SPECIAL RELATIVITY

PROBLEM SET #2 ON WEBWORK – DUE MONDAY AT MIDNIGHT

SPECIAL RELATIVITY

Einstein's procedures and results on the special theory of relativity

Formulas and numerical examples of the effects of length contraction, time dilation, and velocity addition in Einstein's special theory of relativity

The world's most famous patent clerk, c. 1906

EINSTEIN'S SPECIAL THEORY OF RELATIVITY

The special theory of relativity can be reduced to two statements:

1. The laws of physics have the same appearance (mathematical form) within all inertial reference frames, independent of their motions.
2. The speed of light is the same in all directions, independent of the motion of the observer who measures it.

Special = only applies to **inertial reference frames**: those for which the state of motion is not influenced by external forces.

Speed of light: measured to be $c = 2.99792458 \times 10^{10}\,\mathrm{cm/s} = 299{,}792.458\ \mathrm{km/s}$.

SPECIAL RELATIVITY POSTULATE #1

Is special relativity applicable in *our* reference frame?

A. No, because we are not in an inertial reference frame.

B. Yes, because the laws of physics have the same appearance here as in any other frame.

C. Yes, since the non-inertial accelerations that we feel are so small.

SPECIAL RELATIVITY POSTULATE #2

Can someone speed up enough to overtake a beam of light, and thus be in the same inertial reference frame as the light?

A. No, because light always appears to travel at the speed of light no matter what someone's speed is.
B. No, because that would require extreme acceleration and speed, so the reference frame would not be inertial.
C. No, because the speed of light is relative.
D. Yes. Why not?

Anssi Ranki

EINSTEIN'S MOTIVATION FOR SPECIAL RELATIVITY

- Einstein was aware of the results of the Michelson experiments and did not accept the explanation of these results by Lorentz in terms of a force, and associated contraction, exerted on objects moving through the aether.
- However, he was even more concerned about the complicated mathematical form assumed by the four equations of electricity and magnetism (the Maxwell equations) in moving reference frames without such a force by the aether. **The Maxwell equations are simpler and symmetrical in stationary reference frames; he thought that they should be simple and symmetrical under all conditions.**

EINSTEIN'S PROCEDURE IN CREATING SPECIAL RELATIVITY

- Einstein found that he could start from his two postulates and **mathematically show** that in consequence **distance and time are relative rather than absolute**…
  - …and that distances appear contracted when viewed from moving reference frames, exactly as inferred by Lorentz and Fitzgerald for the "aether force." (This is called the **Lorentz contraction,** or Lorentz-Fitzgerald contraction.)
  - …and that the relation between distance and time in differently-moving reference frames is exactly that inferred by Lorentz from the aether-force theory. (This relation is still called the **Lorentz transformation**.)
- He went further to derive a long list of other effects and consequences unsuspected by Lorentz, as follows:

CONSEQUENCES & PREDICTIONS OF EINSTEIN'S SPECIAL THEORY OF RELATIVITY

(A quick preview before we begin detailed illustrations.)

- **Spacetime mixing:** "distance" in each reference frame is a mixture of distance and time from other reference frames
- **Length contraction:** objects seen in moving reference frames appear to be shorter along their direction of motion than the same object seen at rest (**Lorentz-Fitzgerald contraction**)
- **Time dilation:** time intervals seen in moving reference frames appear longer than the same interval seen at rest
- **Velocities are relative,** as before (except for that of light), but add up in such a way that no speed exceeds that of light.
- **There is no frame of reference in which light can appear to be at rest.**

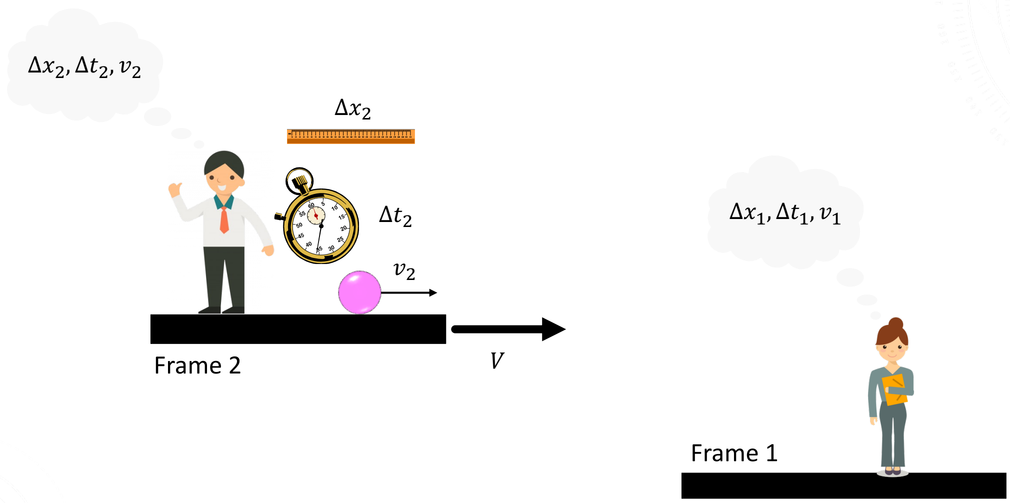
CONSEQUENCES & PREDICTIONS OF EINSTEIN'S SPECIAL THEORY OF RELATIVITY

- **Simultaneity is relative:** events that occur simultaneously in one reference frame do not appear to occur simultaneously in other, differently-moving, reference frames
- **Mass is relative:** an object seen in a moving reference frame appears to be *more massive* than the same object seen at rest; masses approach infinity as reference speed approaches that of light. (This is why nothing can go faster than light.)
- **Mass and energy are equivalent:** Energy can play the role of mass, endowing inertia to objects, exerting gravitational forces, etc. This is embodied in the famous equation $E = mc^2$.

IMPACT OF EINSTEIN'S THEORY OF SPECIAL RELATIVITY

- Einstein's theory achieved the same agreement with experiment as Lorentz without the need of the unseen aether and the force it exerts and with other, testable predictions.
- Einstein's and Lorentz's methods are starkly different.
  - **Lorentz: evolutionary**; small change to existing theories; experimental motivation, but employed "unseen" entities with wait-and-see attitude
  - **Einstein: revolutionary**; change at the very foundation of physics; "aesthetic" motivation; re-interpretation of previous results by Lorentz and others
- Partly because they were so revolutionary, Einstein's relativity theories were controversial for many years, though they continued to pass all experimental tests.

TERMS WE WILL USE IN THE SR FORMULAE



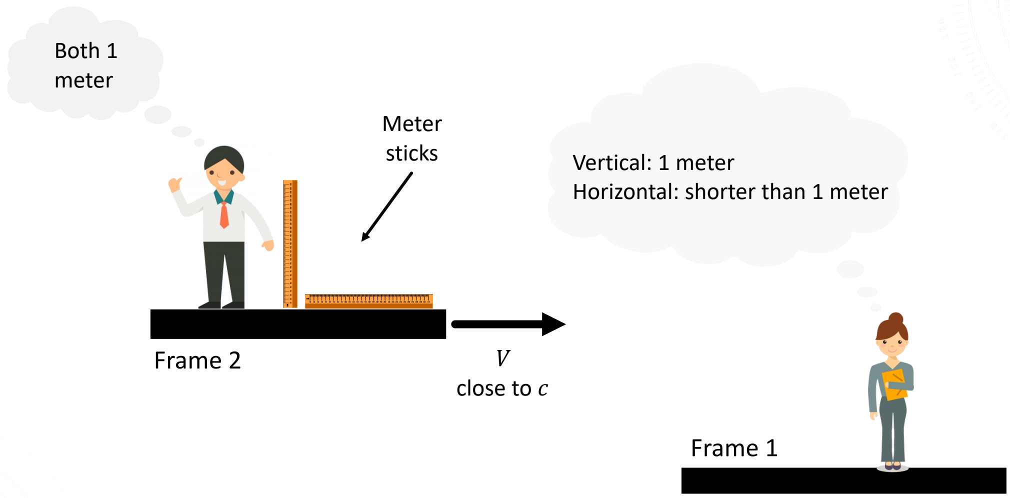
NOMENCLATURE REMINDER

Recall that

- By x_1, we mean the **position** of an object or event along the x axis, measured by the observer in Frame #1 in his or her coordinate system.
- By Δx_1, we mean the **distance** between two objects or events along x, measured by the observer in Frame #1.
- By t_1, we mean the **time** of an object or event, measured by the observer in Frame #1 with his or her clock.
- By Δt_1, we mean the **time interval** between two objects or events, measured by the observer in Frame #1.

If the subscript had been 2 instead of 1, we would have meant measurements by observer #2.

SPECIAL-RELATIVISTIC LENGTH CONTRACTION

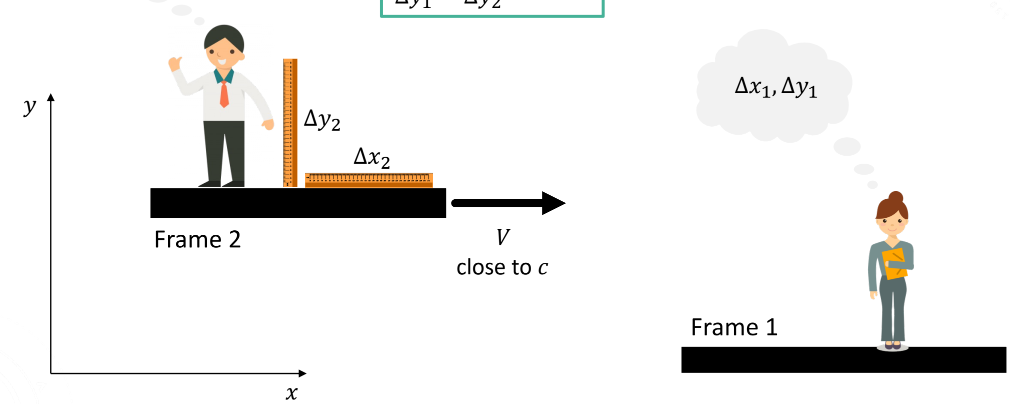


SPECIAL-RELATIVISTIC LENGTH CONTRACTION

$$\Delta x_1 = \Delta x_2 \sqrt{1 - \frac{V^2}{c^2}}$$

$$\Delta y_1 = \Delta y_2$$

Both observers measure lengths **instantaneously**



"GAMMA" - γ

You will frequently see the combination of

$$\gamma = \frac{1}{\sqrt{1-\frac{V^2}{c^2}}} \quad \text{or} \quad \frac{1}{\gamma} = \sqrt{1-\frac{V^2}{c^2}}$$

What is the value of γ if $V = \frac{\sqrt{3}}{2}c = 0.866c$?

"GAMMA" - γ

$$\gamma = \frac{1}{\sqrt{1-\frac{V^2}{c^2}}} \quad \text{or} \quad \frac{1}{\gamma} = \sqrt{1-\frac{V^2}{c^2}}$$

What is the value of γ if $V = 0.995c$? (That is, if $V^2 = 0.99c^2$?)

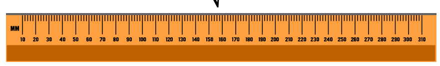
SPECIAL-RELATIVISTIC LENGTH CONTRACTION

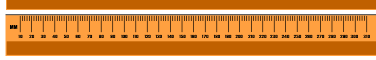
Example: A horizontal meter stick is flying horizontally at half the speed of light. How long does the meter stick look?

Consider the meter stick to be at rest in Frame #2 (its **rest frame**) and us to be at rest in Frame #1:

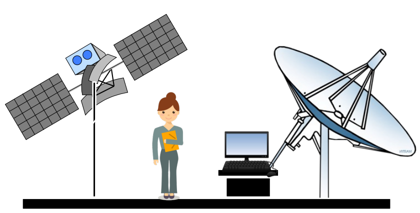
$$\Delta x_1 = \Delta x_2 \sqrt{1 - \frac{V^2}{c^2}} = (100\text{ cm}) \sqrt{1 - \left(\frac{0.5c}{c}\right)^2}$$

$$= (100\text{ cm}) \sqrt{1 - \frac{1}{4}} = \boxed{86.6\text{ cm}}$$

Seen from Frame #2 

Seen from Frame #1 

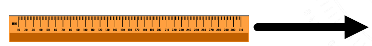
SPECIAL-RELATIVISTIC LENGTH CONTRACTION



Observer measures the length of meter sticks moving as shown.

Δx_1 would always be 1 meter if Galileo's relativity applied.

Meter stick
$\Delta x_2 = 1$ meter

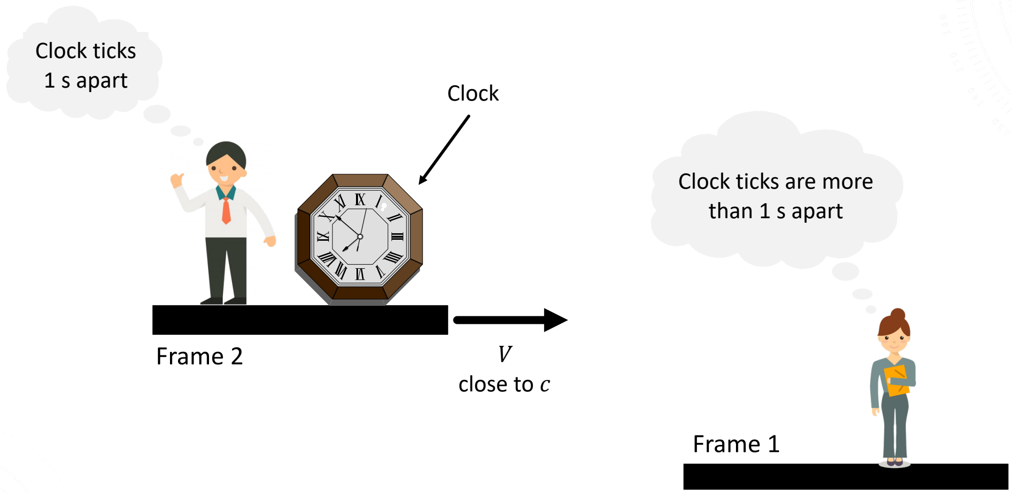


V

Her results:

V	Δx_1
0	1 meter
10 km/s	1 meter
1000 km/s	0.999994 meter
100,000 km/s	0.943 meter
200,000 km/s	0.745 meter
290,000 km/s	0.253 meter

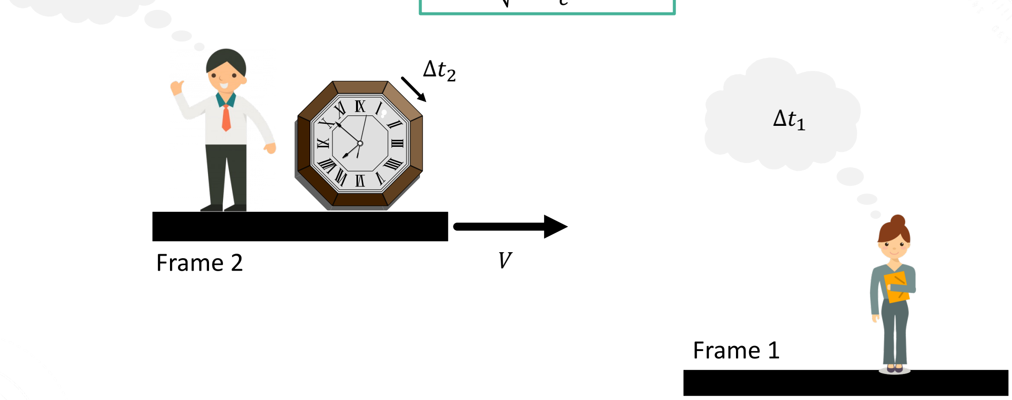
SPECIAL-RELATIVISTIC TIME DILATION



SPECIAL-RELATIVISTIC TIME DILATION

$$\Delta t_1 = \frac{\Delta t_2}{\sqrt{1 - \frac{V^2}{c^2}}} = \gamma \Delta t_2$$

(The clock is at rest in Frame 2.)



SPECIAL-RELATIVISTIC TIME DILATION

Example: A clock with a second hand is flying by at half the speed of light. How much time passes between ticks?

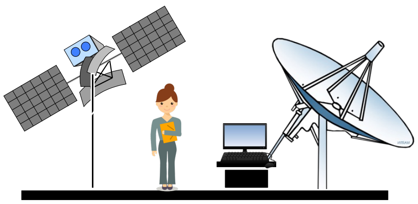
Consider the clock to be at rest in Frame #2 (again, its **rest frame**) and us to be at rest in Frame #1:

$$\Delta t_1 = \frac{\Delta t_2}{\sqrt{1 - \frac{V^2}{c^2}}} = \frac{(1\ \mathrm{s})}{\sqrt{1 - \left(\frac{0.5c}{c}\right)^2}}$$
$$= \frac{(1\ \mathrm{s})}{\sqrt{1 - \frac{1}{4}}} = \boxed{1.15\ \mathrm{s}}$$

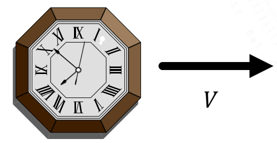
MORE DILATION

A clock, ticking once per second when at rest, flies past us at speed $0.995c$. How long between ticks, in our reference frame?

SPECIAL-RELATIVISTIC TIME DILATION



Clock
$\Delta t_2 = 1$ s between ticks



Observer measures the intervals between ticks on a moving clock, using her own clock for comparison.

Δt_1 would always be 1 second if Galileo's relativity applied.

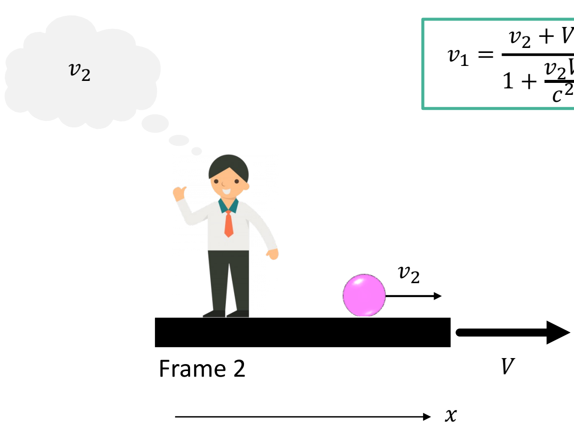
Her results:

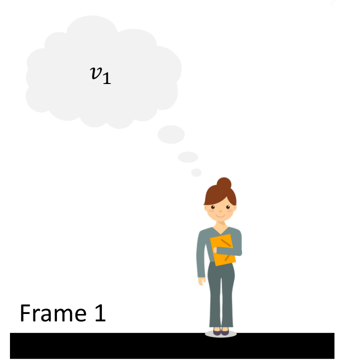
V	Δt_1
0	1 s
10 km/s	1 s
1000 km/s	1.000006 s
100 000 km/s	1.061 s
200 000 km/s	1.34 s
290 000 km/s	3.94 s

SPECIAL-RELATIVISTIC VELOCITY ADDITION – x

$$v_1 = \frac{v_2 + V}{1 + \frac{v_2 V}{c^2}}$$

Note that velocities can be positive or negative.





v_2 is positive → v_2 is negative ←

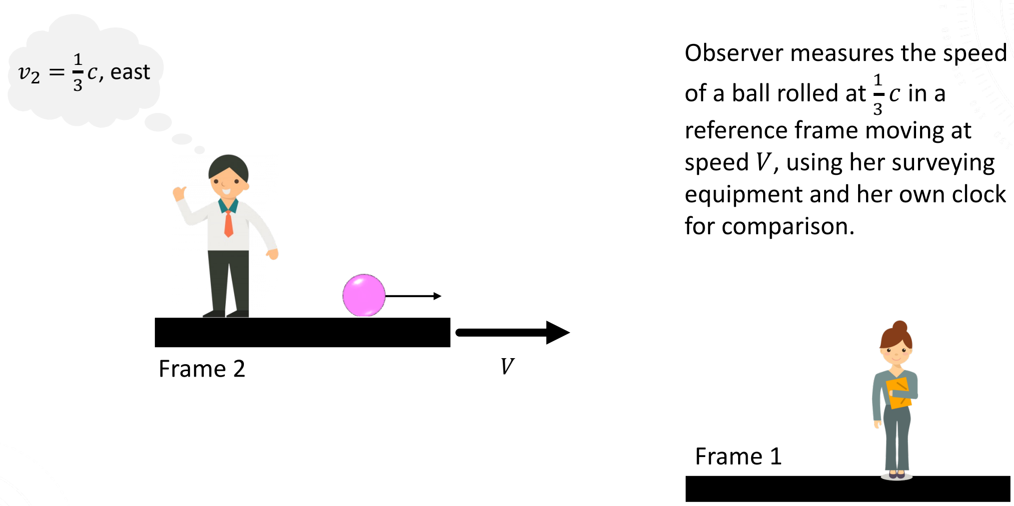
SPECIAL-RELATIVISTIC VELOCITY ADDITION – x

Example: Observer #2 is flying east by Observer #1 at half the speed of light. He rolls a ball at 100,000 km/s $\left(\frac{1}{3}c\right)$ toward the east. What will Observer #1 measure for the speed of the ball?

$$v_1 = \frac{v_2 + V}{1 + \frac{v_2 V}{c^2}} = \frac{\left(\frac{1}{3}c\right) + \left(\frac{1}{2}c\right)}{1 + \frac{\left(\frac{1}{3}c\right)\left(\frac{1}{2}c\right)}{c^2}}$$

$$= \frac{\frac{5}{6}c}{\frac{7}{6}} = \boxed{\frac{5}{7}c} \text{ (east)}$$

SPECIAL-RELATIVISTIC VELOCITY ADDITION – x



Observer measures the speed of a ball rolled at $\frac{1}{3}c$ in a reference frame moving at speed V, using her surveying equipment and her own clock for comparison.

SPECIAL-RELATIVISTIC VELOCITY ADDITION – x

Her results for the speed of the ball, for several values of the speed V of Frame #2 relative to Frame #1 (all towards the east):

V	v_1	
0	100,000 km/s	100,000 km/s
10 km/s	100,008.9 km/s	100,010 km/s
1000 km/s	100,888 km/s	101,000 km/s
100 000 km/s	179,975 km/s	200,000 km/s
200 000 km/s	245,393 km/s	300,000 km/s
290 000 km/s	294,856 km/s	390,000 km/s

If Galileo's relativity applied

SPECIAL-RELATIVISTIC VELOCITY ADDITION – x

Example: Observer #2 is flying east by Observer #1 at half the speed of light. He rolls a ball at 100,000 km/s $\left(\frac{1}{3}c\right)$ toward the **west**. What will Observer #1 measure for the speed of the ball?

$$v_1 = \frac{v_2 + V}{1 + \frac{v_2 V}{c^2}} = \frac{\left(-\frac{1}{3}c\right) + \left(\frac{1}{2}c\right)}{1 + \frac{\left(-\frac{1}{3}c\right)\left(\frac{1}{2}c\right)}{c^2}}$$

$$= \frac{\frac{1}{6}c}{\frac{5}{6}} = \boxed{\frac{1}{5}c} \text{ (east)}$$

VELOCITY ADDITION PRACTICE

Observer #2, flying east at $V = 0.995c$, rolls a ball at $0.995c$ **west**. How fast does the ball appear to Observer #1 to travel?