GENERAL RELATIVITY

PROBLEM SET #3 AVAILABLE ON WEBWORK – DUE WEDNESDAY, 9/25

GENERAL RELATIVITY

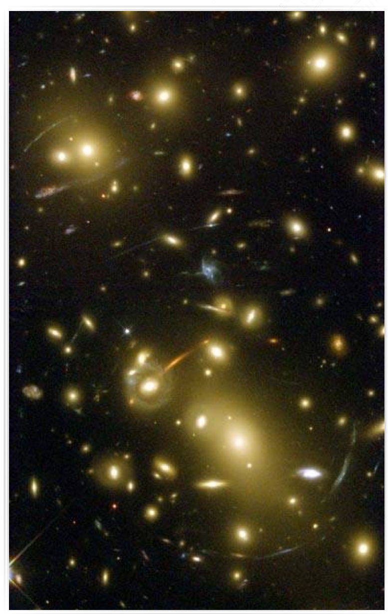
Tests of special relativity

The principle of equivalence

Gravitational time dilation, special-relativistic time dilation, and the Doppler effect

Curved spacetime and the nature of tides

Incorporation of gravity into Einstein's theories: the general theory of relativity



"Giant arc" gravitational lenses in the galaxy cluster Abell 2218 (Andy Fruchter et al., HST/NASA/STScI)

EXPERIMENTAL TESTS OF RELATIVITY

Einstein's theories of relativity represent a rebuilding of physics from the ground up.

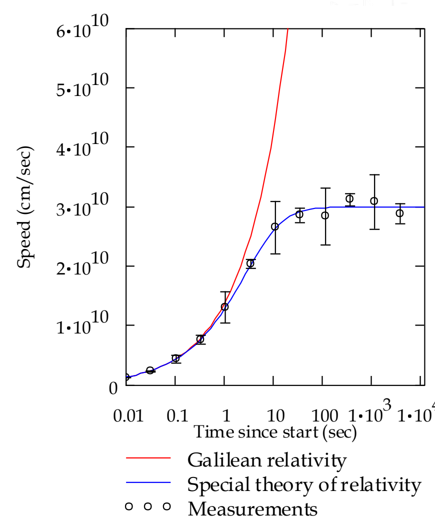
- New postulates and assumptions relate the most basic concepts, such as the relativity of space and time and the "absoluteness" of the speed of light.
- The new theory is logically consistent and mathematically very elegant.
- The new theory *contains* classical physics, as an approximation valid for speeds much smaller than the speed of light.
- Still: it would be worthless if it did not agree with reality (i.e. experiments) better than classical physics.

EXPERIMENTAL TESTS OF SCIENTIFIC THEORIES

No scientific theory is valid unless:

- It is mathematically and logically consistent
- **And** its predictions can be measured in experiments
- ***And*** the theoretical predictions are in precise agreement with experimental measurements

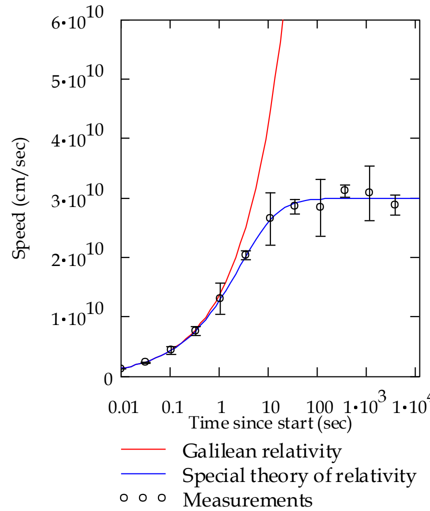
Example: Prediction by the "**old**" and "**new**" relativity theories for the speed of a body accelerated with constant power. Both theories are valid for low speeds, but only special relativity is valid over the whole range of measurements.



EXPERIMENTAL TESTS OF SCIENTIFIC THEORIES

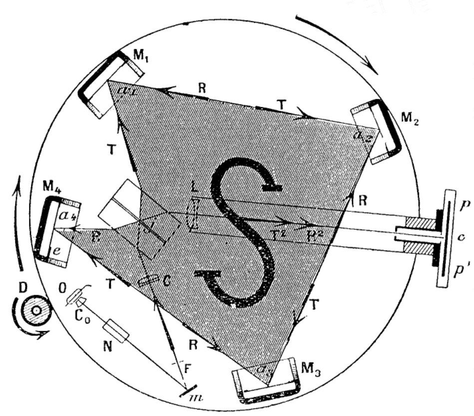
What constitutes a valid scientific experiment?

- Measurements are made with accuracy (the size of potential measurement errors) sufficient to test the predictions of prevailing theories
- **And** the accuracy can be reliably estimated
- ***And*** the measurements are **reproducible**: the same results are obtained whenever, and by whomever, the experiments are repeated



EXPERIMENTAL TESTS OF SPECIAL RELATIVITY

- Michelson-Morley type experiments: speed of light always the same in all directions in all inertial reference frames, to **extremely** high accuracy. Repeated many times, and not just in Cleveland.
- Nuclear reactors/bombs: mass-energy equivalence ($E = mc^2$) via fission

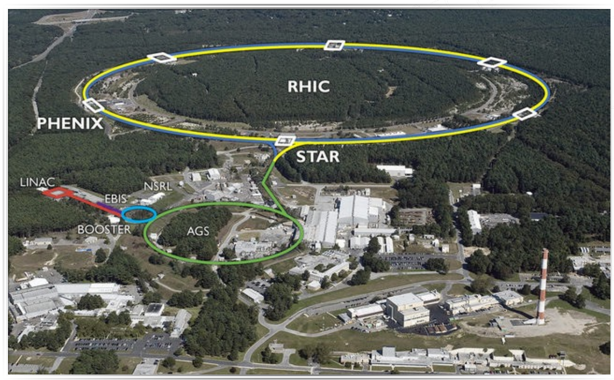


Sagnac interferometer (spins while emitting light that travels in a loop)

EXPERIMENTAL TESTS OF SPECIAL RELATIVITY

High-energy accelerators used in elementary particle physics

- Radioactive particles are seen to live much longer when moving near light speeds than when at rest (direct observation of time dilation).
- Heavy ions get denser (length contraction) as they move faster, and the EM field becomes more intense in the direction of motion.
- Though accelerated particles get extremely close to the speed of light, none ever exceed it.
- Particles become more massive the faster they travel ($E = mc^2$)



Relativistic heavy ion collider at BNL

BEYOND THE SPECIAL THEORY OF RELATIVITY

In 1907, Einstein was asked by the editors of a prominent physics journal to write a review article on relativity. While he was at it, he got to thinking about the **limitations** of special relativity, as well as its successes.

- Special relativity only applies to **inertial** reference frames, and thus could not be used to describe anything but motion in a straight line at a constant speed.
- Therefore, it could not be applied to situations involving **forces,** such as electromagnetic forces involving charges and currents (though it works well for light), or the force of **gravity.**
- Einstein was able to show that no simple modification of the special theory of relativity could fix these problems.

BEYOND THE SPECIAL THEORY OF RELATIVITY

Three new and important ideas occurred to Einstein as he was working on his review article:

- If you **free fall** under the influence of gravity, you feel **weightless.** In this case, your reference frame, though non-inertial, should *act* like an inertial one.
- Gravity *should* itself give rise to time dilation: gravity warps time.
- The phenomenon of tides can equally well be conceived as either a force or a property of curved space: gravity warps space.

These were the first steps in a new relativity theory that could be applied to both inertial *and* non-inertial reference frames.

IDEA #1: THE PRINCIPLE OF EQUIVALENCE

A frame of reference falling freely under the influence of gravity (and in vacuum) is **equivalent** to an inertial reference frame, and special relativity and all other laws of physics appear as usual.

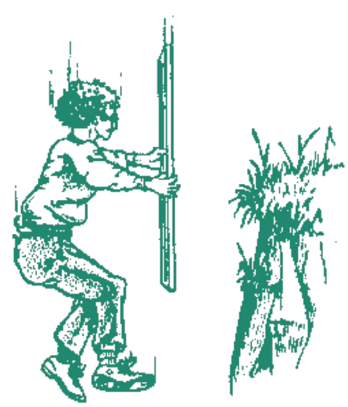
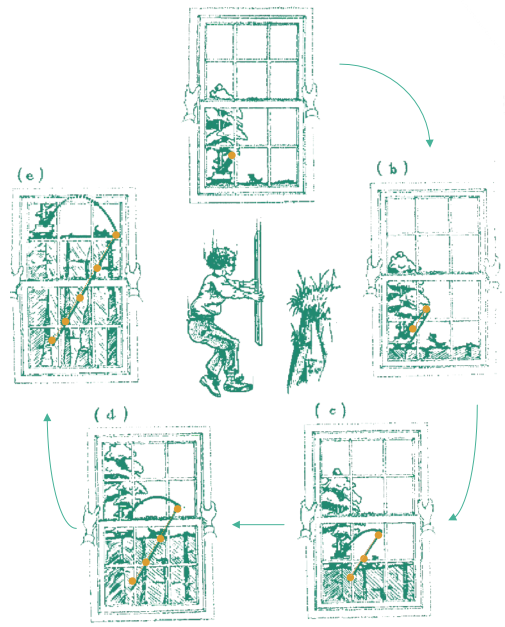


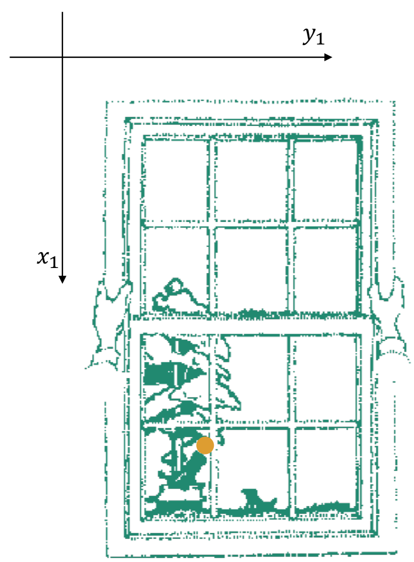
ILLUSTRATION (A) OF THE PRINCIPLE OF EQUIVALENCE

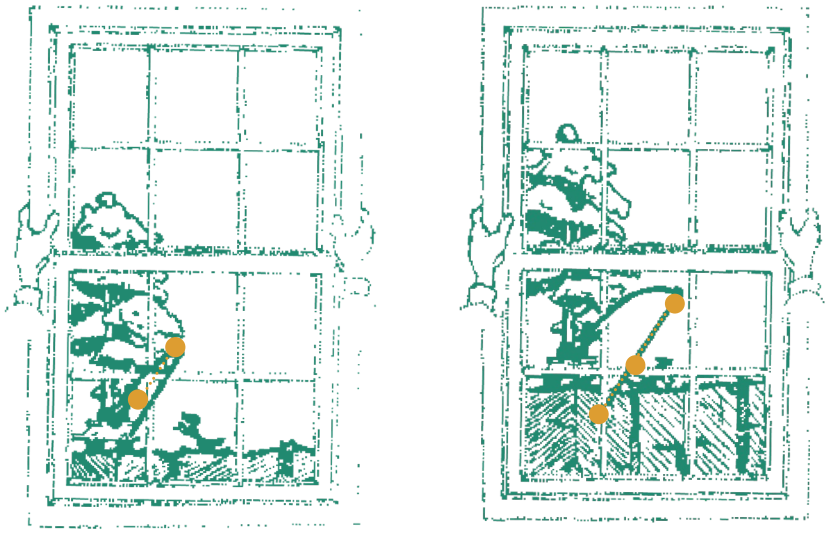
A cannonball is seen by a dog and by a freely-falling observer. **It appears to travel in a straight line to the observer, as it should if both the cannonball and the observer were in inertial reference frames.** (To see the straight line, mark the ball's positions on the window and connect the dots.)



BOTH THE CANNONBALL AND OBSERVER FALL FREELY

Think of the window pane as the falling observer's coordinate system, on which he can mark the position of the cannonball.





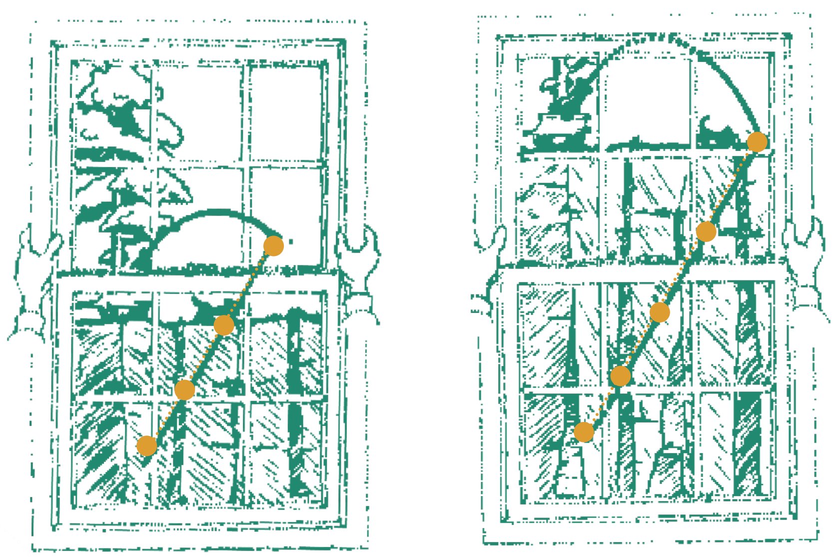
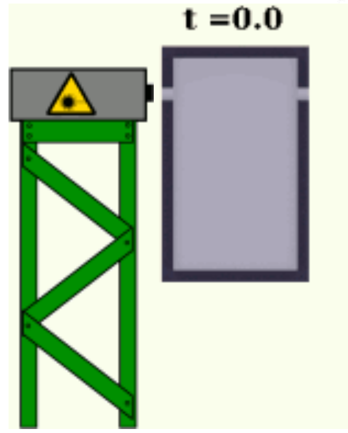


ILLUSTRATION (B) OF THE PRINCIPLE OF EQUIVALENCE

Einstein thought about it more like this: Suppose you are in a BIG elevator with two small windows. I am watching from the outside, and at $t = 0$, I cut the elevator cable and send a **pulse** of laser light aimed through the windows.

- I see the pulse go through both windows, and I see the pulse and elevator respond to the force of gravity and accelerate toward the ground: the light has energy, so it has mass $m = \frac{E}{c^2}$ and will suffer the same gravitational acceleration as the elevator.

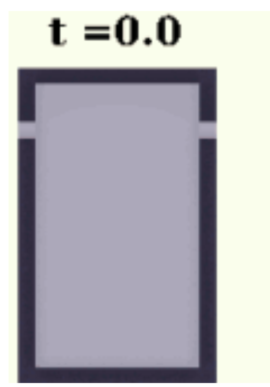


Animation from Einstein Online (Max Planck Institute for Gravitational Physics)

ILLUSTRATION (B) OF THE PRINCIPLE OF EQUIVALENCE

- From inside the elevator, you are oblivious to the acceleration: you feel weightless, and the light pulse seems to you to have traveled in a straight path between the windows. For all you know, you are in an inertial reference frame, since you can feel no external forces nor see any effect on the light pulse.

(Once again, we see how imaginative Einstein was: this seems natural to us, but he did not grow up watching weightless astronauts taking space-walks, and lasers were not invented until the 1950s.)



Animation from Einstein Online (Max Planck Institute for Gravitational Physics)

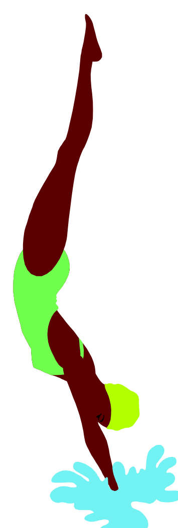
EQUIVALENCE OF INERTIAL AND GRAVITATIONAL MASS

In his thought experiments, Einstein assumed that any two masses subject to a given gravitational force – like the elevator and the light – would exhibit the same acceleration.

- This had been demonstrated in a series of famous experiments (1885-1909) by Loránd Eötvös.
- Specifically: Eötvös's experiments showed that **inertial mass** (the ratio of force to acceleration for forces besides gravity, e.g. that exerted by a spring) and **gravitational mass** (the ratio of gravitational force to acceleration) are equal within an accuracy of one part in 10^8.
- This equivalence of inertial and gravitational mass is often called the **weak equivalence principle,** to distinguish it from Einstein's equivalence principle.

IMPORTANCE OF THE PRINCIPLE OF EQUIVALENCE

- A free-falling reference frame is under the influence of gravity but is **equivalent** to an inertial frame.
- Equivalent means that **the ordinary laws of physics and special relativity apply**.
- Thus, Einstein found one class of reference frames in which gravity existed and in which he knew what the laws of physics were. This made it easier for him to understand how physics works in other non-inertial frames.



QUESTION ON THE EQUIVALENCE PRINCIPLE

You are in the spaceship in which the Course Prologue took place, halfway toward Vega and accelerating at precisely the same rate as the elevator was just accelerating toward the ground. Is this equivalent to an inertial reference frame?

A. Yes, because the ship is moving just as the elevator did.

B. Yes, because the velocity is constant.

C. No, because you feel the force of the spaceship pushing on you.

D. No, because Vega's gravity is larger than Earth's

IDEA #2: GRAVITATIONAL TIME DILATION

To a distant observer, time appears to pass more slowly in places where gravity is strong.

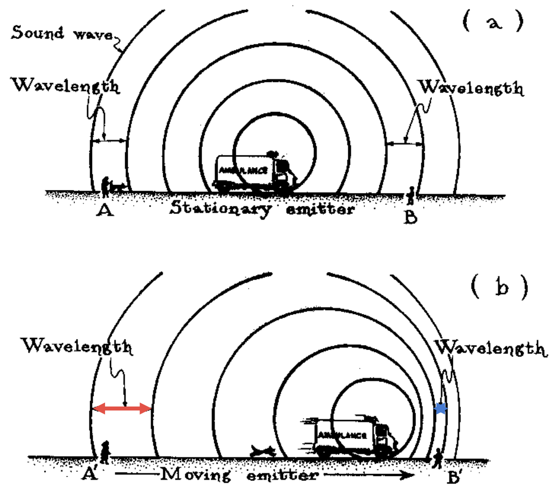
To an observer in a place where gravity is strong, time appears to pass more quickly in places where gravity is weak.

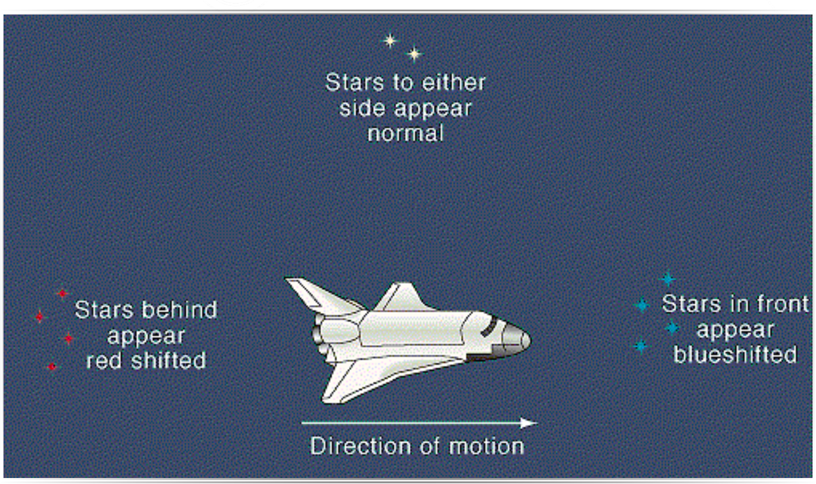
- Both statements embody the idea that **gravity warps time.**
- This sort of time dilation is importantly different from the special-relativistic version of time dilation!
  - In special relativity, two observers in inertial frames moving with respect to each other see the other's clock as moving slowly.
- Einstein thought of gravitational time dilation as an example of the **Doppler effect.**

DOPPLER EFFECT (EXAMPLE OF SOUND WAVES)

The pitch one perceives for a sound changes depending upon the motion of the source: the pitch is higher (**wavelength shorter**) for approaching sources, and lower (**wavelength longer**) for receding sources.

All waves, including light, exhibit the Doppler effect.

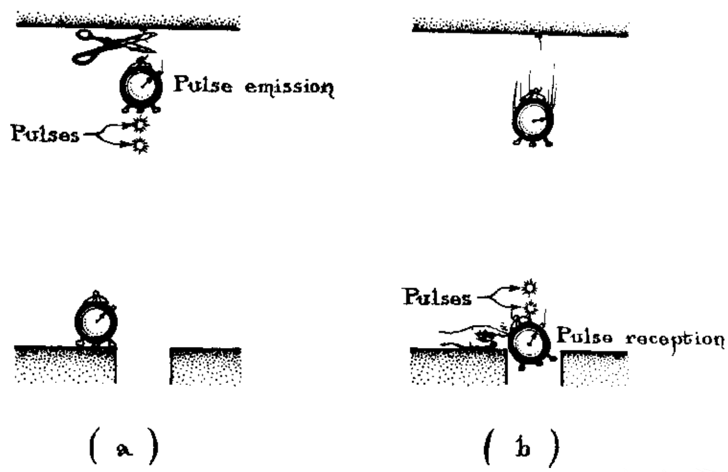




Astronomy Today (Chaisson & McMillan)

DOPPLER EFFECT FOR LIGHT

Approaching objects look bluer, receding objects redder, than their natural colors.



EINSTEIN'S THOUGHT EXPERIMENT

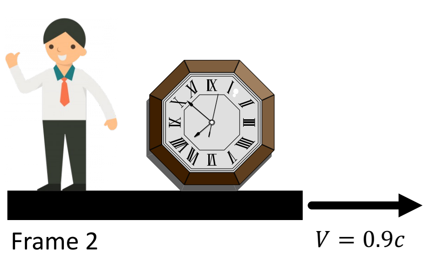
Upper clock sends light pulses (one per tick) just as its cord is cut.

To the lower clock, these pulses are closer together in time than its own ticks – its own ticks take longer than those it sees (from the light pulses) of the upper clock.

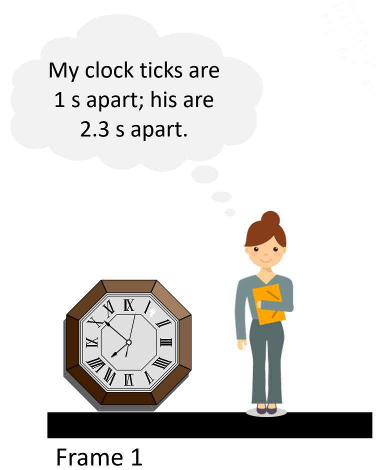
Both are arranged to be in free-falling frames = inertial frames, so that "ordinary" physics and special relativity apply.

SPECIAL-RELATIVISTIC TIME DILATION IS "SYMMETRICAL"

The clocks are identical.

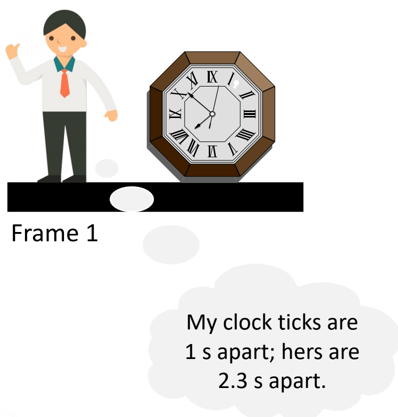


$$\Delta t_1 = \frac{\Delta t_2}{\sqrt{1 - \frac{V^2}{c^2}}} = \frac{1\ \mathrm{s}}{\sqrt{1-(0.9)^2}} = 2.3\ \mathrm{s}$$

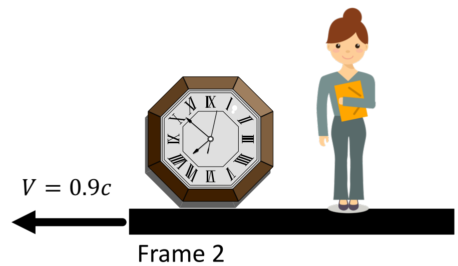


SPECIAL-RELATIVISTIC TIME DILATION IS “SYMMETRICAL”

The clocks are identical.



$$\Delta t_1 = \frac{\Delta t_2}{\sqrt{1-\frac{V^2}{c^2}}} = \frac{1\ \mathrm{s}}{\sqrt{1-(-0.9)^2}} = 2.3\ \mathrm{s}$$



GRAVITATIONAL TIME DILATION IS NOT SYMMETRICAL

The clocks are identical.