SPECIAL RELATIVITY – PART II

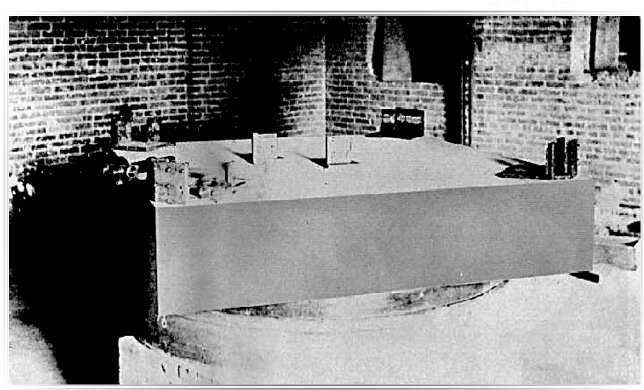
PROBLEM SET #2 ON WEBWORK – DUE MONDAY AT MIDNIGHT

SPECIAL RELATIVITY – PART II

The Lorentz transformation and the Minkowski absolute interval

The mixing of space and time (spacetime) and the relativity of simultaneity: several examples of the use of the absolute interval

Experimental tests of special relativity

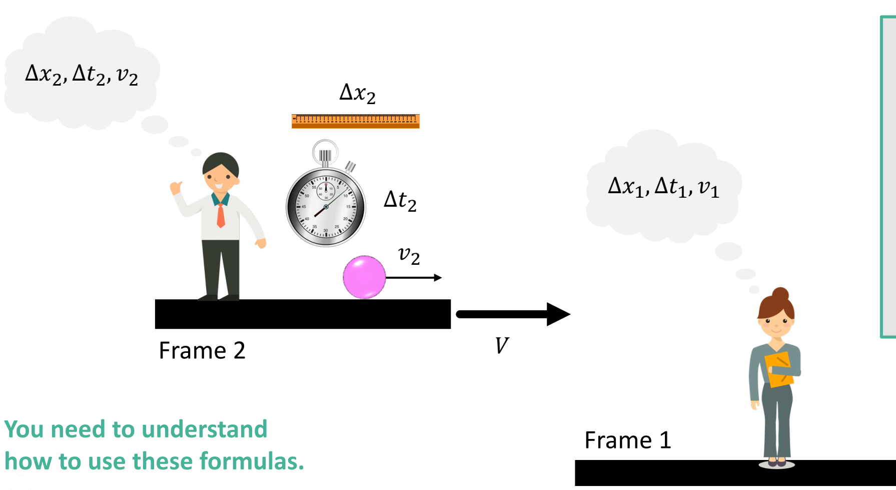


Michaelson-Morley experiment, mounted on a stone slab that is floating in mercury.

CONSEQUENCES & PREDICTIONS OF EINSTEIN'S SR

- **Length contraction (Lorentz-Fitzgerald contraction)**
- **Time dilation**
- **Velocities are relative**, except for that of light, and cannot exceed that of light
- **Spacetime mixing**: "distance" in a given reference frame is a mixture of distance and time from other reference frames
- **Simultaneity is relative.**
- **Mass is relative.**
- **There is no frame of reference in which light can appear to be at rest.**
- **Mass and energy are equivalent.**

EQUATIONS FOR LENGTH CONTRACTION, TIME DILATION, AND VELOCITY ADDITION IN ONE DIMENSION



$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{V}{c}\right)^2}}$$

$$\Delta x_1 = \frac{\Delta x_2}{\gamma}$$

$$\Delta t_1 = \gamma \Delta t_2$$

$$v_1 = \frac{v_2 + V}{1 + \frac{v_2 V}{c^2}}$$

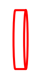
You need to understand how to use these formulas.

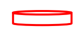
A QUESTION ON LENGTH CONTRACTION

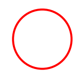
Your friend Tim can throw a baseball at 99.9% of the speed of light. You watch the baseball fly past you; because of length contraction, it looks like

A. A tiny sphere
B. A thin rod, the baseball's diameter in length
C. A thin disk, same diameter as the baseball
D. It did before

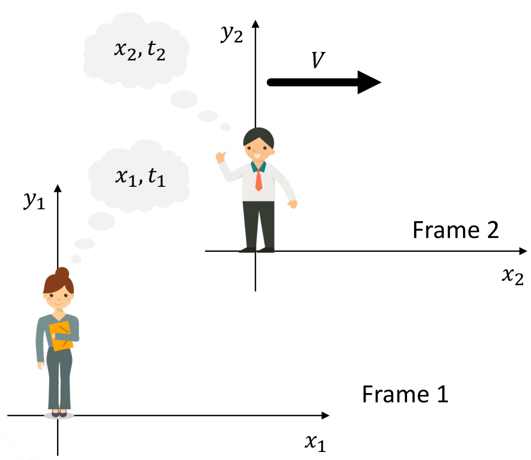
From our point of view, which way is the baseball oriented relative to its motion?

A. 

B. 

C. 

LORENTZ TRANSFORMATION: MIXING OF TIME AND SPACE



"Event" (say, a firecracker exploding)

Observers' coordinate systems coincide ($x_1 = x_2$) at $t_1 = t_2 = 0$. They both see an event, and report where and when it was.

LORENTZ TRANSFORMATION: MIXING OF TIME AND SPACE

The position and time at which the observers see the event are related by the following equations:

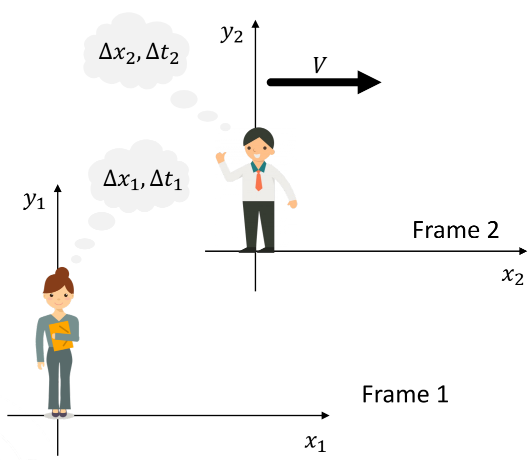
$$x_1 = \frac{x_2 + Vt_2}{\sqrt{1 - \frac{V^2}{c^2}}} = \gamma(x_2 + Vt_2) \qquad t_1 = \frac{t_2 + \frac{Vx_2}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} = \gamma\left(t_2 + \frac{Vx_2}{c^2}\right)$$

Lorentz transformation

- Position in one reference frame is a **mixture** of position and time from the other frame.
- Similarly, time in one reference frame is a mixture of position and time from the other frame.
- The mixture is generally called **spacetime**.

(We will not be using *these* equations on homework or exams.)

MINKOWSKI ABSOLUTE INTERVAL: MIXING OF TIME AND SPACE



Two events (say, two firecrackers exploding)

Each observer sees both events and reports the distance Δx and time interval Δt between them.

MINKOWSKI ABSOLUTE INTERVAL: MIXING OF TIME & SPACE

The distance and time interval each observer measures between the two events turn out to be related:

$$\Delta x_1^2 - c^2 \Delta t_1^2 = \Delta x_2^2 - c^2 \Delta t_2^2$$

Thus, the quantity

$$\begin{aligned}(\text{Absolute interval})^2 &= \Delta x_1^2 - c^2 \Delta t_1^2 = \Delta x_2^2 - c^2 \Delta t_2^2 \\ &= \Delta x^2 - c^2 \Delta t^2\end{aligned}$$

is constant; the same value is obtained with distance Δx and time interval Δt from **any** single frame of reference.

- This can be derived directly from the Lorentz transformation.
- **We will be using this formula for the absolute interval.**

MINKOWSKI ABSOLUTE INTERVAL

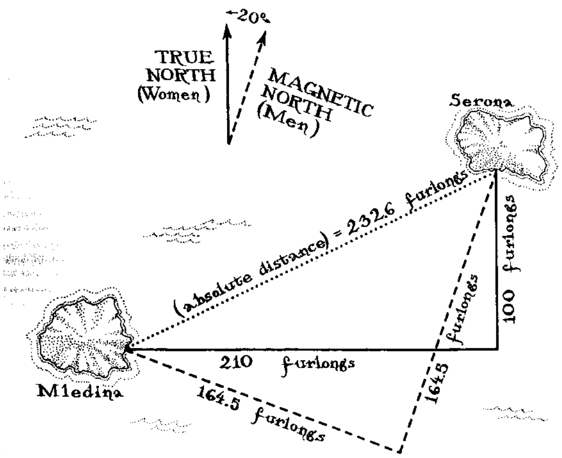
In other words:

$$\begin{aligned}(\text{Absolute interval})^2 &= (\text{distance in Frame 1})^2 - c^2(\text{time interval in Frame 1})^2 \\ &= (\text{distance in Frame 2})^2 - c^2(\text{time interval in Frame 2})^2\end{aligned}$$

Usually, we will have Frame #1 at rest (that makes it “our frame”), and Frame #2 in motion.

Events that occur simultaneously ($\Delta t = 0$) in one frame may not be simultaneous in another; simultaneity is relative.

GEOMETRIC ANALOGY FOR THE ABSOLUTE INTERVAL



Absolute distance (on a map) is covered the same for men and women, even though they take different paths and have different coordinate systems.

The direction the men call North is a mixture of the women's north and east. The direction the women call North is part north, part west, according to the men.

GEOMETRIC ANALOGY FOR THE ABSOLUTE INTERVAL

Absolute distance (on the map) is governed by the Pythagorean theorem:

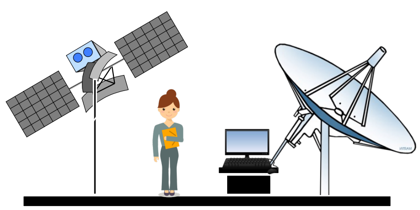
$$(\text{Absolute distance})^2 = (\text{distance north})^2 + (\text{distance east})^2$$

Note the similarity (and the differences) to the Minkowski absolute interval in special relativity:

$$(\text{Absolute interval})^2 = (\text{distance})^2 - c^2(\text{time interval})^2$$

EXAMPLE OF THE RELATIVITY OF SIMULTANEITY

An observer ("I") watches the firecrackers go off and records the time of each explosion.



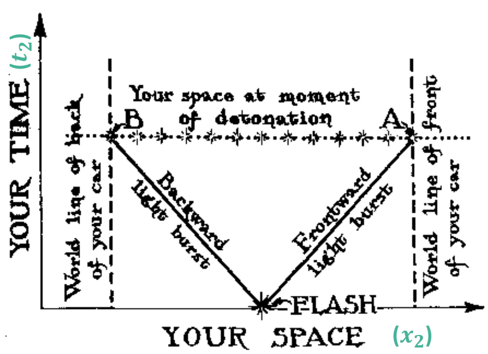
(a)

Firecrackers detonated simultaneously according to the driver ("you").

EXAMPLE OF THE RELATIVITY OF SIMULTANEITY

My results: I see the **rearmost** firecracker detonate first, followed by all the others in sequence.

Your spacetime diagram



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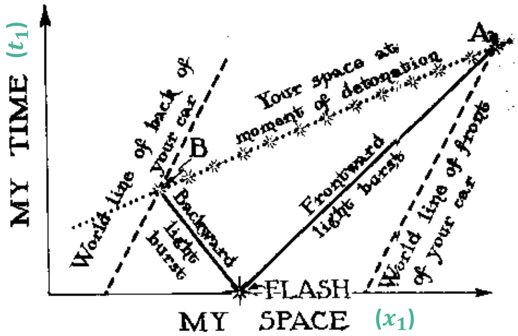

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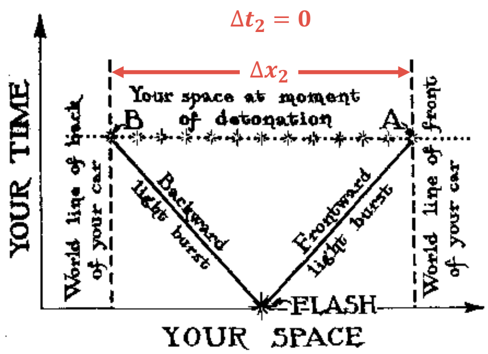
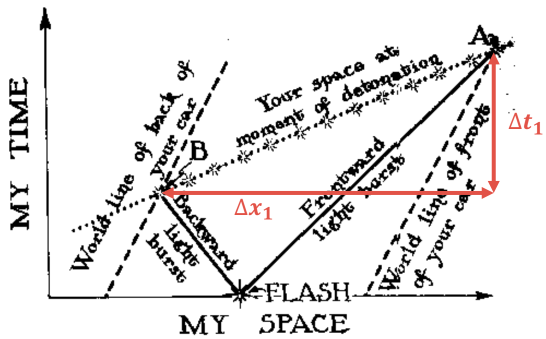


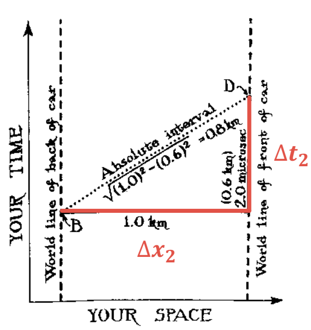
My spacetime diagram

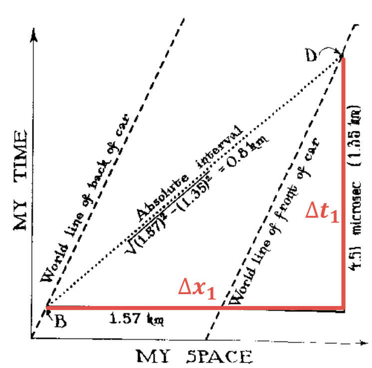


CAR-AND-FIRECRACKER EXPERIMENT #2

Car: 1 km long, moving at $1.62{\times}10^5$ km/s; car backfires (event B), then firecracker on front bumper detonates (event D)

Absolute interval is 0.8 km according to each observer, even though the time delay is $2{\times}10^{-6}$ s according to the observer in the car and $4.51{\times}10^{-6}$ s for the observer standing on the side of the road.





OTHER SIMPLE USES OF THE ABSOLUTE INTERVAL

Suppose that you (standing on the side of the road) were told that events B and D happened simultaneously as viewed from the car, and you measured that they appeared to be separated by 1.19 km along the road (instead of 1 km: longer, because of the motion of the car). **Do you also observe them to be simultaneous?**

The formula for Absolute Interval can be rearranged:

$$\Delta t_1 = \frac{1}{c}\sqrt{\Delta x_1^2 - \Delta x_2^2 + c^2 \Delta t_2^2} = \frac{\sqrt{(1.19\text{ km})^2 - (1\text{ km})^2 + (299792\text{ km/s})^2 (0\text{ s})^2}}{299792\text{ km/s}}$$

$$= 2.14 \times 10^{-6}\text{ s}$$ **No**

OTHER SIMPLE USES OF THE ABSOLUTE INTERVAL

Suppose that you (standing on the side of the road) saw the events B and D happen simultaneously, separated by 0.84 km along the road (shorter than 1 km: this is exactly the setup for Lorentz length contraction). **What was the time interval between events B and D as seen in the car?**

Again, we rearrange the formula for the absolute interval:

$$\Delta t_2 = \frac{1}{c}\sqrt{\Delta x_2^2 - \Delta x_1^2 + c^2 \Delta t_1^2} = \frac{\sqrt{(1\text{ km})^2 - (0.84\text{ km})^2 + (299792\text{ km/s})^2 (0\text{ s})^2}}{299792\text{ km/s}}$$

$$= 1.80 \times 10^{-6}\text{ s}$$

OTHER SIMPLE USES OF THE ABSOLUTE INTERVAL

Suppose the events B and D both happen at the same spot on the car (the tailpipe, for example), separated by 2×10^{-6} s as seen from the car. You see them to be 2.38×10^{-6} s apart (longer: this is exactly the setup for time dilation). **How far apart along the road do events B and D appear to you to be?**

Again, we rearrange the formula for the absolute interval:

$$\Delta x_1 = \sqrt{\Delta x_2^2 - c^2\Delta t_2^2 + c^2\Delta t_1^2}$$

$$= \sqrt{(0\ \mathrm{km})^2 - (299792\ \mathrm{km/s})^2(2\times10^{-6}\ \mathrm{s})^2 + (299792\ \mathrm{km/s})^2(2.38\times10^{-6}\ \mathrm{s})^2}$$

$$= 0.387\ \mathrm{km}$$

RECOGNIZE WHAT CONCEPT TO USE

What kind of problem is this? What formula should you use?

One type of radioactive particle decays in 2×10^{-6} s on average, if it is at rest. How long does it take if it is moving at $0.995c$?

A. Length contraction
B. Time dilation
C. Velocity addition
D. Absolute interval

RECOGNIZE WHAT CONCEPT TO USE

What kind of problem is this? What formula should you use?

In my car, 1 km long and moving at 99% of the speed of light, I flash my headlights and taillights simultaneously; you see the flashes delayed by 4×10^{-6} s. How far apart along the road are the spots when the flashes appeared to you to occur?

A. Length contraction
B. Time dilation
C. Velocity addition
D. Absolute interval

RECOGNIZE WHAT CONCEPT TO USE

What kind of problem is this? What formula should you use?

I throw a meter stick so that it moves parallel to its length; it looks to you to be only half a meter long. How fast is it moving, relative to us?

A. Length contraction
B. Time dilation
C. Velocity addition
D. Absolute interval