ELECTRON DEGENERACY PRESSURE & WHITE DWARFS

PROBLEM SET #4 ON WEBWORK – DUE WEDNESDAY, 10/9, AT MIDNIGHT

ELECTRON DEGENERACY PRESSURE & WHITE DWARFS

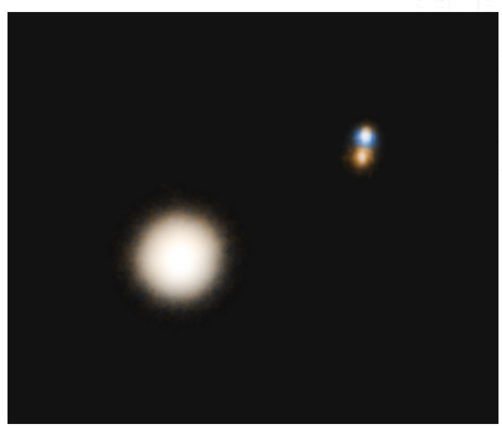
Electron degeneracy pressure, relativistic degeneracy pressure, and white dwarf stars

The relativity of mass

The origin of white dwarf stars

Planetary nebulae

Prevention of black holes, but only those less massive than $1.44M_{\odot}$

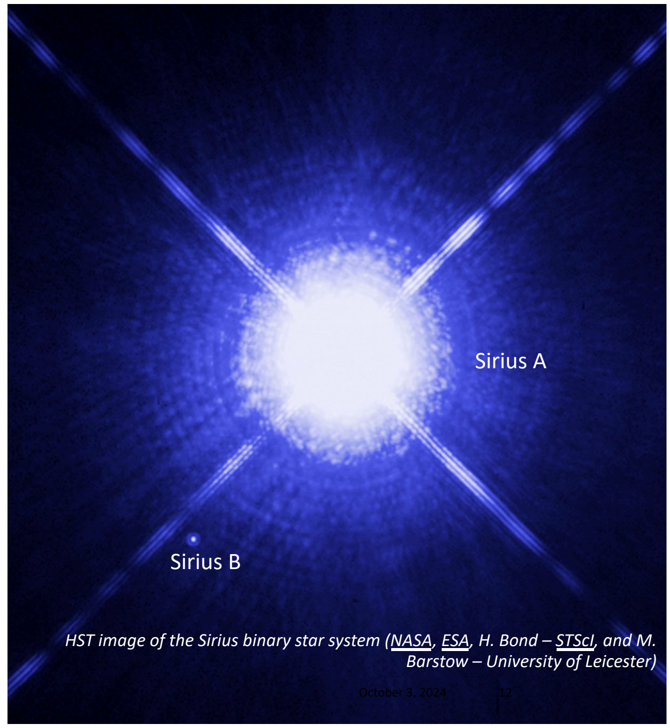


Triple-star system 40 Eridani (Mark Johnston). The uppermost star, 40 Eri B, is a white dwarf.

WHITE DWARF STARS

White dwarfs are stars similar in mass and temperature to normal stars but are much fainter and much smaller – the size of planets. Discovered in 1862, they were a hot topic in astronomy in the 1920s. Hundreds of thousands are known today.

Sirius, the brightest star is the sky, has a companion star which is a white dwarf. They orbit each other with a period of about 50 years. Sirius A is vastly brighter than Sirius B at visible wavelengths.

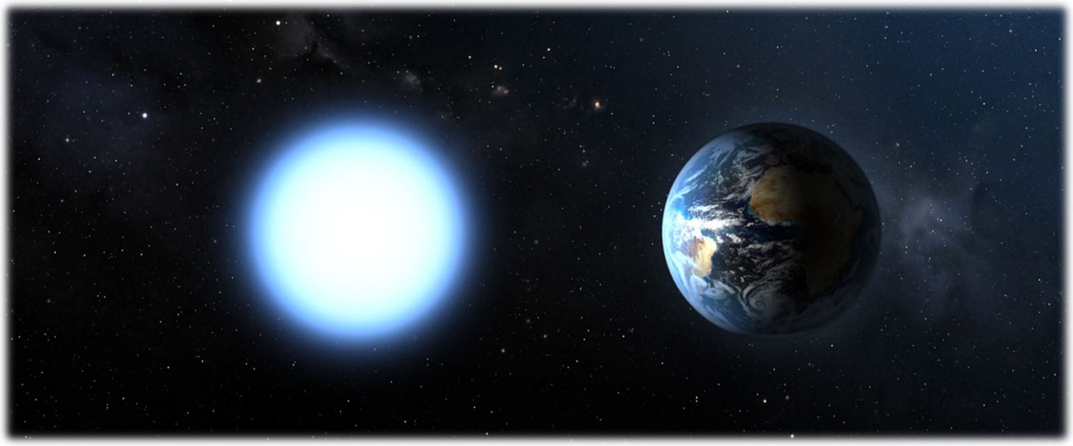


HST image of the Sirius binary star system (NASA, ESA, H. Bond – STScI, and M. Barstow – University of Leicester)

SIRIUS B: A FAIRLY TYPICAL WHITE DWARF

- From its measured distance from Sirius A and their orbital period (plus Newton's laws), we know that the mass of Sirius B is $1.02M_\odot$.
- From its observed color (blue-white), we know that its temperature is rather high: $T = 25{,}000$ K (compared to 5800 K for the Sun and 9940 K for Sirius A).
- Its luminosity is only $0.056L_\odot$, much less than that of Sirius A ($25.4L_\odot$).
- From all this information, astronomers can work out the diameter of Sirius B; the result is 1.2×10^4 km, slightly smaller than that of the Earth (1.3×10^4 km).

The mass of a star in the size of a planet.



SIRIUS B

The mass of a star in the size of a planet

THEORY OF WHITE DWARFS

Soon after he invented it (1926), Fowler applied his theory of degeneracy pressure to white dwarf stars. His result:

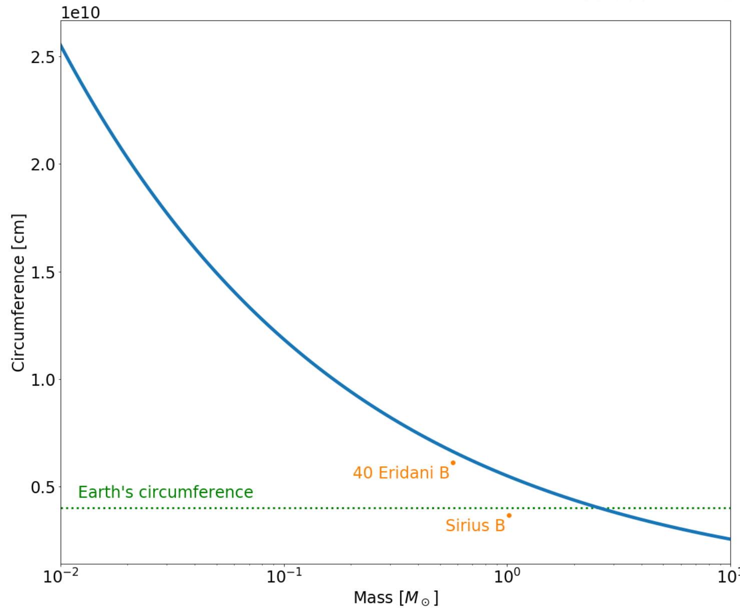
- Stars supported by degeneracy pressure instead of gas pressure would have sizes close to that determined from astronomical observations of Sirius B.

Shortly after, Edmund Stoner (1929) noticed that this result implied a peculiar relation between mass and size:

- **Higher-mass degenerate stars are smaller than lower mass ones,** the opposite of how normal stars behave.
    - Reason: more mass means more pressure is required to balance gravity, and more degeneracy pressure requires more tightly-confined electrons (smaller star).
- For stars heavier than about a solar mass, Stoner (prompted – incorrectly – by Anderson) noticed from his theory that the confinement imparted so much energy to the electrons in the center of the star that the electron speeds are close to the speed of light.

STONER'S FIRST WHITE DWARF THEORY

Stoner's theory is close, but it does not quite agree with the data.



THEORY OF WHITE DWARFS

- Fowler's theory of degenerate matter did not take Einstein's theory of relativity into account; therefore Stoner (and, independently, Frenkel) started over to combine relativity and quantum mechanics into a new theory of **relativistic degeneracy pressure**.
  - Recall that electrons, like everything else, cannot move faster than light.
- The more massive the degenerate star, the closer the electron speeds get to the speed of light.
- The closer the speeds get to c, the harder it is to further accelerate the electrons.
- Thus, the electron degeneracy pressure does not keep increasing as much with tighter confinement: the electrons reach a point where they cannot move any faster. There is a **maximum** to the electron degeneracy pressure, and a corresponding maximum weight – and maximum mass, about $1.7M_{\odot}$ - that degeneracy pressure can support (Stoner 1930, 1932; independently by Landau 1931, Chandrasekhar 1931).
- If the weight cannot be supported by electron degeneracy pressure, the degenerate star will collapse to smaller sizes.

THEORY OF WHITE DWARFS

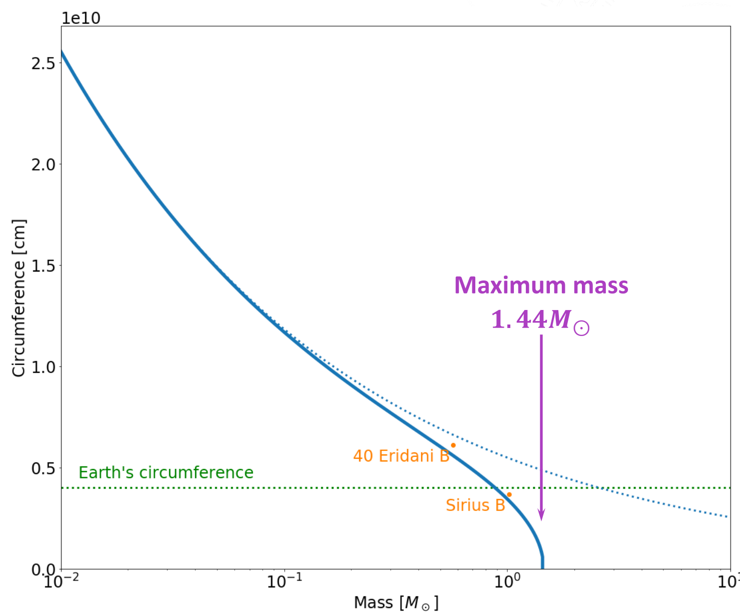
Meanwhile, Subrahmanyan Chandrasekhar appeared on the scene, initially as Fowler's graduate student.

- He combined Stoner's theory of degeneracy pressure with a much more detailed theory of stellar structure and, armed only with an adding machine, produced a numerical solution correct for all white-dwarf masses (Chandrasekhar 1935).
- The result confirmed that white-dwarf circumferences decrease smoothly to zero as mass increases to a maximum, which turns out to be $1.44M_{\odot}$.

Chandrasekhar (AIP)

CHANDRASEKHAR'S WHITE DWARF CALCULATIONS

By including relativity, Chandrasekhar was able to better predict the data with his theory.



LEARN TO READ THESE GRAPHS

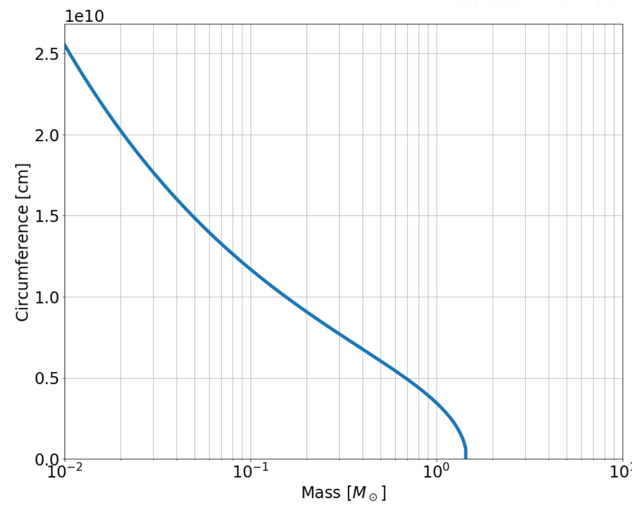
What is the mass of a white dwarf with circumference 2×10^{10} cm?

A. $0.01M_{\odot}$

B. $0.011M_{\odot}$

C. $0.02M_{\odot}$

D. $0.025M_{\odot}$



LEARN TO READ THESE GRAPHS

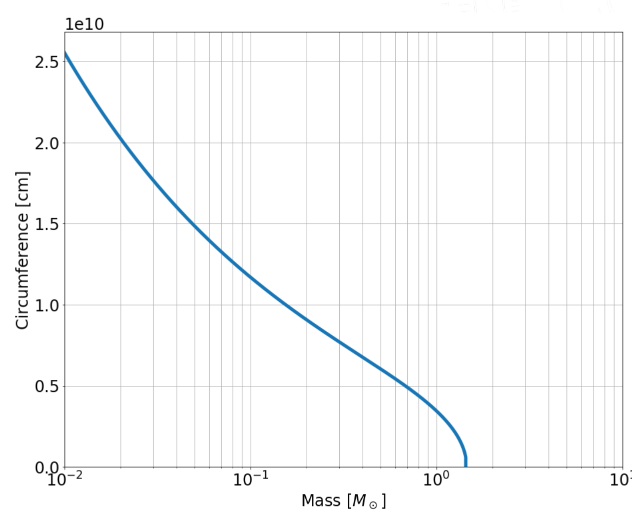
What is the mass of a white dwarf with circumference 1×10^{10} cm?

A. $0.1M_{\odot}$

B. $0.11M_{\odot}$

C. $0.15M_{\odot}$

D. $0.2M_{\odot}$



THE RELATIVITY OF MASS, THE SPEED OF LIGHT LIMIT, & MAXIMUM DEGENERACY PRESSURE

Why is the speed of light the maximum speed that can be reached by electrons? (Remember, this was not part of the two original axioms from which Einstein started…)

- Because of the **relativity of mass**: if a body with rest mass m_0 moves at speed V with respect to an observer, the observer will measure a mass

$$m = \frac{m_0}{\sqrt{1 - \frac{V^2}{c^2}}} = \gamma m_0$$

- Note the similarity to the formula for time dilation: in particular, that the denominator approaches 0, and thus **m approaches infinity, if V approaches c**.

RELATIVITY OF MASS

What is the mass of an object that has a rest mass of 1 gram and flies by at $0.99c$?

RELATIVITY OF MASS

How fast would an object have to fly for its mass to be a factor of 2 larger than its rest mass?

THE RELATIVITY OF MASS, THE SPEED OF LIGHT LIMIT, & MAXIMUM DEGENERACY PRESSURE

Suppose you have a body moving at very nearly the speed of light, and you want it to exceed the speed of light. What can you do?

- It needs to **accelerate** for its speed to increase.
- You need to exert a force on it in order to make it accelerate.
- The force required is, essentially, proportional to the product of mass and acceleration in your reference frame. (Nonrelativistic version of this statement is Newton's second law: force = mass times acceleration, or $F = ma$.)
- But the mass approaches infinity as V approaches c, and thus an infinite force is required to accelerate it further. There is no such thing as an infinite force, **so c is the ultimate speed limit**.

HOW TO MAKE A WHITE DWARF STAR

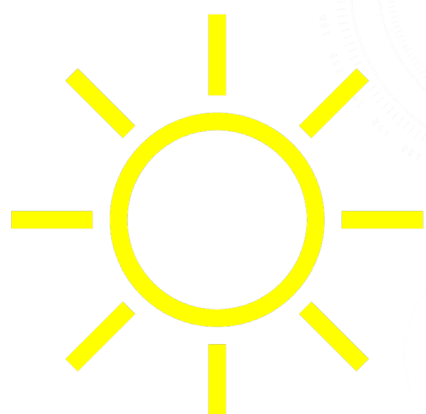
Start with a normal star like the Sun. Fusion of protons into helium in the star's center generates heat and pressure that can support the weight of the star. The Sun was mostly made of hydrogen (1 proton + 1 electron) when it was born and started with enough hydrogen to last like this for more than 10 billion years.

The Sun is not really yellow, but it is popular to draw it that way.

QUESTION!

What color is the Sun?

A. White
B. Yellow, no matter what you just said.
C. Orange
D. Red

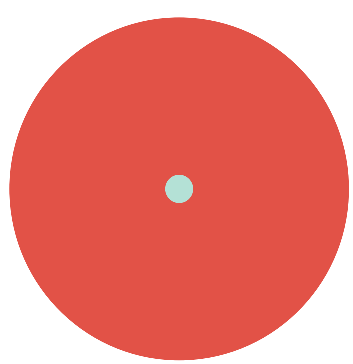


HOW TO MAKE A WHITE DWARF STAR

When it begins to run out of hydrogen in its center, not enough heat and pressure are generated to balance the star's weight, so the core of the star gradually begins to collapse.

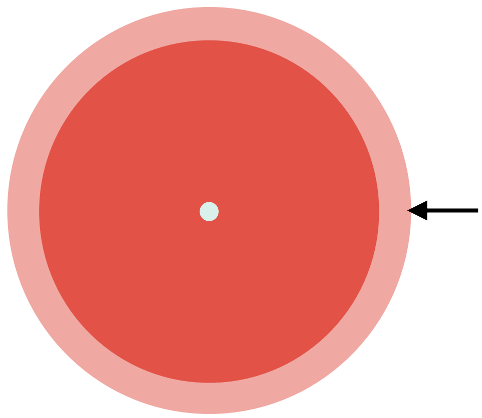
HOW TO MAKE A WHITE DWARF STAR

As the core collapses, it gets hotter even though no extra heat has been generated – just because it compresses. It gets so hot that light from the core causes the outer parts of the star to expand and get less dense, causing the star to look cooler from the outside. The star is becoming a **red giant**.



HOW TO MAKE A WHITE DWARF STAR

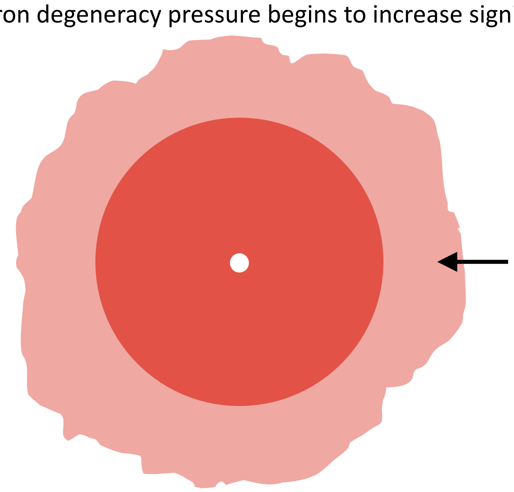
Eventually, the core gets so hot that it is possible for helium to fuse into carbon and oxygen. Extra heat and pressure are once again generated and the core stops collapsing; it is stable until the helium runs out, which takes a few million years. The outer parts of the star are not very stable, though.



Matter flowing away from the star

HOW TO MAKE A WHITE DWARF STAR

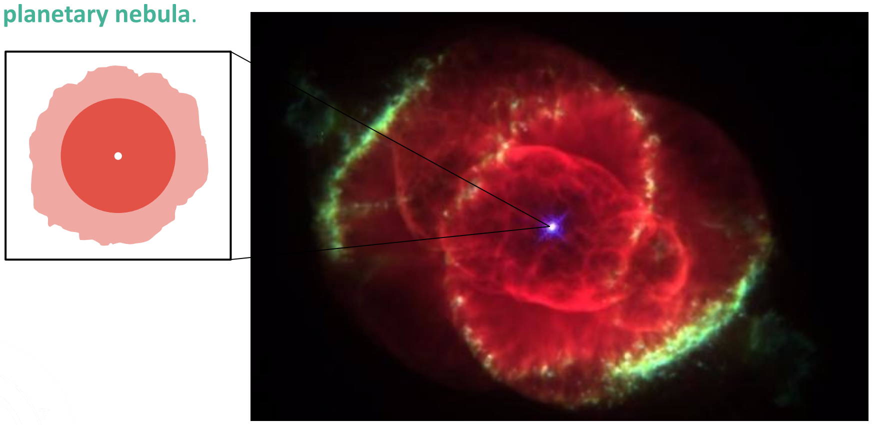
Eventually, the core is all carbon and oxygen, no additional heat and gas pressure is generated, and the core begins to collapse again. This time the density is so large – the electrons are so close together – that electron degeneracy pressure begins to increase significantly as the collapse proceeds.



Matter flowing away from the star

HOW TO MAKE A WHITE DWARF STAR

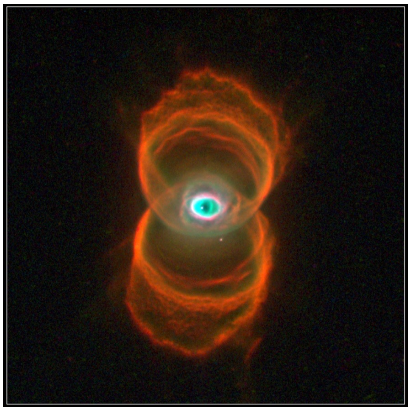
Electron degeneracy pressure eventually brings the collapse of the core to a halt, before it gets hot enough to fuse carbon and oxygen into magnesium and silicon. The unstable outer parts of the star completely fall apart; they are ejected and ionized by light from the core, producing a **planetary nebula**.

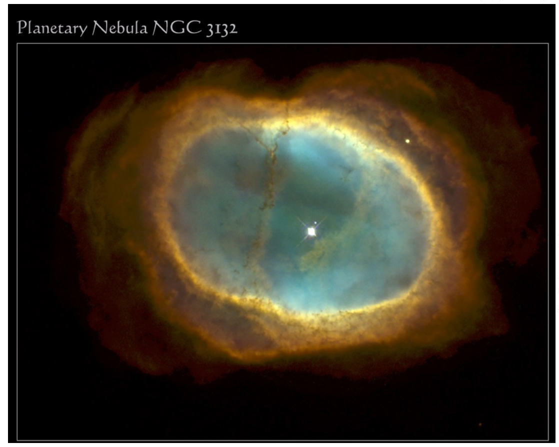


PLANETARY NEBULAE

The death of solar-type stars produces some of the most photogenic nebulae. Here are HST images of some pretty ones; many others can be found at Astronomy Picture of the Day by using Search and entering "planetary nebula." *(Credits: Hubble Heritage Team and Hubble Helix Nebula Team, STScI/NASA)*

Hourglass Nebula, MyCn18

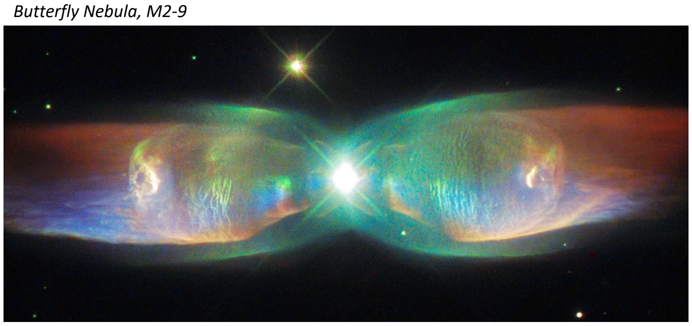


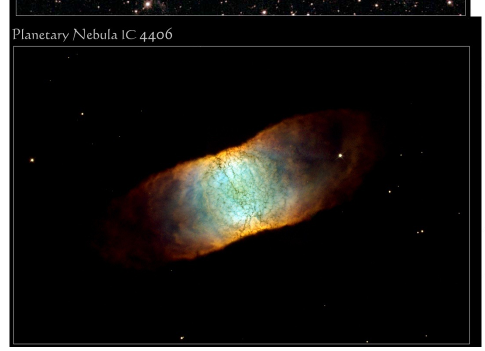


PLANETARY NEBULAE

Credits: R. Sahai, J. Trauger, B. Balick (JPL/U. Washington/NASA)

Butterfly Nebula, M2-9





PLANETARY NEBULAE

Credits: Hubble Heritage Team and R. Sahai (STScI/JPL/NASA)

HOW TO MAKE A WHITE DWARF STAR

The planetary nebula's material expands away from the scene in a few thousand years, leaving behind the hot, former core of the star, now about the size of Earth. Its weight supported against further collapse by electron degeneracy pressure, it will do nothing but sit there and cool off, for eternity.

HOW TO MAKE A WHITE DWARF STAR

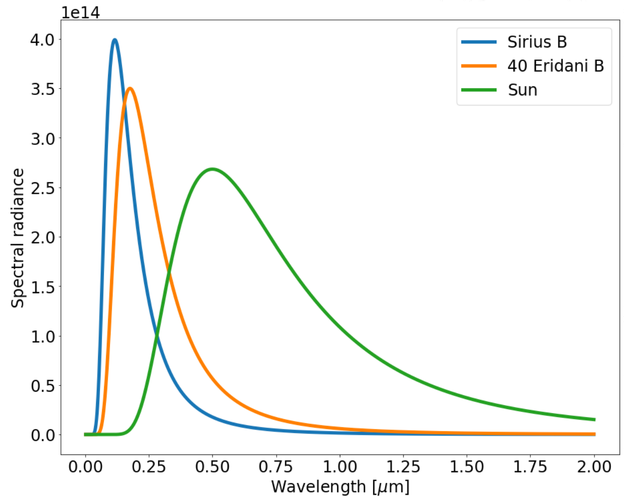
When brand new, this degenerate star is quite hot and looks white (like Sirius B) or even blue in color, leading to the name **white dwarf.** The oldest "white dwarfs" in our galaxy, age about 12 billion years, have had enough time to cool down to temperatures in a few thousands of degrees and thus look red. (Despite this, they are still called white dwarfs.)

RELATIONSHIP BETWEEN COLOR AND TEMPERATURE IN A STAR

Stars are almost perfect **blackbodies**: objects which emit light in a way that depends only on their temperature.

- The higher the temperature, the shorter the peak wavelength of the emission.

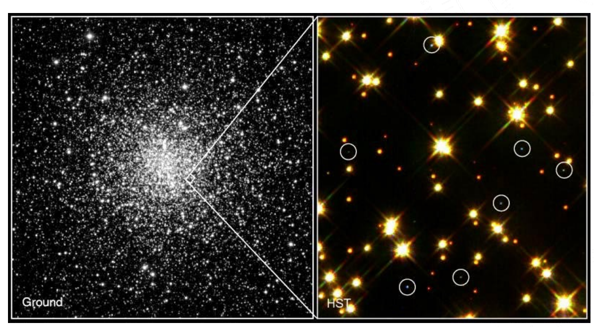
Hotter blackbodies emit light at shorter (bluer) wavelengths, while cooler blackbodies emit light at longer (redder) wavelengths.



CONFIRMATION OF THE THEORY OF WHITE DWARFS

We know of hundreds of thousands of white dwarf stars today. Sure enough, all stellar masses under $1.44 M_\odot$ are represented, but no white dwarf heavier than this has ever been found.

Nominally for this work, Chandrasekhar was awarded a Nobel Prize in Physics (1983). The NASA Chandra X-ray Observatory (CXO) is named in his honor. And Stoner is, unfortunately, largely forgotten.



Seven white dwarfs (circled) in a small section of globular cluster M4 (by H. Richer and M. Bolte; Left: Kitt Peak National Observatory 36"; Right: Hubble Space Telescope)

THEORY OF WHITE DWARFS

- Important result of the theory: maximum mass for white dwarfs, which turns out to be $1.44M_{\odot}$. Electron degeneracy pressure cannot hold up a heavier mass.
- Implication: for stars with a core mass less than $1.44M_{\odot}$, core collapse is stopped by electron degeneracy pressure before the horizon size is reached.
- However, for stars with cores more massive than $1.44M_{\odot}$, the weight of the star overwhelms electron degeneracy pressure, and the collapse can keep going. **What can stop heavier stars from collapsing all the way to become black holes after they burn out?**

FINAL COLLAPSE OF BURNED-OUT STARS: WHITE DWARF OR BLACK HOLE?

The Sun and smaller stars will become white dwarfs after they burn out and, lacking the gas pressure generated by their nuclear heat source, collapse under their weight. What about Sirius A, which weighs a good deal more than the limit?