A HYPOTHETICAL JOURNEY TO SOME BLACK HOLES

PROBLEM SET #1 IS ON WEBWORK – DUE WEDNESDAY AT MIDNIGHT

(WEBWORK LOGIN INFO: USERNAME = NETID, PASSWORD = STUDENT ID #)

JOURNEY TO HADES

Hades is a hypothetical black hole near the bright star Vega (in the constellation Lyra), 26 light years from Earth.

Travel in a starship: acceleration = Earth's gravity ("$1g$"), speed close to the speed of light for most of the time. **Nothing traveling through physical space can go faster than the speed of light.**

The trip takes 6 years as measured on the starship, but 26 years as measured by an observer on Earth.

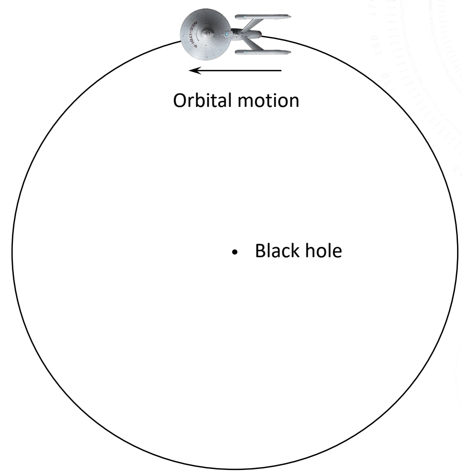
- This difference is a prediction of Einstein's theory of relativity: **length contraction.** The distance to Hades looks shorter from the moving starship than from the stationary Earth (or Hades). (We will discuss this in detail next week.)

Enter orbit above Hades: orbit circumference $= 10^6$ km (half of the Moon's orbit), revolution period = 5 minutes and 46 seconds (speed in orbit = 2890 km/s).

A NOTE ABOUT ORBITS

When the orbit is much larger than the black hole, orbiting a black hole is just like orbiting a planet or star.

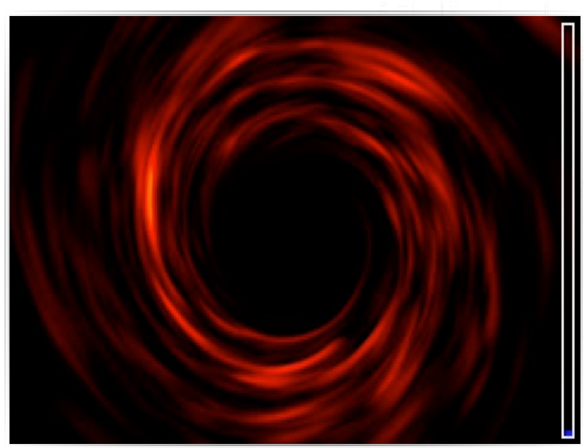
- No thrust is required to **stay** in an orbit.
- In orbit, the gravitational force provides the **force required** for circular motion.
- There is one proper velocity, v, for each orbital radius.
- If the starship has the *wrong* speed for the orbit, it will drift into a different orbit which is a match for the speed. This is called orbital **stability**.



THE BLACK HOLE ITSELF

The special feature of black holes is that there is a distance closer than which nothing can get away.

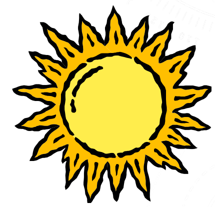
- That special distance traces out a surface that we call the **event horizon** or simply the **horizon**.
- The horizon is just a set of points in space; there is no matter there. It is not the physical edge of anything.
- If we are outside the horizon, we can see other things outside the horizon, but we cannot see anything inside the horizon.



Black hole and swirl of infalling gas, seen from a range of angles (Phil Armitage and Chris Reynolds, U. Colorado)

WORK, HEAT, AND ENERGY IN ASTRONOMY

Hydrogen atom binding energy	1.6×10^{-12} erg
Dietary calorie	4.2×10^{10} erg
Burn 1 kg anthracite coal	4.3×10^{14} erg
Detonate H bomb (1 megaton)	4.2×10^{22} erg
Earth-Sun binding energy	5.3×10^{40} erg
Sun's fuel supply at birth	2×10^{51} erg
Supernova (exploding star)	10^{53} erg



UNITS OF ENERGY

In ASTR 102, our usual unit of energy will be the **erg**:

$$1\ \mathrm{erg} = \frac{1\ \mathrm{gram} \times (1\ \mathrm{cm})^2}{(1\ \mathrm{s})^2} = \mathrm{g\ cm^2\ s^{-2}}$$

This is the unit of energy in the CGS (centimeter-gram-second) system of units.

You might be more familiar with the SI (International System, a.k.a. MKS for meter-kilogram-second) unit of energy, the **joule**:

$$1\ \mathrm{joule} = \frac{1\ \mathrm{kg} \times (1\ \mathrm{m})^2}{(1\ \mathrm{s})^2} = \mathrm{kg\ m^2\ s^{-2}} = 10^7\ \mathrm{erg}$$

The others we have listed will find some uses as well.

LUMINOSITY (TOTAL POWER OUTPUT) IN ASTRONOMY

For the astronomical objects, the power is mostly emitted in the form of light, hence the name.

	ergs per second [erg/s]	Watts [W = J/s]	Solar luminosities [$L_\odot$]
100 W light bulb	1.0×10^{9}	100	
150 horsepower car engine	1.2×10^{12}	1.2×10^{5}	
Large city	10^{15}	10^{8}	
H bomb (1 megaton, 0.01 seconds)	4.2×10^{21}	4.2×10^{14}	1.1×10^{-12}
Sun	3.8×10^{33}	3.8×10^{26}	1
Largest stars	4×10^{38}	4×10^{31}	10^{5}
Milky Way galaxy	8×10^{43}		2×10^{10}
3C 273 (a typical quasar)	4×10^{45}		10^{12}

TYPICAL LUMINOSITIES & IMPORTANT CONVERSIONS

- Normal stars: around one solar luminosity $[L_\odot]$
- Giant stars: thousands to hundreds of thousands of $L_\odot$
- Normal galaxies: $10^9 - 10^{10} L_\odot$
- Quasars: $10^{12} - 10^{13} L_\odot$
- $1 L_\odot = 3.8\times10^{33}$ erg/s = luminosity of the Sun
- 1 watt $= 10^7$ erg/s

Example: Vega, the brightest star in the Northern summer sky, has a luminosity of about 1.9×10^{35} erg/s. What is its luminosity in units of the solar luminosity?

$$L = 1.9\times10^{35}\ \frac{\text{erg}}{\text{s}} \times \frac{1\ L_\odot}{3.8\times10^{33}\ \text{erg/s}} = 50\ L_\odot$$

RATES

Speed and luminosity are examples of rates.

- Speed v is the rate of change of position x with time t:

$$v = \frac{\Delta x}{\Delta t} \implies \Delta x = v \Delta t \text{ if } v \text{ is constant}$$

Position *interval* (distance)

Time *interval*

- Luminosity L is the rate of change of energy E with time t:

$$L = \frac{\Delta E}{\Delta t} \implies \Delta E = L \Delta t \text{ if } L \text{ is constant}$$

NOW YOU TRY

There are 8 furlongs in a mile and 2 weeks in a fortnight. Suppose that we take the furlong to be our unit of length, and a fortnight to be our unit of time.

What is the corresponding unit of speed?

A. Furlong fortnights

B. Fortnights per furlong

C. Furlongs per fortnight

D. Furlongs per second

AND AGAIN…

There are 8 furlongs in a mile and 2 weeks in a fortnight. Suppose that we take the furlong to be our unit of length, and a fortnight to be our unit of time.

What is the NYS Thruway speed limit in this new system of units?

A. 1.5 furlongs per fortnight
B. 1.7×10^{5} furlongs per fortnight
C. 8×10^{-4} furlong fortnights
D. 42 fortnights per furlong

LUMINOSITY AS A RATE

Example: How long could the Sun survive at its current luminosity, considering the fuel supply with which it was born?

$$\Delta t = \frac{\Delta E}{L} = \frac{2\times10^{51}\ \mathrm{erg}}{3.8\times10^{33}\ \mathrm{erg/s}} = 5.3\times10^{17}\ \mathrm{s}\times\left(\frac{1\ \mathrm{yr}}{3.16\times10^{7}\ \mathrm{s}}\right) = 1.7\times10^{10}\ \mathrm{yr}\ (17\ \text{billion years})$$

It has already lived 4.56 billion years…

HOW BIG IS THAT?

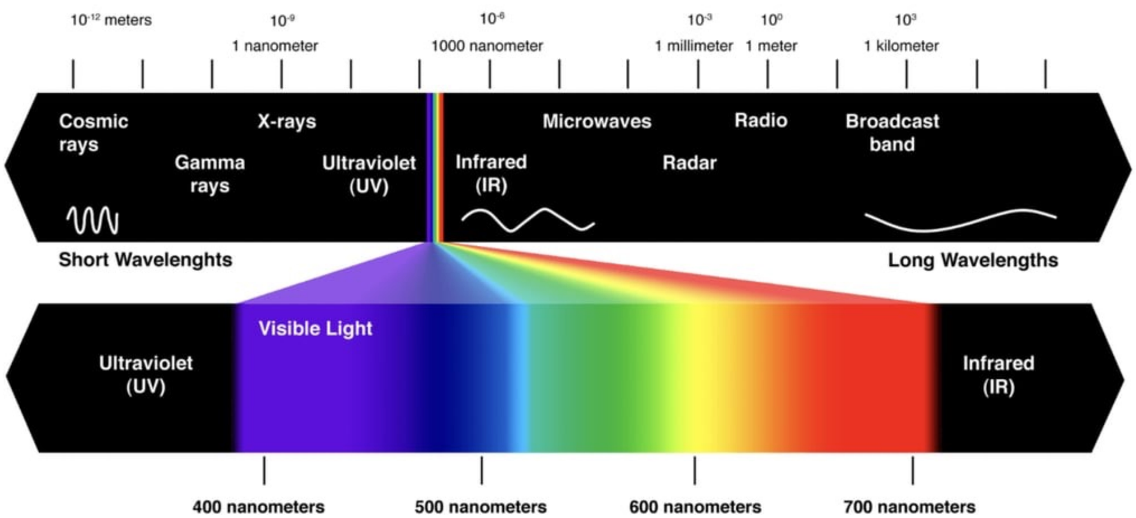
Many important physical quantities that we will use frequently are collected on the "How Big is That?" sheet, found under the **Home** page section of our ASTR 102 website.

- You will always have access to this page while you are working on homework assignments and exams. You do not need to memorize the numbers.
- However, to effectively use the sheet and to understand our astronomical discussions, you must become familiar enough with them to know how big most of them are.
  - It would do you good to memorize at least the "typical" values of things as discussed on the previous slides.

EXPLORING THE NEIGHBORHOOD OF HADES

Drop Arnold the robot into the hole, having him send laser signals back to the ship as he falls.

- The laser light shifts to longer and longer wavelengths as he falls: the **Doppler shift** caused by his high speed with respect to you.

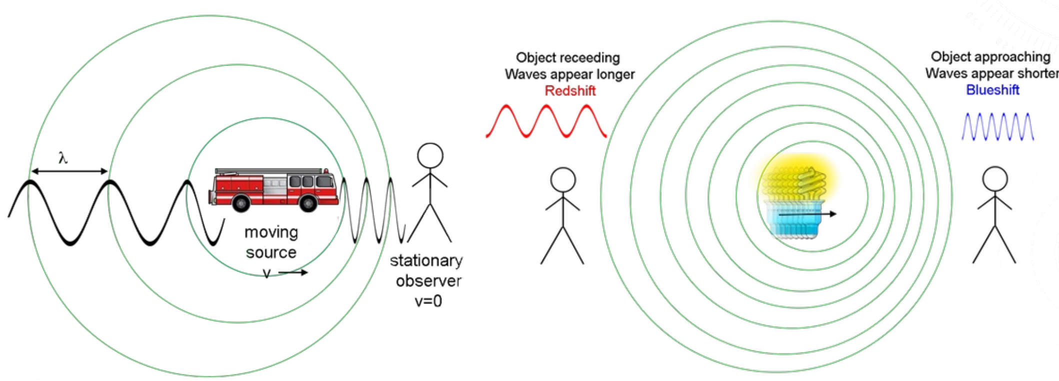


THE ELECTROMAGNETIC SPECTRUM (LIGHT)

Visible light is only a tiny part of the spectrum of light.

THE DOPPLER SHIFT

When the source of a wave moves towards or away from you (or similarly, you are moving towards or away from the source), the wave's frequency either compresses or stretches.



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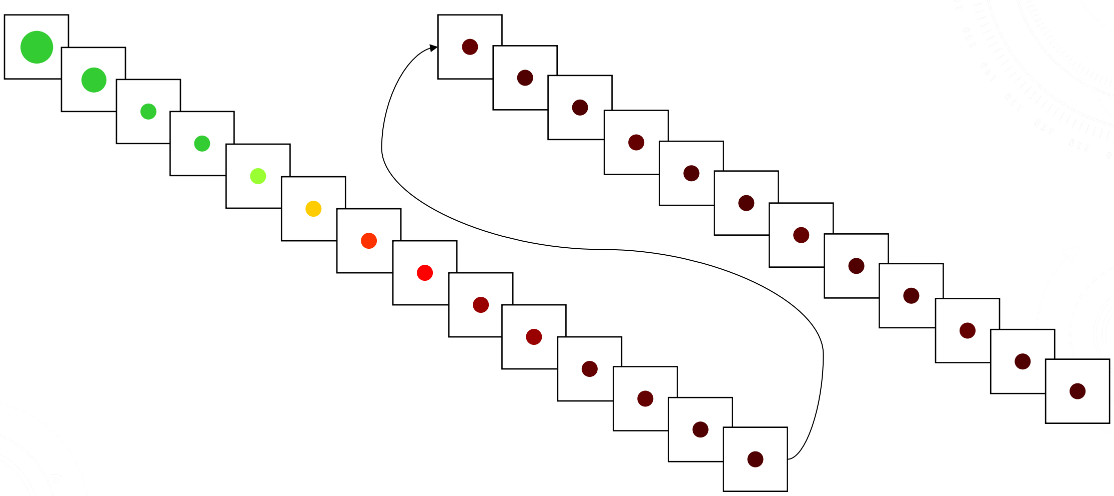
- Instead of winking out abruptly as he crosses the hole's horizon, signals keep arriving at gradually increasing intervals for forever: from the outside, it *looks* like it takes Arnold an infinite amount of time to cross the horizon, even though he has already fallen in (according to him).

THE LASER SPOT AS THE ROBOT FALLS TOWARD THE BLACK HOLE



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- Instead of winking out abruptly as he crosses the hole's horizon, signals keep arriving at gradually increasing intervals for forever: from the outside, it *looks* like it takes Arnold an infinite amount of time to cross the horizon, even though he has already fallen in (according to him).
- This is a consequence of **time being warped** by the black hole's gravity!
- **Arnold *cannot* get out once he falls past the horizon!**

IN ARNOLD'S VIEW

Suppose that you were also shining a green laser at Arnold as he fell. How do you think our laser spot looked to him?

A. Just like his did to us: gets redder ever more slowly without ever disappearing.
B. Gets steadily redder without bound, until Arnold crosses the horizon.
C. Gets bluer ever more slowly without ever disappearing.
D. Gets steadily bluer without bound, until Arnold crosses the horizon.

IN THE EARTH'S VIEW

During our trip, the distance from Earth to Hades appeared to be much shorter than when we were at rest.

An observer watching from Earth would also see an element of the scene to be shorter during our trip. What element?

A. The distance from Earth to Hades, just like us.
B. Our spaceship
C. Hades itself
D. The Earth

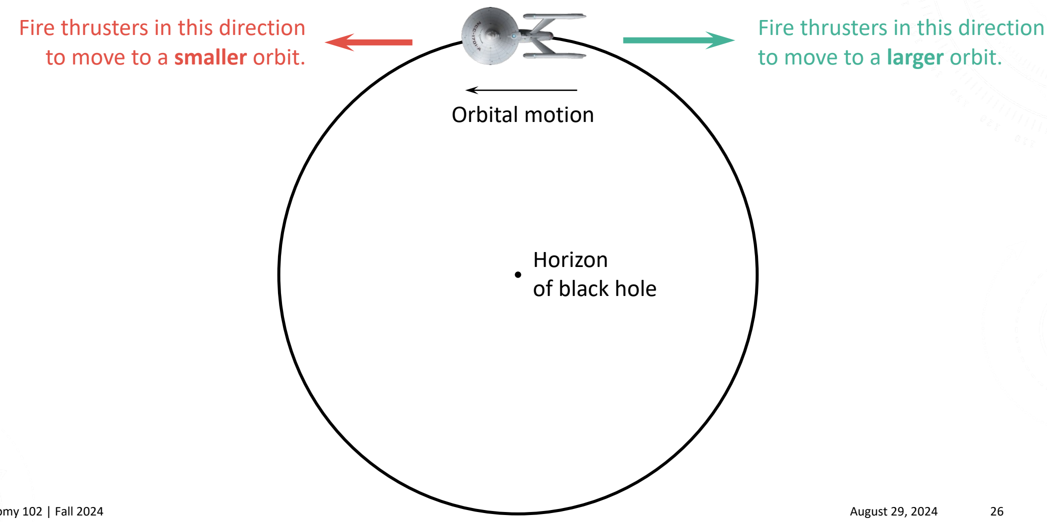
CHANGING ORBITS AROUND HADES (FOR ORBITS MUCH LARGER THAN THE HORIZON)

To move from orbit to orbit, our spaceship must obey the laws of physics: in particular, the conservation of energy and angular momentum.

- Smaller-radius orbits have faster orbital speeds and higher kinetic energy (energy of motion), but lower angular momentum.
- So, to move from a larger- to a smaller-radius orbit, we need to **put on the brakes** to reduce the angular momentum and **fall** toward the smaller orbit, picking up speed again without adding angular momentum.
- In space, the "brakes" are applied when we fire the thrusters straight ahead.
- *Vice-versa* to transfer to a larger orbit: fire the thrusters straight behind.

These are the same rules that apply for spacecraft orbiting planets and stars.

CHANGING ORBITS AROUND HADES (FOR ORBITS MUCH LARGER THAN THE HORIZON)

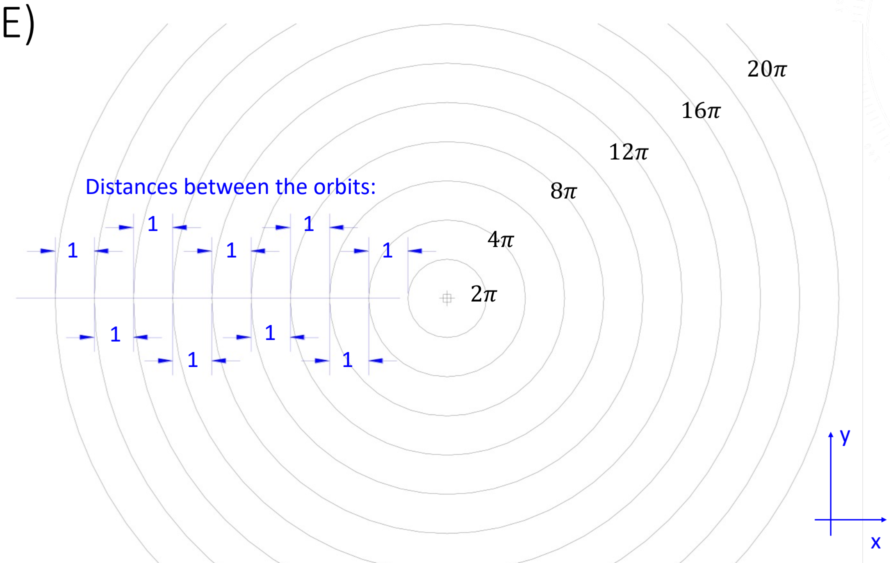


EXPLORING THE HORIZON OF HADES

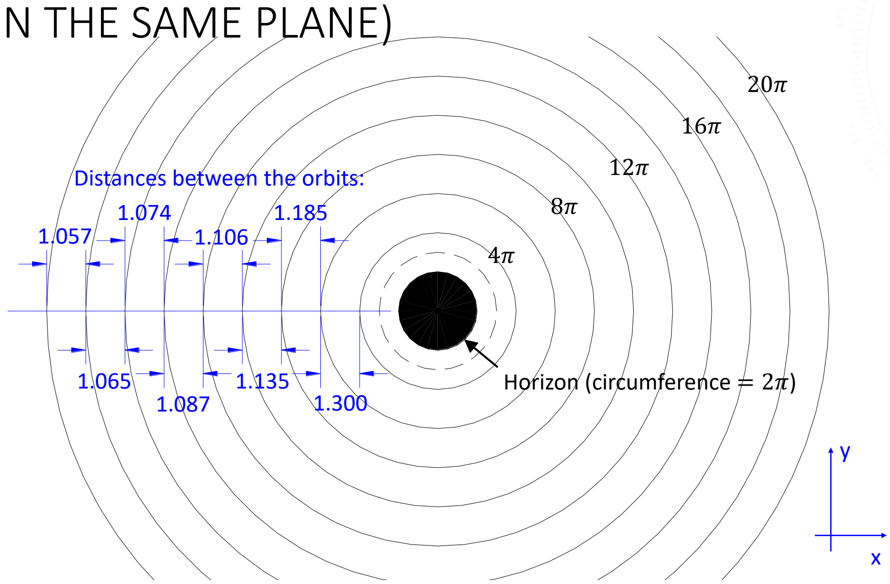
Take a small shuttle into orbits progressively smaller in circumference, trying to reach an orbit 1.0001 times larger than the horizon.

- As you approach the horizon, the circumference of the orbit becomes noticeably less than 2π times the radius (**warped space**: “non-Euclidean geometry”)

CIRCULAR ORBITS IN FLAT SPACE (ALL IN THE SAME PLANE)

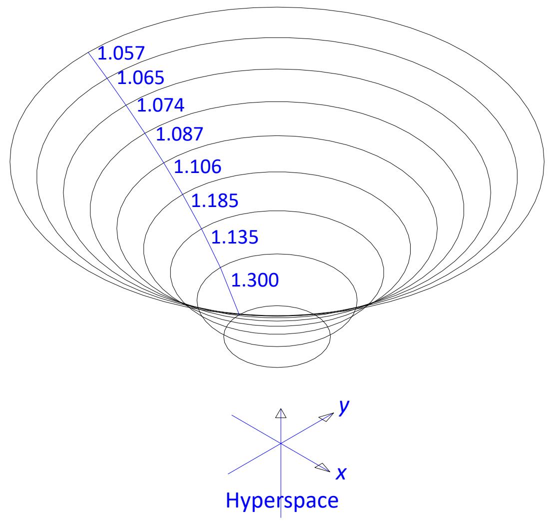


CIRCULAR ORBITS SPACE WARPED BY A BLACK HOLE (ALL IN THE SAME PLANE)



HYPERSPACE: ONE WAY TO VISUALIZE WARPED SPACE

To connect these circles with segments of these "too long" lengths, we can consider them to be offset from one another along some **imaginary dimension** that is perpendicular to x and y ***but is not z***. (If it were z, the circles would not appear to lie in the same plane!) Such additional dimensions comprise **hyperspace**.



HOW MANY HYPERSPACE DIMENSIONS?

To describe space warping in a plane (two dimensions), we have just made use of one dimension of hyperspace.

For three dimensions, like real 3D space, how many dimensions of hyperspace would we need to describe space warping?

A. 1

B. 2

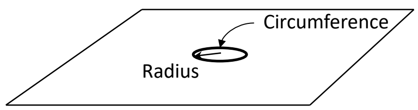
C. 3

D. 4

"RUBBER SHEET" ANALOGY OF WARPED SPACE

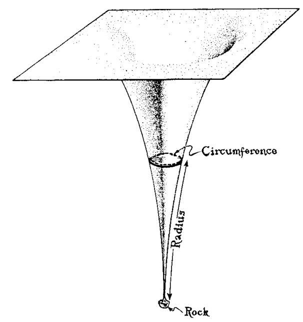
Flat space

Circle drawn (here, in perspective) on a flat rubber sheet has circumference and radius that obey $C = 2\pi r$.



Warped space near a black hole

A circle with the same circumference has a much larger radius than before, and the two no longer obey $C = 2\pi r$, as if the rubber sheet were stretched by a heavy rock placed at the circle's center. **The direction of the stretch, though, is in hyperspace (NOT physical space).**



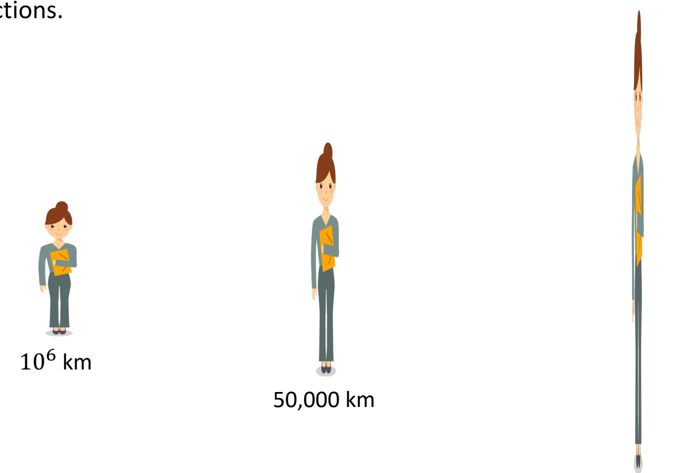
EXPLORING THE NEIGHBORHOOD OF HADES

Take a capsule into orbits progressively smaller in circumference, trying to reach an orbit 1.0001 times larger than the horizon.

- As you approach the horizon, the circumference of the orbit becomes noticeably less than 2π times the radius (**warped space**: "non-Euclidean geometry")
- In a 100,000 km-circumference orbit, you feel strong **tidal forces**: head and feet are pulled apart by a force equal to one-eighth of Earth's gravity ("$\frac{1}{8}g$")
- By 30,000 km, your head and feet are pulled apart by $4g$; by 20,000 km (still more than a factor of 100 larger than the horizon circumference), it is $15g$.

TIDAL STRETCHING AT VARIOUS ORBITS AROUND HADES

A body would be stretched along the direction toward the black hole and squeezed in the perpendicular directions.



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- By 30,000 km, your head and feet are pulled apart by $4g$; by 20,000 km (still more than a factor of 100 larger than the horizon circumference), it is $15g$.
- Give up and return to the ship! We are clearly not going to make it close enough to the horizon; we need a more massive black hole.