty relations" (PRA **87**, 062103) [5].

  Do better uncertainty relations make better steering inequalities?

  - "EPR-steering inequalities with quantum memories" (arXiv: in submission) [6]

Thanks for listening!

Works Cited:

(1) M. D. Reid, Phys. Rev. A **40**, 913 (1989).

(2) S. P. Walborn, A. Salles, R. M. Gomes, F. Toscano, and P. H. Souto Ribeiro, Phys. Rev. Lett. **106**, 130402 (2011).

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(4) P. B. Dixon, G. A. Howland, J. Schneeloch, and J. C. Howell, Phys. Rev. Lett. **108**, 143603 (2012).

(5) J. Schneeloch, C. J. Broadbent, S. P. Walborn, E. G. Cavalcanti, and J. C. Howell, Phys. Rev. A **87**, 062103 (2013).

(6) “EPR-steering inequalities with quantum memories” (arXiv: in submission)

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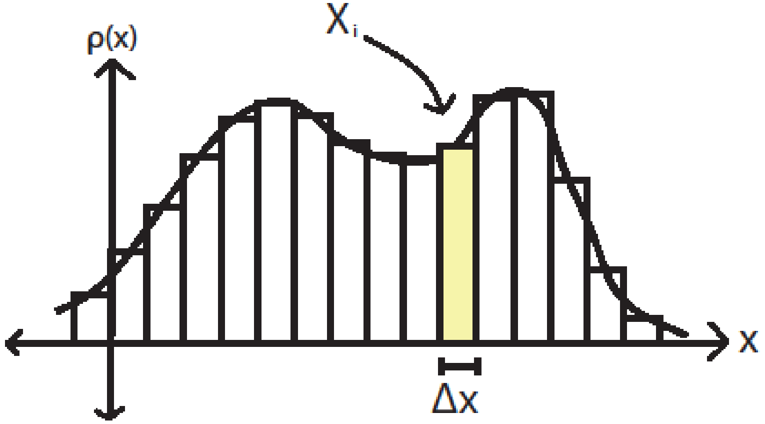
Notes: Our First steering inequality

The uncertainty (i.e. entropy) of the discrete approximation is never less than of the thing itself.

$$\boxed{h(x) = \sum_i P(X_i)\, h_i(x) + H(X)}$$



$$h(x) \leq \log(\Delta x) + H(X)$$

$$h(x, y) \leq \log(\Delta x \Delta y) + H(X, Y)$$

- What we show is that this true for conditional entropies as well (not obvious).

$$h(x|y) \leq \log(\Delta x) + H(X|Y)$$

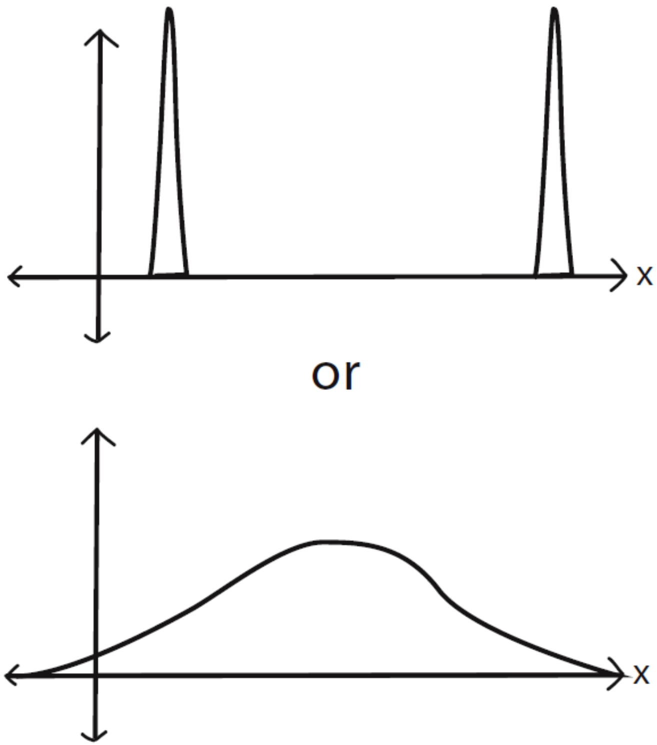
… which gives us our first inequality.

Notes: Why entropy?

- Entropy is a more sensitive measure of uncertainty than standard deviation.
  - e.g. sharp bimodal distributions

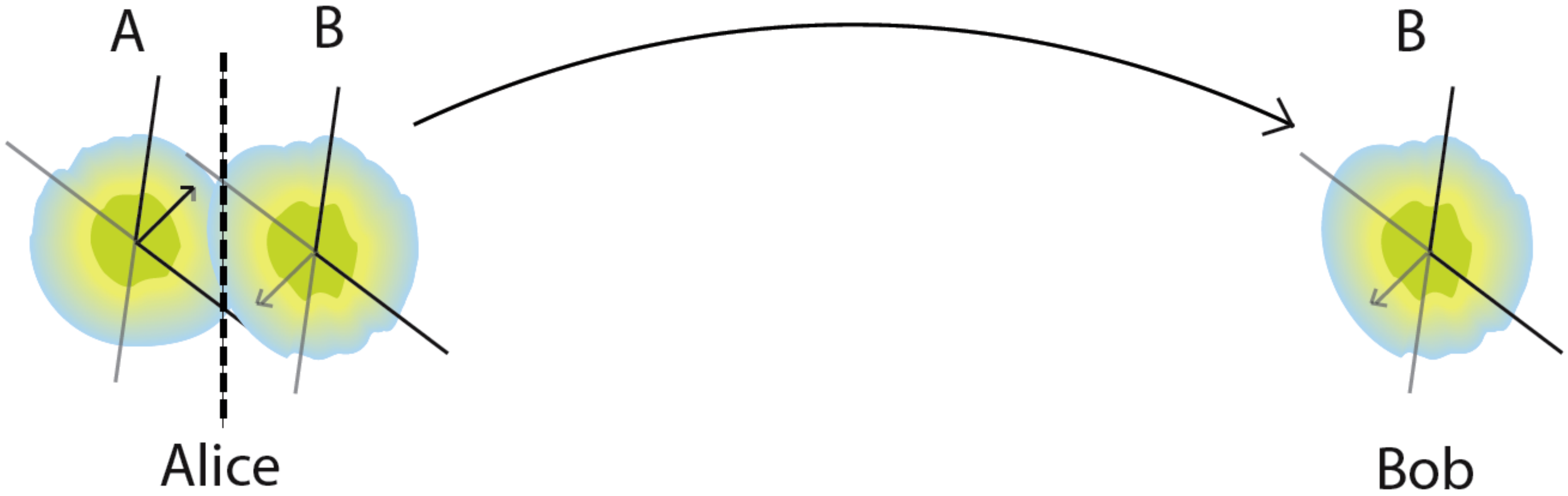
$$H(X) \equiv -\sum_i P(X_i)\log_2(P(X_i))$$

- Entropy is the fundamental concept used in information theory.
  - And its many applications!



which has more uncertainty?

Notes: Why EPR-"*steering*"?



- Alice and Bob share subsystems A and B, entangled in $\vec{x}$ and $\vec{k}$.
  - If Alice measures $\overrightarrow{x_A}$ …
  - If Alice measures $\overrightarrow{k_A}$ …

  What happens to the state of B?

She can "steer" the distribution of states Bob measures, but this still means..

NO SIGNALLING!