GRAVITATIONAL RADIATION, GENERAL RELATIVITY & THE EXISTENCE OF BLACK HOLES

PROBLEM SET #3 (ON WEBWORK) DUE WEDNESDAY AT MIDNIGHT

REVIEW SESSION WEDNESDAY (9/25)

EXAM #1 ON THURSDAY!

- 1 hr 15 min in-class exam, open book and open notes
- Things you should DEFINITELY bring with you:
  - Writing utensil (pencil or pen – blue or black ink)
  - Calculator
- Things you should PROBABLY bring with you:
  - Lecture notes
  - Laptop or tablet (so that you can access the WeBWorK homework problems and the “How Big Is That?” sheet on the course website)
- REVIEW SESSION – Wednesday, 9/25, at 7:30pm in B&L 372

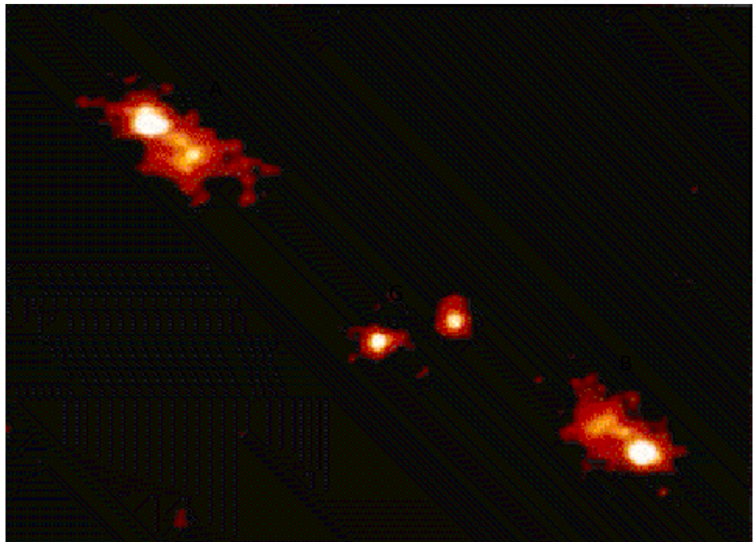
GRAVITATIONAL RADIATION, GENERAL RELATIVITY & THE EXISTENCE OF BLACK HOLES

Experimental tests of general relativity

The Hulse-Taylor pulsar and the discovery of gravitational radiation

Schwarzschild solves Einstein's equations applied to stars, and finds black holes in the results

Singularities in physics: why no one worried about Schwarzschild's black holes for awhile.



Gravitational lens in galaxy cluster AC114 (Smail et al. 1995). Note that the "quasars" A and B are mirror images of each other; they are both images of a single quasar lying far behind the cluster to which galaxy G belongs.

DIRECT DETECTION OF GRAVITATIONAL RADIATION

How can we directly detect gravitational waves?

- **Bar detectors**: Make very precise length measurements of a solid bar, which will stretch back and forth when a gravitational wave passes by, as did the bricks in our previous example.
- **Laser interferometers**: Ultra-precise "bar length" measurements, capable of bypassing some of the limitations of the ordinary bar detectors.
  - LIGO (the Laser Interferometer Gravitational-wave Observatory) and Virgo are based upon this technology.

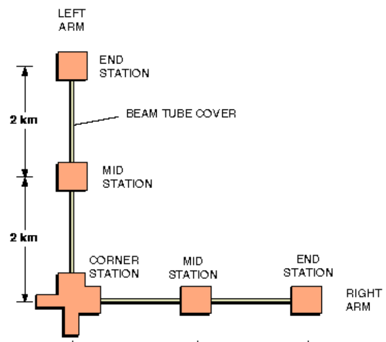
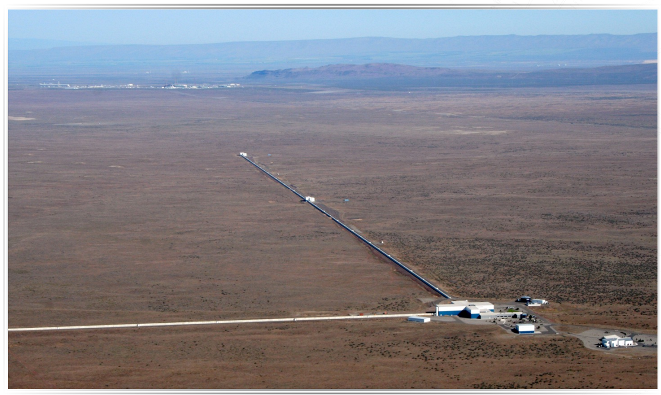


Fig. 1: Schematic layout of LIGO Site at Hanford, WA
(Installation at Livingston, LA has no mid-stations)

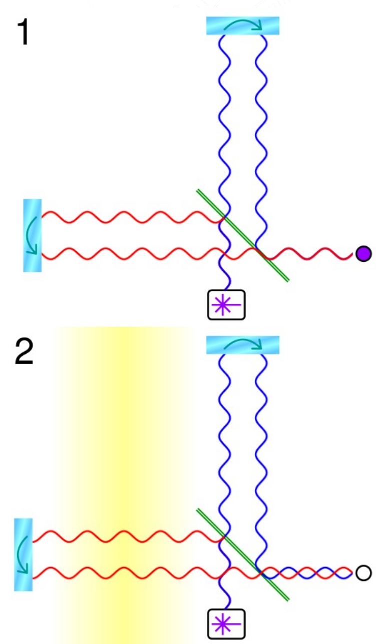


LIGO in Hanford, WA (Similar facility in Livingston, LA)

DIRECT DETECTION OF GRAVITATIONAL RADIATION

Even though gravitational waves are extremely weak (as Einstein predicted), we have detected 96 that have passed through Earth since 2015.

DETECTING GRAVITATIONAL WAVES

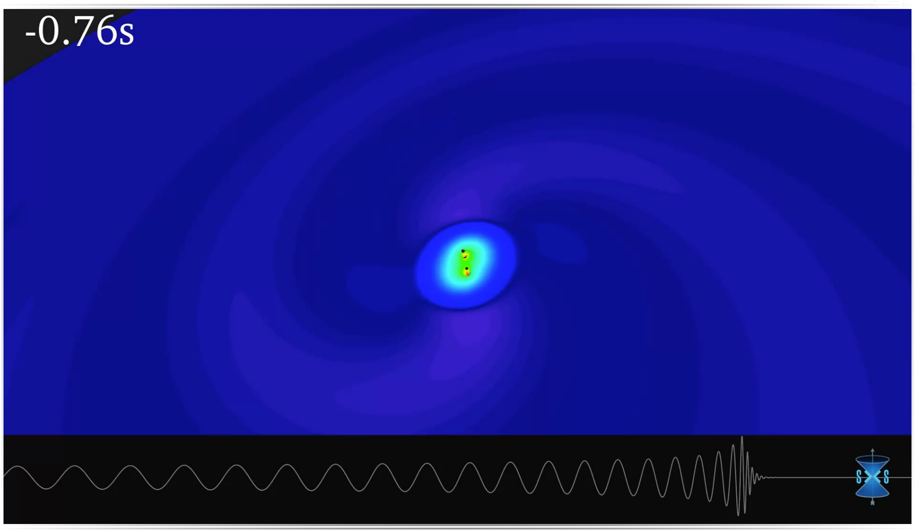


The easiest way to detect gravitational waves is by observing them stretch and squeeze the mass around us.

LIGO and Virgo are massive interferometers that are between 3 and 4 km long.

- Interferometers use light to detect differences in the path length.
- Light is a wave – if you add two waves that are in phase, they will **constructively** interfere; if they are out of phase, they will **destructively** interfere.
- LIGO is sensitive enough to detect a path difference of < 0.0001 of the diameter of a proton.

MERGING OF TWO BLACK HOLES



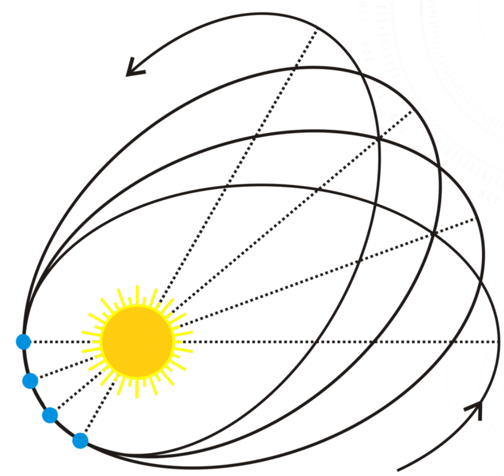
EXPERIMENTAL TESTS OF GENERAL RELATIVITY

Until the discovery of black holes, general relativity was only tested with rather weak gravitational fields, but the variety of validations possible were impressive. The following lists the most important experiments:

- Precession of the "perihelion" of Mercury's orbit: matter following gravity-induced curves in spacetime (Einstein 1915, explaining the observation first made by Le Verrier 1859).

PRECESSION OF MERCURY

- Mercury's orbit rotates with each revolution, resulting in the semimajor axis processing around the Sun.
- Originally observed by LeVerrier in 1859, all but 43"/century was explained by the gravitational perturbations of the other planets in the Solar System via Kepler and Newton's laws.
- The last 43"/century precession is exactly predicted by Einstein's GR: spacetime is warped close to the Sun, and Mercury travels close enough for its path to be affected.



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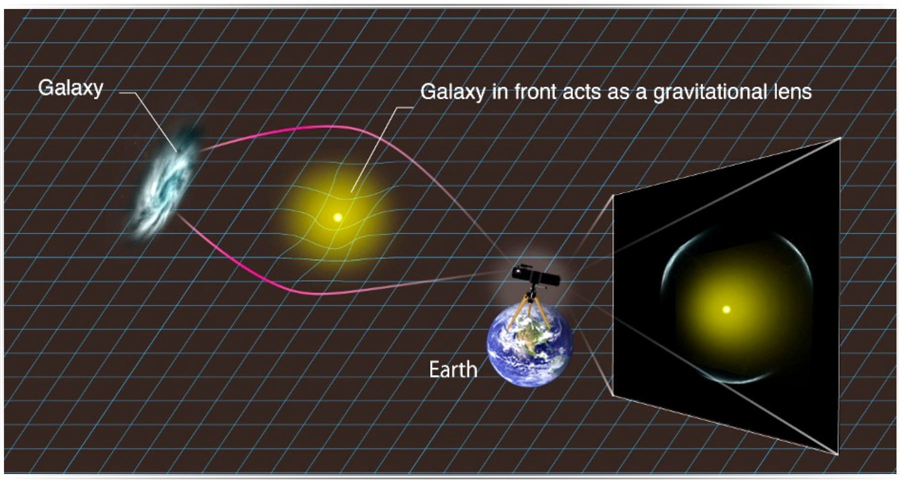
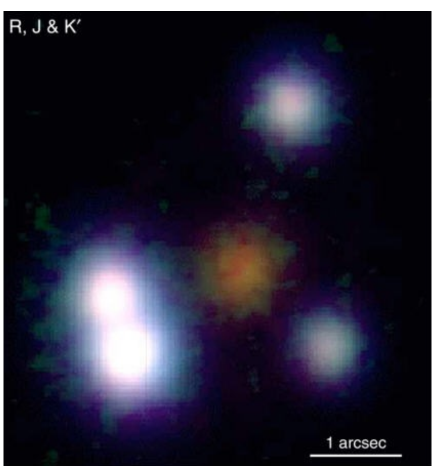
  - Light visible from distant quasars deflected by galaxies (Walsh, Carswell, & Weymann 1979)



GRAVITATIONALLY-LENSED GALAXIES

Light from distant galaxy follows warped space around galaxy cluster; we (at Earth) see images of the lensed galaxy in different places on the sky.

GRAVITATIONAL LENSES



PG 1115+080: gravity of the nucleus of an unusually massive galaxy (red) produces four images (blue) of a much more distant quasar. (CISCO, Subaru Telescope, NAOJ)

GC 0024+1654: gravity of a galaxy cluster (orange-ish) produces several images of a more distant, ring-shaped galaxy (blue). (W.N. Colley et al., HST/NASA/STScI)

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- Gravitational redshifts in the spectra of stars (Adams 1925) and on Earth (Pound & Rebka 1959): direct observation of gravitational time dilation
- The "geodetic effect:" precession of a gyroscope in orbit (NASA LAGEOS and Gravity Probe B satellites, 1995-2005)

EXPERIMENTAL TESTS OF GENERAL RELATIVITY

Of special importance among the weak-field validations of general relativity, though, is the discovery of gravitational radiation, which is therefore worth illustrating in a little more detail (Hulse & Taylor 1975).

SCIENTIFIC THEORIES

Is it possible for scientific theories to be proven wrong?

A. Yes, by good experimental results that contradict their predictions
B. Yes, by consensus of the best workers in the field
C. No, as it is possible that they will eventually agree with experiments.
D. No, as long as any reputable researcher believes in them ("veto power")

DISCOVERY OF GRAVITATIONAL WAVES

In 1974, Princeton professor Joe Taylor and his graduate student Russell Hulse discovered and extensively observed a binary pulsar, now known as PSR 1913+16.

- As the name implies, the binary pulsar consists of two **neutron stars** revolving around each other, one of which is a **pulsar**. (Note: we will be studying neutron stars in a few weeks.)
- Pulse arrivals can be timed with exquisite accuracy. The pulse arrival times in PSR 1913+16 exhibit a periodic delay/advance resulting from the orbital motion.
- With high-precision pulse timing, Hulse and Taylor were able to derive the size of the orbit, the masses of the stars, and their velocities very accurately. By watching for a long time, they observed that the **orbit is shrinking**.

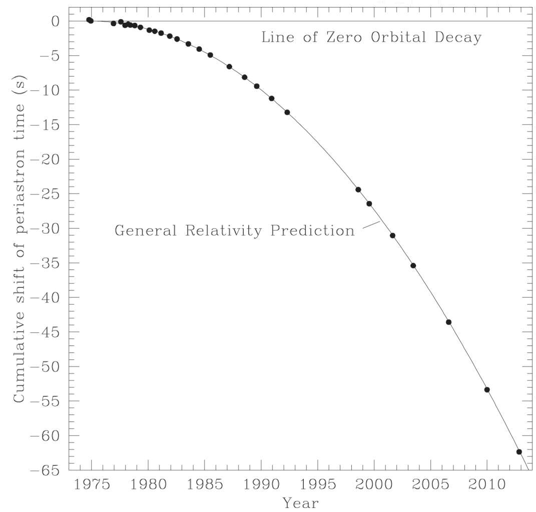
Because the orbit is shrinking, the binary system is somehow losing energy. Hulse and Taylor realized that this loss could be gravitational radiation: the neutron stars accelerate as they orbit.

- So, they calculated the gravitational-radiation loss expected from general relativity for the determined stellar masses, orbital size, and speed.

DISCOVERY OF GRAVITATIONAL WAVES

- The result from general relativity is in precise agreement with their results.
- This observation therefore constitutes the **discovery of gravitational radiation,** and an important experimental verification of general relativity. The 1993 Nobel Prize in Physics went to Hulse and Taylor for this work.

The shift in "periastron time," an indicator of the distance of closest approach of the two neutron stars in PSR 1913+16, as a function of time. From Weisberg & Huang (2016).

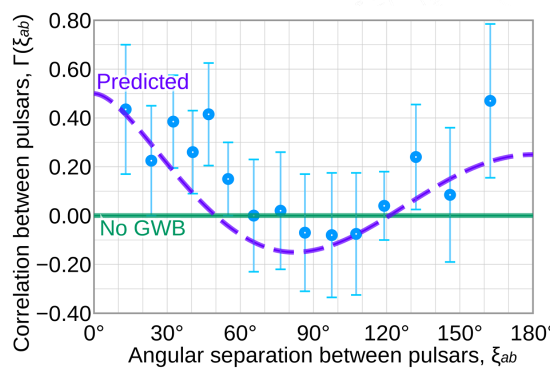


ADDITIONAL GRAVITATIONAL WAVES DETECTED

Expanding on the work by Hulse and Taylor, Hellings and Downs postulated that an array of pulsars could be used to detect a gravitational wave (with a wavelength much larger than any discovered by terrestrial interferometers).

- The presence of a gravitational wave will stretch/squeeze the distance between us and a pulsar, changing the pulsar signal's travel time to us.

In June 2023, NANOGrav released their precise timing measurements of 67 pulsars over 15 years that reveals a stochastic background of gravitational waves passing through our Galaxy.



CMG Lee

EXPERIMENTAL TESTS OF GENERAL RELATIVITY

Results of experiments:

- All reproducible experiments to date have confirmed the predictions of Einstein's relativity theories.
- Few scientific theories are so well-supported by experimental fact.
- We keep using the theory to predict new effects. Those effects involving conditions within those for which the theory has been tested are very likely to be real.
- Experimental tests of these newly-predicted effects are, in many cases, even stricter tests of the theories.

Black holes were among the first of these "new effects" predicted by the general theory of relativity, though this was not recognized at the time.

KARL SCHWARZSCHILD'S WORK

Schwarzschild read Einstein's paper on general relativity in 1916. He was interested in the physics of stars and had a lot of spare time between battles on the Russian front, so he solved Einstein's field equation for the region outside a massive spherical object.

His solution had many interesting features, including

- Prediction of space warping in strong gravity, and invention of embedding diagrams to visualize it
- Demonstration of gravitational time dilation, just as Einstein had pictured it
- Prediction of black holes, though this was not recognized at the time

Karl Schwarzschild

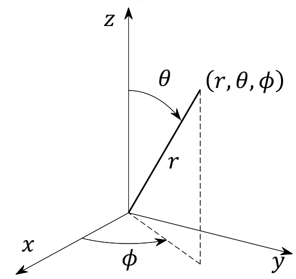
PART OF SCHWARZSCHILD'S SOLUTION

In case you are interested (i.e. not on the exam)…

- In spherical coordinates (at right), Schwarzschild's absolute interval Δs between two events that occur very close to a circle (with circumference C) around the star is

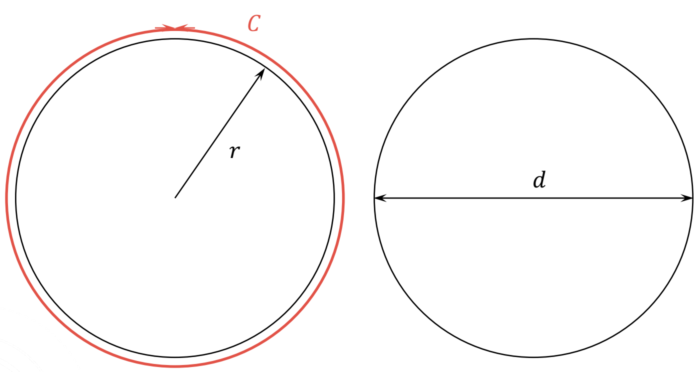
$$\Delta s^2 = \frac{\Delta r^2}{1 - \frac{4\pi GM}{Cc^2}} + \frac{C^2 \Delta\theta^2}{4\pi^2} + \frac{C^2 \sin^2\theta\, \Delta\phi^2}{4\pi^2} - c^2\left(1 - \frac{4\pi GM}{Cc^2}\right)\Delta t^2$$

Note that some of the terms either blow up (the denominator equals zero) or are zero if $C = \frac{4\pi GM}{c^2}$.



EMBEDDING DIAGRAMS

Embedding ("rubber sheet") diagrams provide an **analogy** by which to envision how four-dimensional spacetime can be warped by the introduction of additional, imaginary dimensions. These additional dimensions have been introduced before, under the name **hyperspace**. Schwarzschild can be considered the inventor of hyperspace.

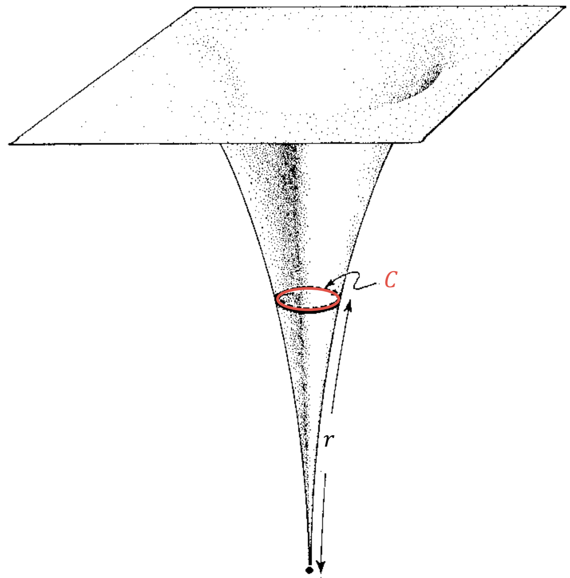


First, consider circles in flat **2D** space

$$C = 2\pi r = \pi d$$

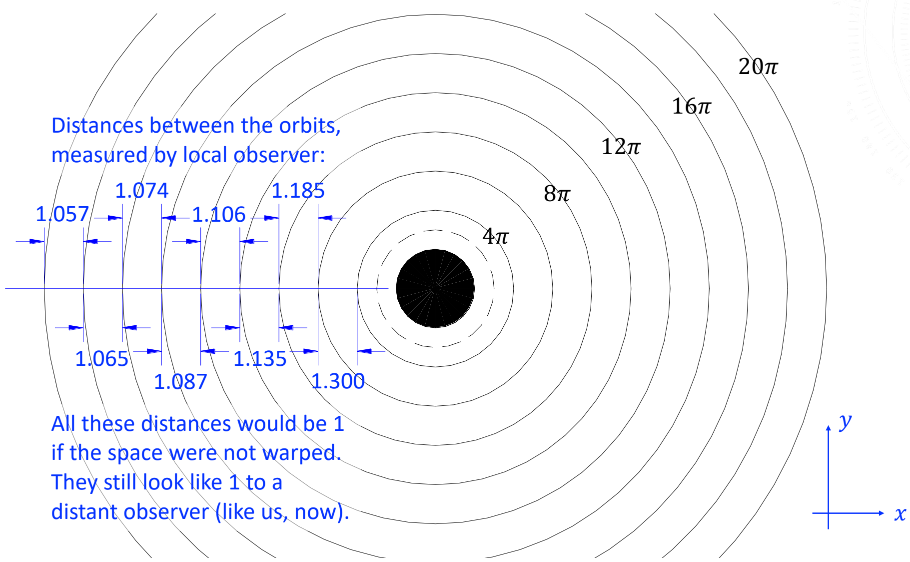
EMBEDDING DIAGRAMS

An embedding diagram: Circles in curved space still **look** like 2D figures, but they **behave** as if their centers are much further away than $C/2\pi$. We can picture this as a stretching of space in a direction (in hyperspace!) perpendicular to the circle.



EMBEDDING DIAGRAM

Calculated from Schwarzschild's solution for an object with $M = c^2/2G$



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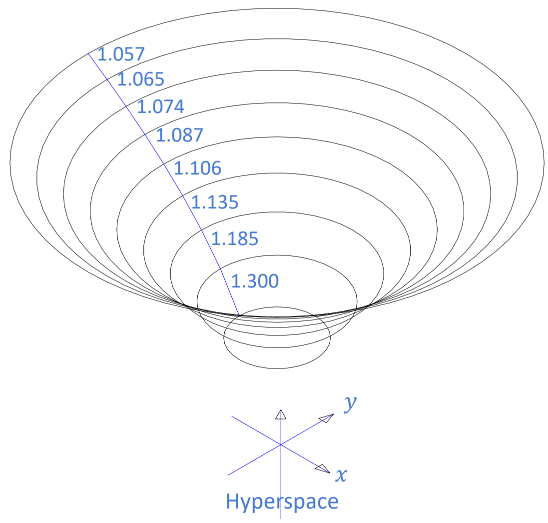
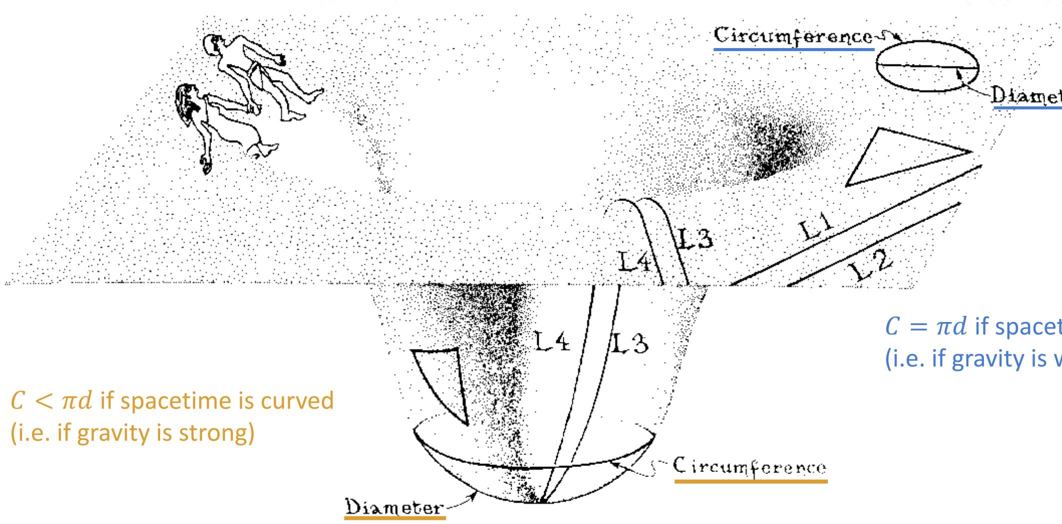


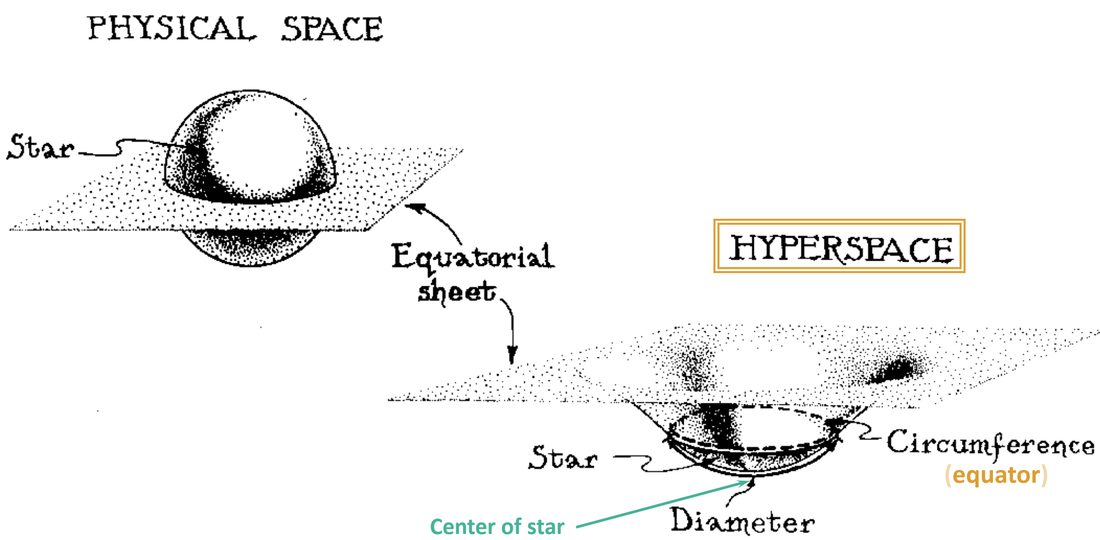
To connect these circles with segments of these "too long" lengths, they can be considered offset from one another along some **imaginary** dimension that is **perpendicular to x and y but is not z.** (If it were z, the circles would not appear to lie in the same plane!) Such additional dimensions comprise hyperspace.



$C = \pi d$ if spacetime is flat (i.e. if gravity is weak)

$C < \pi d$ if spacetime is curved (i.e. if gravity is strong)

EMBEDDING DIAGRAMS



SCHWARZSCHILD'S VIEW OF THE EQUATORIAL PLANE OF A STAR

NEAR THE HORIZON

You are in an orbit around a black hole with a mass $M = \frac{c^2}{2G}$, with a circumference that is **10 times that of the black hole's horizon**, and you can measure distances with 10% accuracy. By switching to orbits very nearby, and measuring circumferences and distances between orbits, can you reveal the space-warping effects of the black hole?

A. Yes – the black hole severely warps space here.

B. Yes, but just barely.

C. No – insufficient accuracy

D. No – the warp is essentially zero here