GENERAL RELATIVITY & GRAVITATIONAL RADIATION

PROBLEM SET #3 ON WEBWORK – DUE WEDNESDAY AT MIDNIGHT

EXAM #1 NEXT THURSDAY (9/26) IN CLASS

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- 1 hr 15 min in-class exam, open book and open notes
- Things that you should DEFINITELY bring with you:
  - Writing utensil (pencil or pen – blue or black ink)
  - Calculator
- Things that you should PROBABLY bring with you:
  - Lecture notes
  - Laptop or tablet (so that you can access the WeBWorK homework problems and the "How Big Is That?" sheet on the course website)
- REVIEW SESSION – Wednesday, 9/25 at 7:30pm in B&L 372

GENERAL RELATIVITY & GRAVITATIONAL RADIATION

Curved spacetime and the nature of tides

Incorporation of gravity into Einstein's theories: the general theory of relativity

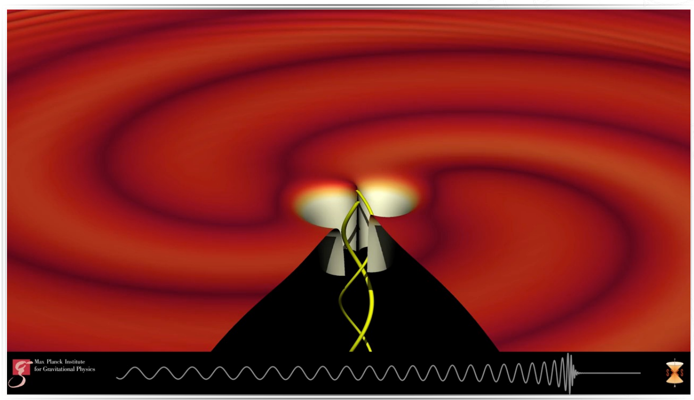
The Einstein field equation

More details about light

Gravitational radiation – gravity's counterpart to light

Experimental tests of general relativity

The Hulse-Taylor pulsar and the discovery of gravitational radiation



Gravitational waves and spacetime diagram of two converging black holes

TIMING RADIO SIGNALS

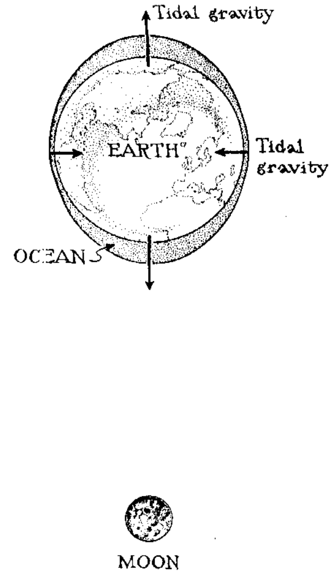
You receive radio signals of clock ticks from a distant friend of yours; instead of the 1s tick interval your friend advertised, you see them to be 2s apart. Your friend is

A. Moving fast, toward you
B. Moving fast, away from you
C. At rest, but near a black hole
D. Can't tell!

Then, you check the wavelength of the radio signal you are receiving, and it turns out to be half as long as your friend said it is. Your friend is

A. Moving fast, toward you
B. Moving fast, away from you
C. At rest, but near a black hole
D. Can't tell!

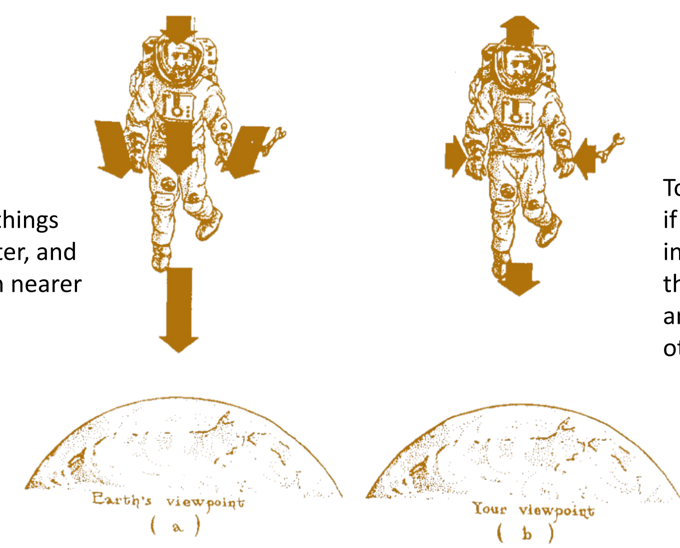
IDEA #3: TIDES



Tides can be thought of as an effect of gravitational force (as Newton did), or as the effect of a curvature, warping or stretching of space created by the presence of the gravitating mass ("space is warped by gravity").

TIDAL GRAVITY

Earth pulls all things toward its center, and pulls harder on nearer things.

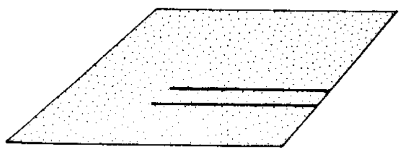


To you, it seems as if you are stretched in the direction of the Earth's center and squeezed in the other direction.

ODD GEOMETRICAL PROPERTIES OF CURVED SPACE

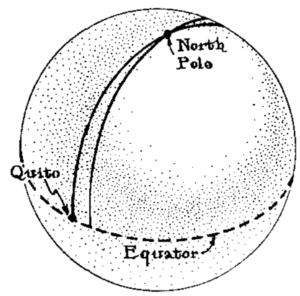
Flat space

Parallel lines never intersect.

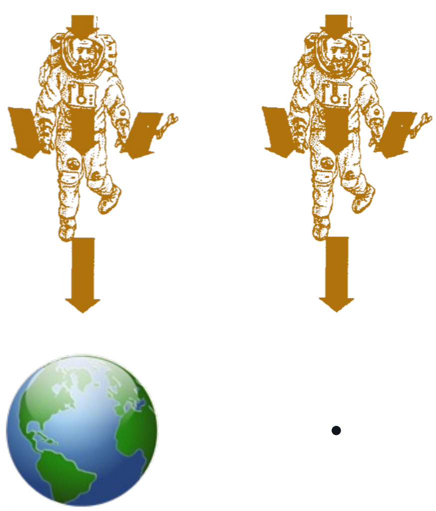


Curved space

Parallel lines *can* intersect. Consider lines of longitude, which are obviously parallel when they cross the equator, but they meet at each Pole.



PREVIEW: TIDES FROM BLACK HOLES



It turns out that the formulae describing tidal acceleration are exactly the same in general relativity as in classical (Newtonian) physics.

Thus, tides are the same at a given distance (greater than the planet's radius) away from a planet as they are at the same distance away from a black hole with the same mass as the planet.

GENERAL THEORY OF RELATIVITY

Einstein's deductions from these ideas led to the General Theory of Relativity.

- Particles and light follow **geodesics**: the shortest paths between two points. These are straight lines, if space and time are not curved.
- In general, **space and time are warped,** so that the geodesics are generally not straight lines.
- Masses and energies present in space and time determine how space and time are warped. This process is what we call **gravity**.

The general theory of relativity can be summed up in one statement:

Spacetime, with its curvature, tells masses how to move; masses tell spacetime how to curve.

HIEROGLYPHS – EINSTEIN'S FIELD EQUATION

The field equation is the ultimate mathematical expression of Einstein's theory of general relativity.

Astronomy 102 version:

"With its curvature, spacetime tells masses how to move; masses tell spacetime how to curve."

Physics 413 / Astronomy 554 version:

$$\frac{1}{2}\left[\frac{\partial^2 g^{\lambda}_{\lambda}}{\partial x^{\kappa}\partial x^{\mu}} - \frac{\partial^2 g_{\mu\lambda}}{\partial x^{\kappa}\partial x_{\lambda}} - \frac{\partial^2 g_{\lambda\kappa}}{\partial x_{\lambda}\partial x^{\mu}} + \frac{\partial^2 g_{\mu\kappa}}{\partial x^{\lambda}\partial x_{\lambda}}\right] + g_{\eta\sigma}\left[\frac{\partial x^{\eta}}{\partial \xi^{\alpha}}\frac{\partial^2 \xi^{\alpha}}{\partial x^{\lambda}\partial x_{\lambda}}\frac{\partial x^{\sigma}}{\partial \xi^{\alpha}}\frac{\partial^2 \xi^{\alpha}}{\partial x^{\mu}\partial x^{\kappa}} - \frac{\partial x^{\eta}}{\partial \xi^{\alpha}}\frac{\partial^2 \xi^{\alpha}}{\partial x^{\kappa}\partial x_{\lambda}}\frac{\partial x^{\sigma}}{\partial \xi^{\alpha}}\frac{\partial^2 \xi^{\alpha}}{\partial x^{\mu}\partial x^{\lambda}}\right]$$

$$-\frac{1}{2}g_{\mu\kappa}\left(\frac{1}{2}\left[2\frac{\partial^2 g^{\lambda}_{\lambda}}{\partial x^{\mu}\partial x_{\mu}} - \frac{\partial^2 g^{\mu}_{\lambda}}{\partial x^{\mu}\partial x_{\lambda}} - \frac{\partial^2 g_{\lambda\mu}}{\partial x_{\lambda}\partial x_{\mu}}\right] + g_{\eta\sigma}\left[\frac{\partial x^{\eta}}{\partial \xi^{\alpha}}\frac{\partial^2 \xi^{\alpha}}{\partial x^{\lambda}\partial x_{\lambda}}\frac{\partial x^{\sigma}}{\partial \xi^{\alpha}}\frac{\partial^2 \xi^{\alpha}}{\partial x_{\mu}\partial x^{\mu}} - \frac{\partial x^{\eta}}{\partial \xi^{\alpha}}\frac{\partial^2 \xi^{\alpha}}{\partial x^{\mu}\partial x_{\lambda}}\frac{\partial x^{\sigma}}{\partial \xi^{\alpha}}\frac{\partial^2 \xi^{\alpha}}{\partial x_{\mu}\partial x^{\lambda}}\right]\right)$$

$$= -8\pi G \sum_{n} p_{n\mu}\frac{dx_{n\kappa}}{dt}\delta^3(x - x_n)$$

WHAT YOU GET WHEN YOU SOLVE THE FIELD EQUATION

In case you are interested (i.e., **not on the exam**)…

- The solution to the field equation is a function called the **metric tensor** (g, in the field equation). This function tells how much distance or time is displaced in each dimension per unit displacement in a given dimension.
  - The metric tensor describes all the details of the curvature of spacetime that correspond to the mass distribution entered on the right side of the equation.
- Accordingly, the metric is related to the **absolute interval**. Each different solution to the field equation corresponds to a different absolute interval.

GRAVITATIONAL RADIATION (AKA GRAVITATIONAL WAVES)

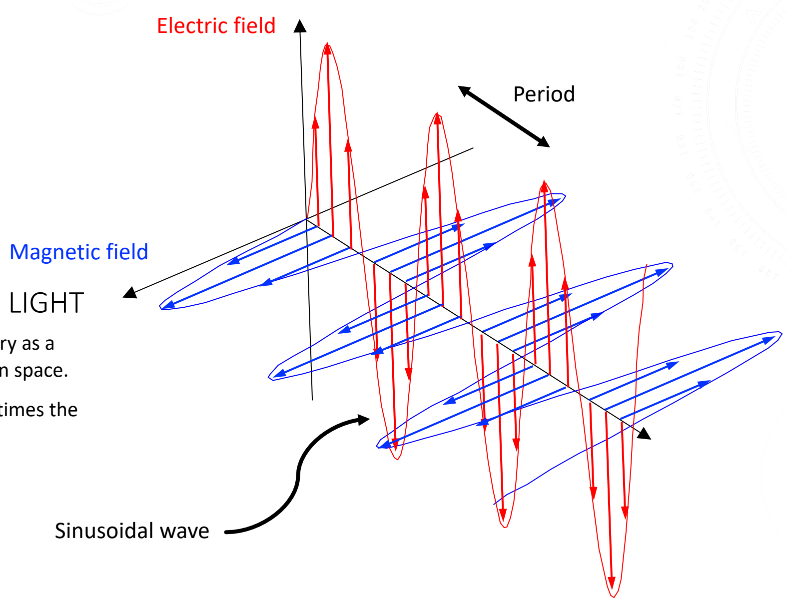
One of the first results Einstein obtained from his new general theory of relativity was that there should be a gravitational analog of light.

- By writing the field equation for spacetime that contains no masses, an equation is generated that has **waves** of curvature as its solution.
  - Specifically: the components of the metric tensor g vary in a periodic, repeating manner as the wave passes by a given point in space.
- These waves would propagate through empty spacetime at the same speed as light.
- Einstein noted that the effects of such a wave would be very weak, though, and doubted that gravitational radiation would ever be observed.

INTERLUDE: LIGHT

Practically all information that humanity has collected about celestial objects has arrived in the form of light.

- Like every other elementary form of energy, light exhibits both wave and particle properties depending on what experiment is being performed on it.
- In its **wave** guise, it consists of waves of electric and magnetic fields.
- This was first inferred by Maxwell in the 1860s: By writing the Maxwell equations for space that contains no electric charges or currents, and combining the results, equations are generated for the electric and magnetic field that have **sinusoidal** waves of electric and magnetic fields as their solution.

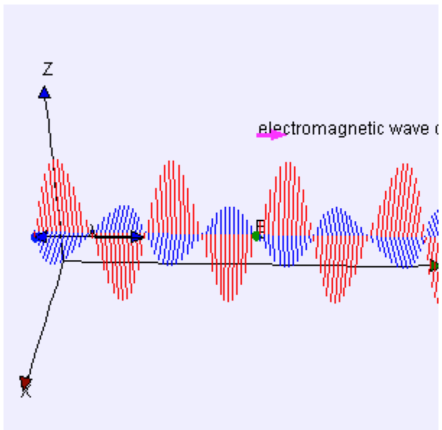


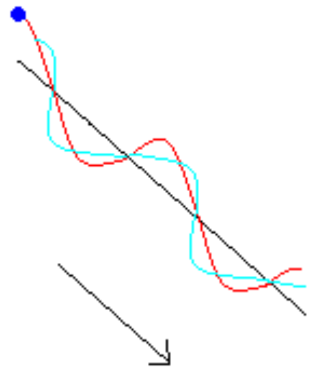
A "PLANE WAVE" OF LIGHT

Electric and magnetic fields vary as a function of time at one point in space.

The **wavelength** is the period times the speed of light.

A "PLANE WAVE" OF LIGHT

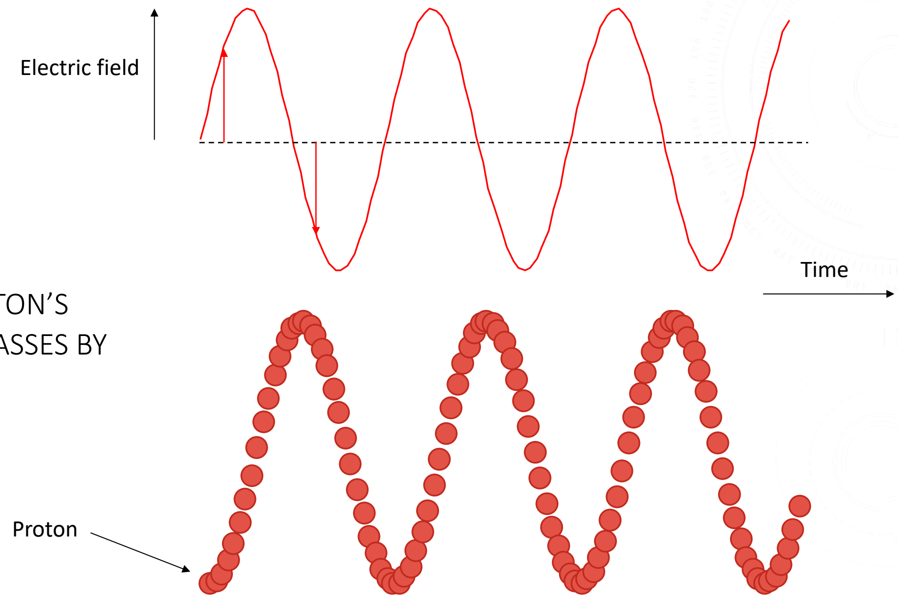




SOME PROPERTIES OF LIGHT

- The ripples of electric and magnetic field that comprise light travel through empty space at the speed of light.
- An electric field exerts a force on electric charges in the direction of the field. A magnetic field exerts a weaker force on a moving charge in the direction perpendicular to both the field and the velocity.
  - Individual electric charges – like protons or electrons – will accelerate in response to a passing light wave.
- If charges are accelerated – perhaps by some other force – they emit light.
- Light represents the transport of electromagnetic energy through empty space without involving the transport of electric charges or current.

SNAPSHOT OF A PROTON'S POSITION AS LIGHT PASSES BY



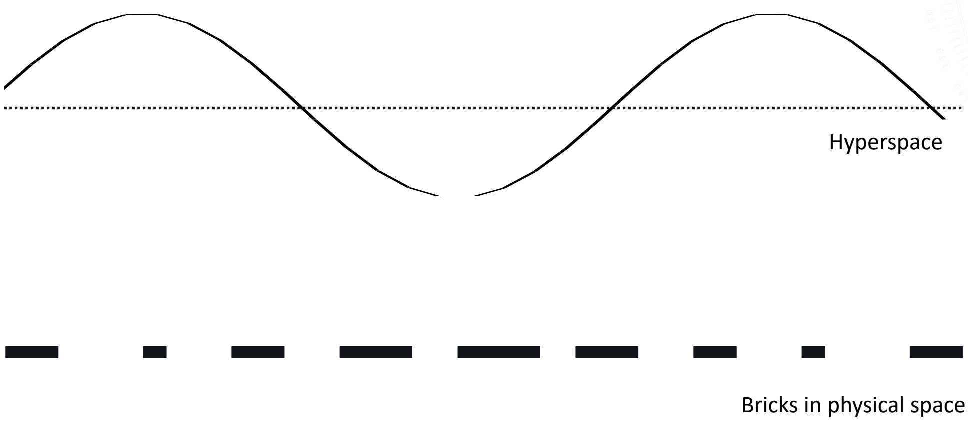
SOME PROPERTIES OF GRAVITATIONAL RADIATION

- A gravitational field exerts force on masses in the direction of the field. Alternatively, you can also think of this as changing the curvature of spacetime, which leads to the motion of masses.
  - Spacetime will warp (masses will accelerate) in response to a passing gravitational wave.
- If spacetime is warped (or masses are accelerated), then gravitational radiation is produced.
- Gravitational radiation represents the transport of gravitational energy through empty space without involving the transport of (rest) masses.

Note the direct analogy of gravitational waves and light, and of masses and electric charges/currents.

NO GRAVITATIONAL WAVE

GRAVITATIONAL WAVE



FIND THE INCORRECT STATEMENT

A. Gravitational waves are solutions to the Einstein field equation, as light waves are to the Maxwell equations.

B. A gravitational wave makes a mass bob up and down in physical space, as light makes electric charges do.

C. Gravitational waves travel through vacuum at speed *c*, just like light.

D. Gravitational waves are traveling bundles of gravitational field, just as light waves are traveling bundles of electric and magnetic fields.