RELATIVITY

PROBLEM SET #2 ON WEBWORK – DUE MONDAY, 9/16 AT MIDNIGHT

RELATIVITY

- Visit a spinning $45M_{\odot}$ black hole
- Measurement of physical quantities, reference frames, and space-time diagrams
- Relative and absolute physical quantities
- Classical physics and Galileo's theory of relativity
- An apparently small problem in classical physics leads Einstein to a revolutionary solution

Albert Einstein, circa 1905

PROPERTIES OF YOUR NEW BLACK HOLE

- Mass $= 45M_{\odot}$
- Horizon circumference = 533 km
- Rotation period = 0.0037 seconds
- 20% of the black hole's total energy lies in the **swirl of space** just outside the horizon, on the equator: 10^4 times as much energy as the Sun radiates in its entire lifetime.

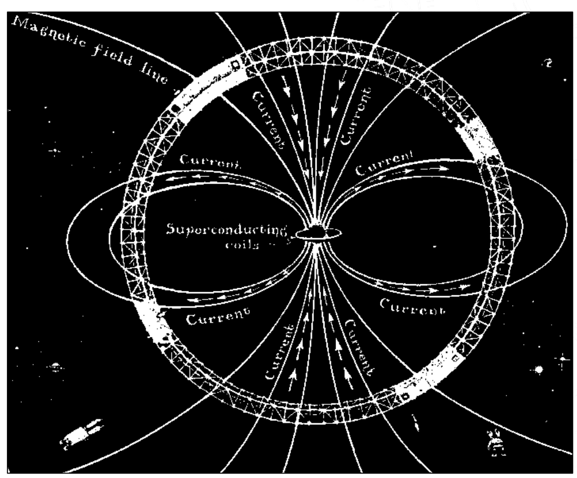
Your new crew builds a giant girder-work, 5×10^6 km in circumference (2.2 times that of the Moon's orbit around the Earth), rotating once every half hour to provide $1g$ gravity on its inner and outer surfaces.

The energy of the "swirl" is tapped to fuel the new city on the girder-work.

THE "SWIRL OF SPACE" NEAR THE HORIZON

The extreme warping of space near a horizon is an indication that **space itself is stuck to the horizon,** in the view of a distant observer.

- Like time stopping at the horizon, in the view of a distant observer. (Remember Arnold's fall?)
- Thus, if the horizon moves, it drags space along with it. A rotating horizon winds space around the horizon: this is the "swirl of space."
- Upshot: any part of any object placed in the "swirl" will be dragged around with it. Connect that "crank" to a machine and the rotating black hole will turn the crank, usually for a very long time.



SUMMARY: WEIRD PROPERTIES OF BLACK HOLES AND RELATIVITY (THAT WE NEED TO EXPLAIN)

Lengths appear to contract when travelling at near-light speeds.

No speed can exceed light's.

Nothing can escape from within a black hole's horizon.

High-energy light is given off by ions falling into a black hole.

The "swirl of space" around a rotating black hole

Gravitational waves

Near a black-hole horizon:

- Space warps (e.g., orbit radius much larger than $\frac{C}{2\pi}$)
- Time warps (to a distant observer, time seems to slow down and stop near a horizon)
- Gravitational redshift (same as time warp)
- Very strong gravity and tidal forces
- Gravity and tidal forces very near horizon are *less* for a heavier black hole than for a lighter one.
- Orbits smaller than 3 horizon circumferences: "reverse thrust" required to change, resulting in instability
- No orbits smaller than 1.5 horizon circumferences
- Sky compressed into a small circle overhead

INTRODUCTION TO EINSTEIN'S THEORIES OF RELATIVITY

Over the next five lectures, we will discuss Einstein's relativity theories and some interesting and important predictions made from them. Our goals are:

- To understand the nature of the theories and the properties of space and time at high speeds and in strong gravity
- To understand these things well enough so that you make the basics of relativity understandable to others who have not taken a course like this

Einstein at Caltech, 1933

INTRODUCTION TO EINSTEIN'S THEORIES OF RELATIVITY

Specifically, the special theory of relativity (1905) and the general theory of relativity (1915), by which the existence and properties of black holes are predicted.

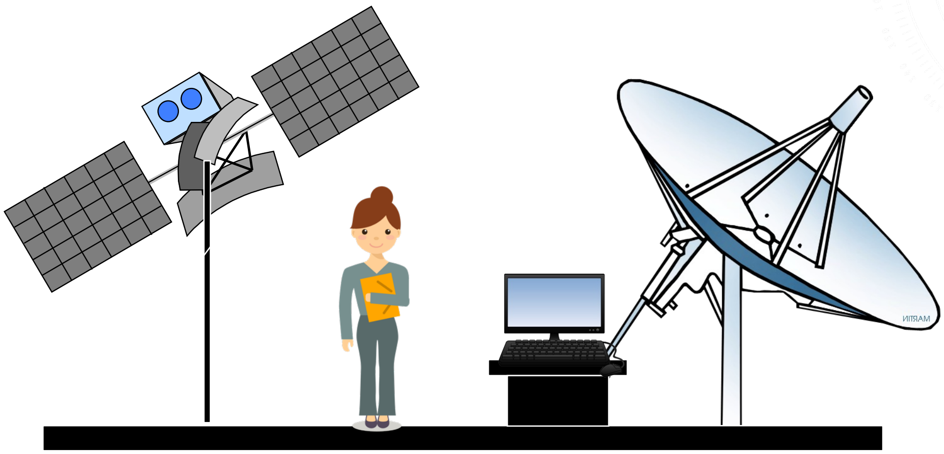
We shall immediately dispel two popular misconceptions about Einstein's theories:

- **Einstein did not mean that "everything is relative."** The theories leave many physical quantities absolute; for example, the speed of light.
- **Relativity existed before Einstein.** Galileo and Newton had another theory of relativity. The differences between Einstein's relativity and that which preceded his theories are simply which physical quantities the theories take to be relative or absolute.

REFERENCE FRAMES

A **reference frame** (or "frame of reference") consists of an observer, like you, and a **hypothetical** bunch of instruments that can measure length, time, etc., all at the same state of motion (not moving with respect to each other).

A REFERENCE FRAME

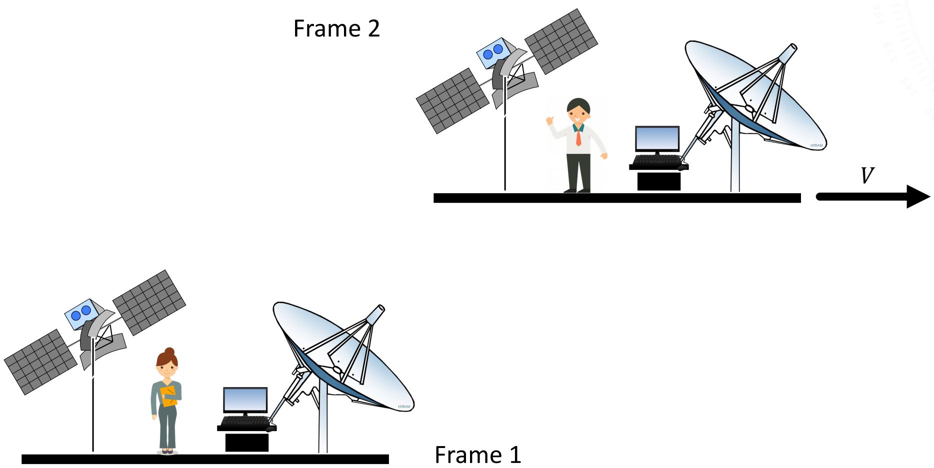


REFERENCE FRAMES

A **reference frame** (or "frame of reference") consists of an observer, like you, and a **hypothetical** bunch of instruments that can measure length, time, etc., all at the same state of motion (not moving with respect to each other).

- An **inertial** reference frame is one whose state of motion is not influenced by any external forces.
- Observers in different reference frames can, by use of their instruments, measure things they see in each other's frame and report their results to each other.

TWO REFERENCE FRAMES IN RELATIVE MOTION

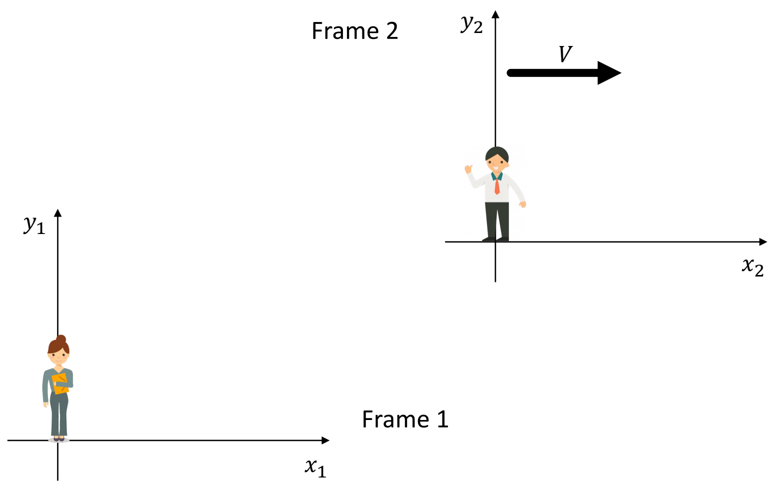


REFERENCE FRAMES

A **reference frame** (or “frame of reference”) consists of an observer, like you, and a **hypothetical** bunch of instruments that can measure length, time, etc., all at the same state of motion (not moving with respect to each other).

- An **inertial** reference frame is one whose state of motion is not influenced by any external forces.
- Observers in different reference frames can, by use of their instruments, measure things they see in each other’s frame and report their results to each other.
- It is often useful to assign **coordinate systems** to each observer, who sits at the origin of his or her system.

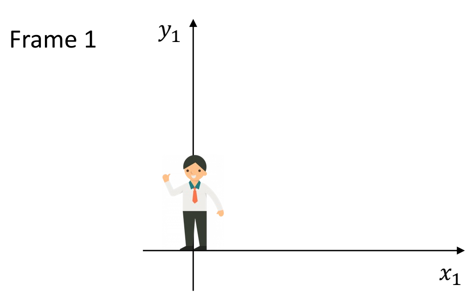
COORDINATE SYSTEMS FOR REFERENCE FRAMES

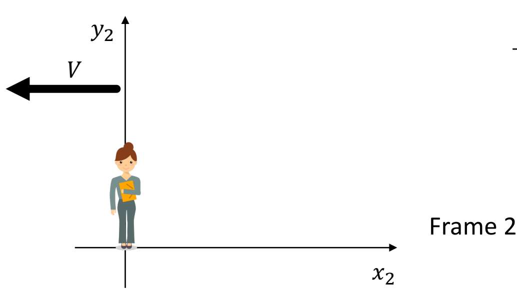


COORDINATE SYSTEMS FOR REFERENCE FRAMES

It does not matter which frame is labeled 1 and which is labeled 2.

This picture is physically identical to the previous one.





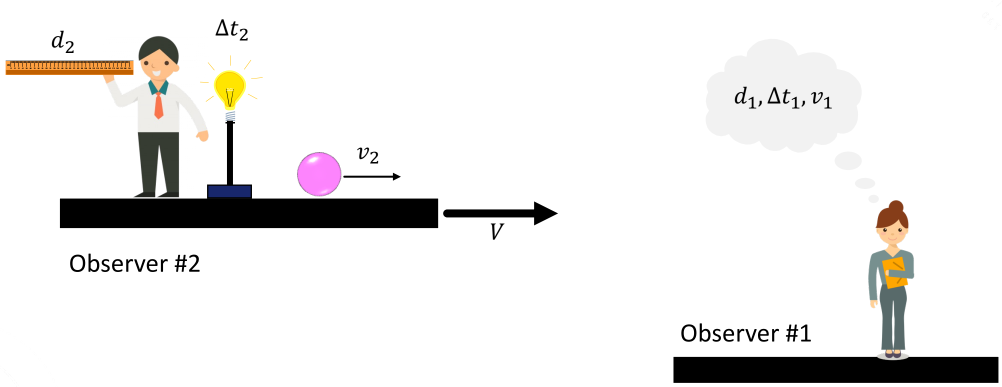
EXAMPLES: MEASUREMENTS BY TWO OBSERVERS IN DIFFERENT REFERENCE FRAMES, MOVING WITH RESPECT TO EACH OTHER

Observer in Frame 1 sees Frame 2 moving east at speed V (similarly, the observer in Frame 2 sees Frame 1 moving west at speed V).

- Observer in Frame 2 holds up a meter stick (horizontally), flashes a light at the beginning and end of a certain time interval, and rolls a ball horizontally across his floor.
- Observer in Frame 1 takes pictures, measures time intervals, etc. and determines the length of the meter stick, the length of the time interval, and how fast the ball appears to her.

What do you think that she will find? (It turns out that the answers given by Galileo and Newton differ from those given by Einstein if V is large enough.)

TO OBSERVER #1: HOW LONG DOES THE METER STICK LOOK, HOW LONG BETWEEN THE FLASHES OF LIGHT, AND HOW FAST DOES THE BALL APPEAR TO ROLL?



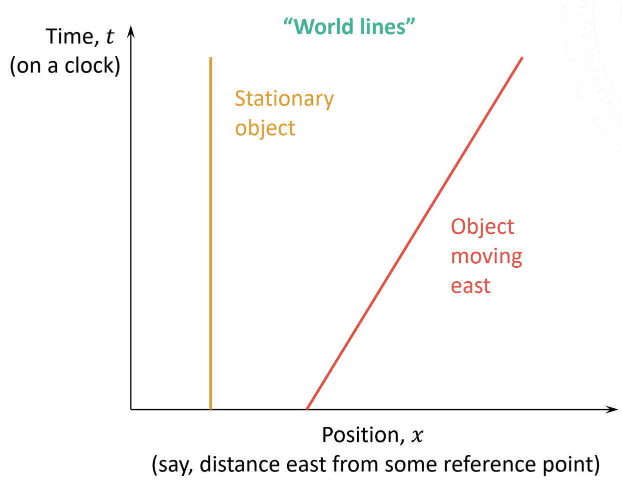
MATH SYMBOLS

The use of some common mathematical shorthand is just too convenient to avoid.

- **Subscripts** will be used to denote an observer by whom a measurement is made; for example, d_1 is a distance measured by observer 1, and v_2 is a velocity measured by observer 2.
- Capital delta (Δ) will be used to denote a **change** in some quantity: for example, Δt_1 is an interval of time t measured by observer 1, Δx and Δy are distances measured along the x and y directions, respectively, and ΔE is a change in the energy E of some system.
- x and y give the **location** of a point in the coordinate system: the distance of that point from the y and x axis (the origin of the coordinate system).

SPACE-TIME DIAGRAMS

A way to report distance *and* time measurements made within one reference frame



ABSOLUTE & RELATIVE PHYSICAL QUANTITIES

Suppose that two observers are moving with respect to each other and can measure the same set of physical quantities, like our Observers #1 and #2.

- For some physical quantities, their results – numerical values and units – will be the same. These quantities are called **absolute**: measurements of them give the same result no matter how the observer is moving.
- If the observers measure **different** values of a given physical quantity, that quantity is called **relative**: measurements yield different results for observers in different states of motion.

CHOOSE THE ABSOLUTE QUANTITY

In Einstein's relativity, one of the following quantities will turn out to be absolute, and the other three relative. Can you guess which one will be absolute?

A. The order in which cause and effect occur
B. Time itself
C. Simultaneity of two events
D. The mass of the Sun

OLD ("CLASSICAL") PHYSICS

Three main fields:

- Mechanics and gravitation: Newton's laws
- Electricity, magnetism, and light: Maxwell's equations
- Heat and thermodynamics: Gibbs and Boltzmann

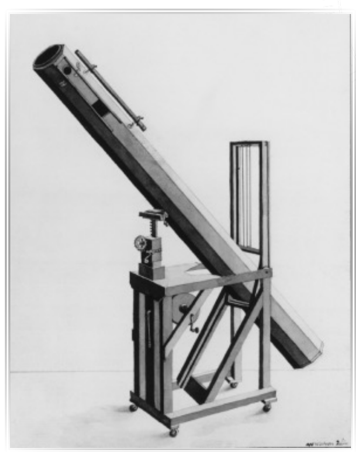
Each consists of a small number of laws, mostly expressed as mathematical formulas. By mathematically manipulating these formulas, and plugging numbers in, the results of experiments can be predicted, or new effects can be envisioned.

Sir Isaac Newton, c. 1702

OLD ("CLASSICAL") PHYSICS

These theories were fantastically accurate and were successfully used to predict important new discoveries and to invent new technologies. Examples include:

- Discoveries of Uranus and Neptune
- Discovery of radio waves
- Invention of various heat engines (Carnot, Diesel,…)



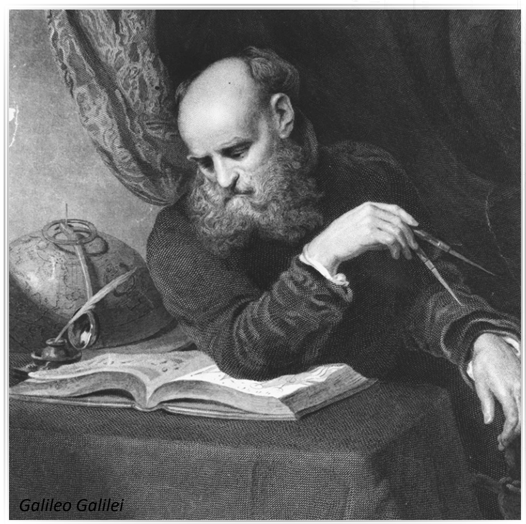
Replica of William Herschel's 6.2 in diameter reflecting telescope, which he used to discover Uranus in 1781.

OLD ("CLASSICAL") PHYSICS

These theories were all built on the same principle of relativity (Galileo's): **distance and time are absolute.** Given a meter stick and a clock in one reference frame,

- The meter stick looks one meter long from all other frames of reference
- The clock ticks seem to take one second from all other frames of references

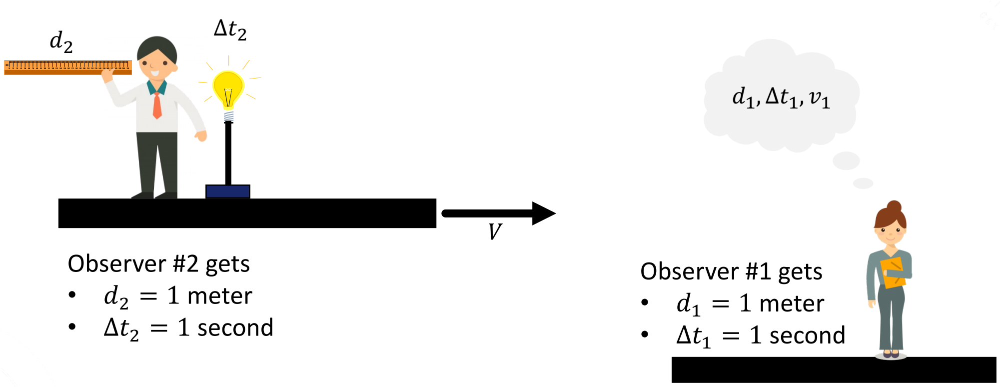
And **velocities** are relative: if Frame 2 looks like it moves east at speed V to Frame 1, and an observer in Frame 2 rolls a ball east at speed v (according to him), then the ball will appear to be rolling east at $v + V$ to the observer in Frame 1.



Galileo Galilei

GALILEO'S RELATIVITY: ABSOLUTE DISTANCE & TIME

Both observers measure the length of the same meter stick and the duration between flashes of the same light. Their results are identical, to high accuracy, no matter what V is.

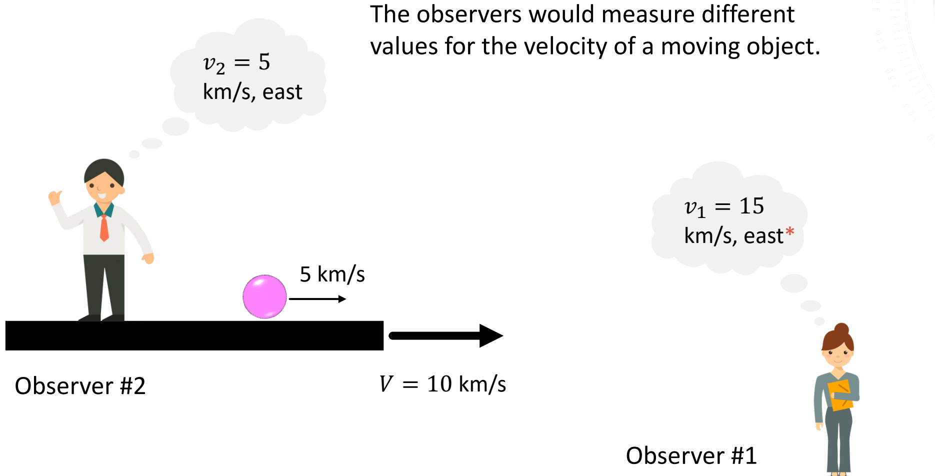


Observer #2 gets
- $d_2 = 1$ meter
- $\Delta t_2 = 1$ second

Observer #1 gets
- $d_1 = 1$ meter
- $\Delta t_1 = 1$ second

GALILEO'S RELATIVITY: VELOCITIES ARE RELATIVE

The observers would measure different values for the velocity of a moving object.



*That's just $v + V = 15$ km/s.

IF, ON THE OTHER HAND…

…Observer #2 had rolled the ball at 5 km/s east, and Observer #1 had measured the ball to be moving at 15 km/s **west**, then what is the velocity V of #2 with respect to #1?

A. 10 km/s west
B. 10 km/s east
C. 20 km/s west
D. 20 km/s east
E. None of the above

ELECTRICITY & MAGNETISM, LIGHT, AND THE "AETHER"

In the late 1800s, physicists were intensely studying electromagnetism, especially the properties of light.

Electromagnetism is accurately described by a set of four simple equations, called **Maxwell's equations,** that relate the values of the electric and magnetic **fields.** (Fields are simply those quantities that tell what the force on an electric charge would be.)

One of the many great successes of Maxwell's equations is that they naturally explain light: they can be combined to produce an equation describing how light travels as **bundles of electric and magnetic fields** through space that looks just like the equation that Newton's laws give for how **vibrations of a string** would travel along that string.

- Vibration of what? This analogy tempted physicists to think of electric and magnetic fields as vibrations in an all-pervasive, universal, but *hitherto unobserved* medium that underlies the matter from which the things around us are made.
- This hypothetical medium was called the **aether.**

LIGHT AND THE "AETHER"

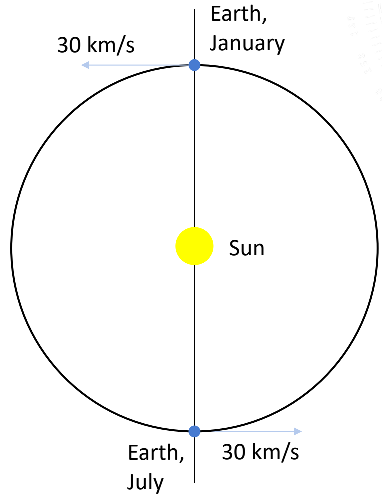
- Since it was supposed to pervade all space in the universe, the aether provided a natural definition of "rest." The aether was considered to be stationary, providing an **absolute reference frame** in which to observe other motions.
- According to Galileo's relativity, light would appear to move at different speeds for observers in different states of motion, because light would always move at the same speed with respect to the aether.
- **But:** to those same observers, the equations of electromagnetism are not exactly Maxwell's equations but a *much* more complicated set of equations that can be mathematically obtained from Maxwell's equations and the observer's velocity through the aether.

The differences in the equations are **small**, though: they would only get large if an observer were to move through the aether at a speed approaching that of light.

CAN WE DETECT THE AETHER?

The American physicist Albert Michelson decided to measure these tiny effects on the speed of light and thereby **detect** the hitherto undetected aether.

- By himself, and in partnership with Morley, Michelson built a series of very clever devices that could measure the differences in the speed of light resulting from 30 km/s motion through the aether (Earth's orbital motion).

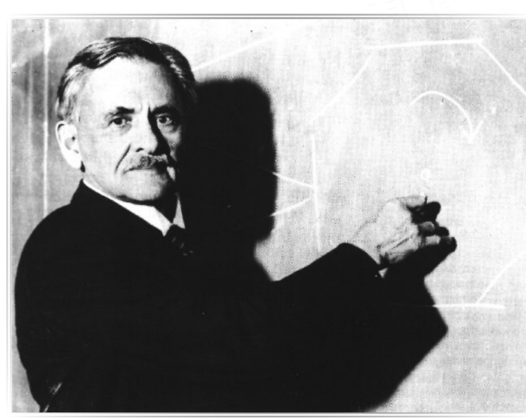


The speed of light measured in January and July should differ by 60 km/s.

CAN WE DETECT THE AETHER?

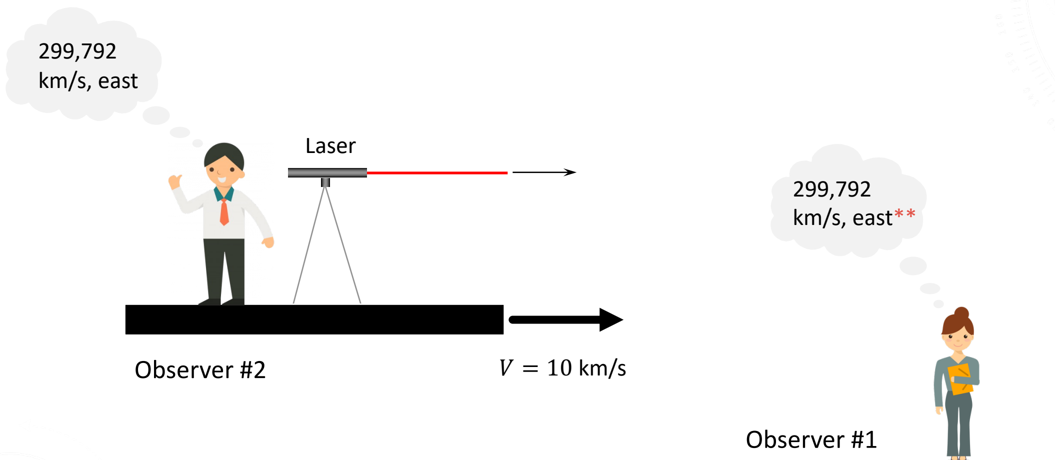
- Result: **the speed of light is constant – always the same, no matter what direction the Earth moves.**
- Thus, either Cleveland (where Michelson did his experiments) is always at rest in absolute space, or light is not related to the aether.
  - But, then, what good is the aether? After all, it was only invented to explain the propagation of light.

Many prominent physicists joined in the struggle to try to explain this puzzling result by seeking small and reasonable corrections to the theory of electromagnetism.



Albert A. Michelson

HOW FAST DOES LIGHT APPEAR TO TRAVEL?

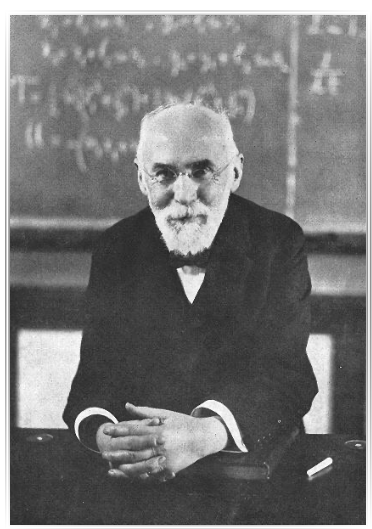


**Not 299,802 km/s!

CAN WE DETECT THE AETHER?

Two of the most famous:

- Fitzgerald and Lorentz: The experiments can be explained if a force is exerted which makes objects shorter along the direction of their motion through the aether.
- Lorentz: Accounting for this contraction, Maxwell's equations are the same in all reference frames.



Hendrik A. Lorentz

ENTER EINSTEIN

Albert Einstein had recently received his PhD in physics (at ETH, Zurich) and was working as an examiner in the Swiss patent office.

- He was greatly worried about the theoretical problems of electromagnetism with the aether and even more greatly worried that this problem might require a more fundamental solution than those "band-aids" proposed by Lorentz and Fitzgerald.
- So, he started reworking the theories starting from the very bottom: Galileo's theory of relativity, which had not been questioned for ages.
- He found that a change in relativity, **all by itself**, removed all the funny problems of the form of the Maxwell equations in different reference frames.

EINSTEIN'S CONCLUSIONS

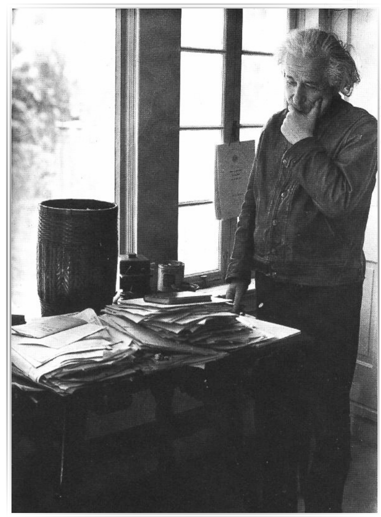
- There is no such thing as the aether.
- The length contraction, and the related "time dilation," are real but are not caused by any unknown force; rather, **length and time are relative,** and results of measurements of them depend upon the frame of reference.
- Velocities of moving objects are still relative, but the relation is no longer as before, owing to the relativity of length and time.
- The speed of light is special, though: it is **absolute,** independent of the reference frame.

Published in 1905, this solution is generally called the **special theory of relativity**.

EINSTEIN’S SPECIAL THEORY OF RELATIVITY

The special theory of relativity can be reduced to two statements:

1. The laws of physics have the same appearance within all inertial reference frames, independent of their motions.
2. The speed of light in vacuum is the same in all directions, independent of the motion of the observer who measures it.



Einstein at Princeton, 1937