ASTR 102 MOVIE NIGHTS

Don’t forget to VOTE!

We will have two movie nights during the next couple of weeks, to view

- *Interstellar* (2014)
- *Star Trek* (2009)

We will watch the films in the above order on the first night, and the reverse order on the second night. Screenings on both nights will start at 6pm. Refreshments will be provided!

If you haven’t yet done so, please take a minute to fill out the Google form with your availability: https://forms.gle/YFkiREULbujT29m4A

BLACK HOLE EVAPORATION

EXAM #2 ON THURSDAY

EXAM #2 ON THURSDAY

- 1 hr 15 min exam in class, open book and open notes
- Things that you should DEFINITELY bring with you:
  - Writing utensil (pencil or pen – blue or black ink)
  - Calculator
- Things that you should probably bring with you:
  - Lecture notes
  - Laptop or tablet (so that you can access the WeBWorK homework problems and the "How Big is That?" sheet on Blackboard)
- Practice exam available on WeBWorK – email me when you want solutions
- REVIEW SESSION – Tomorrow (Wednesday, 11/6) at 7:30pm in B&L 372

BLACK HOLE EVAPORATION

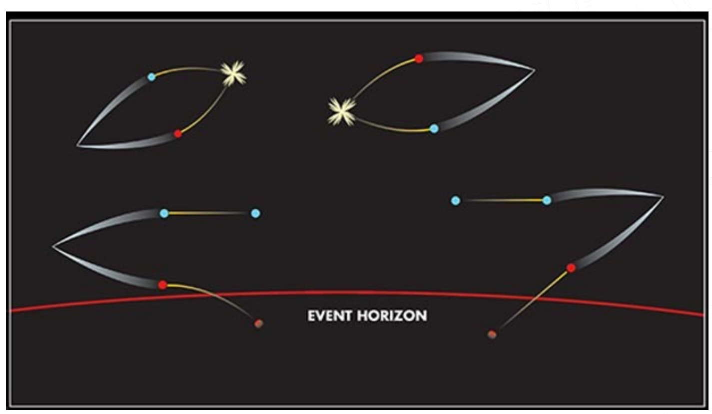
Hawking's "area increase" theorem

Entropy and the area of the event horizon: the thermodynamics of black holes

Quantum-mechanical vacuum fluctuations and the emission of light by black hole: "Hawking radiation"

Evaporation of black holes

Exotic matter



Virtual particles near a black hole's horizon (S.W. Hawking)

THE HORIZON AREA THEOREM

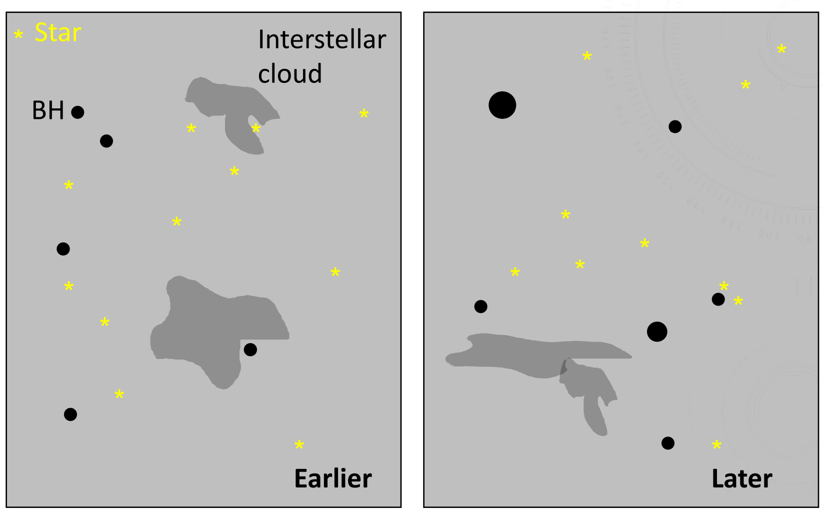
In 1970, Stephen Hawking used general relativity to prove a useful rule called the **horizon area theorem**:

> The total horizon area in an enclosed system containing black holes never decreases. It can only increase or stay the same.

Increases in total horizon area come from growth of black holes by collapse of stars or accretion of "normal" matter, and by the coalescence of black holes.

ILLUSTRATION OF HAWKING'S HORIZON-AREA THEOREM

A closed-off part of the universe. As time goes on, the total area of all the horizons in this closed system increases, owing to the growth of black holes by collapse, accretion, and coalescence.



WHY SHOULD THAT SEEM STRANGE?

The horizon-area theorem is simple (and intuitively obvious), but it represents a puzzle at a deeper level when you consider the **heat** and **disorder** in the matter that forms or falls into a black hole.

- Before: the matter is hot, and there are many particles sharing the heat among themselves in the form of their random motions. A complete description of the system would thus have different entries for position and velocity for each particle – a vast number of *quantities* required.
- After: the system can be completely described by only three quantities: its mass, spin, and charge. It is orderly!

The problem: In all other natural processes, **matter is never seen to go from a disorderly state to an orderly one all by itself.** In fact, this is a law of thermodynamics…

ENTROPY & THE SECOND LAW OF THERMODYNAMICS

Compare these two statements:

- The **horizon area theorem**: The total horizon area in a closed system never decreases.
- The **second law of thermodynamics**: The total entropy of a closed system never decreases.

Entropy = the logarithm of the number of ways the atoms and molecules in a system can be rearranged without changing the system's overall appearance. A larger entropy means that the system is more disorderly, or more "random."

Do black holes really have entropy as low as they seem to? Does horizon area have anything to do with entropy?

DEFINITION OF ENTROPY

(Not on the exams or homework)

Officially, the definition of entropy includes an additional multiplicative factor:

$$S = k \ln \Omega$$

where

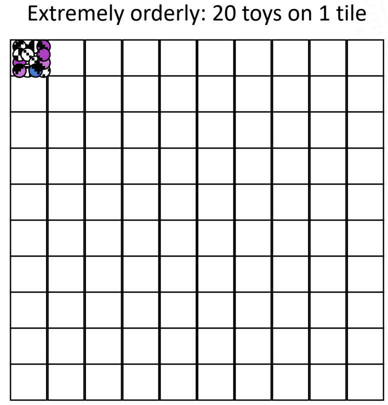
$$S = \text{entropy}$$
$$k = 1.381 \times 10^{-16}\ \text{erg/K}, \text{Boltzmann's constant}$$
$$\Omega = \text{number of equivalent rearrangements}$$

For ASTR 102: **Entropy is some number times the logarithm of the number of equivalent rearrangements.**

ENTROPY EXAMPLE: TOYS IN A PLAYROOM

This playroom floor has 100 tiles on which the children can arrange 20 different toys. Parents prefer the toys to be kept in an extremely orderly configuration, with all the toys piled on one tile in one corner (as shown). There is only one such arrangement; the entropy of this configuration is therefore the logarithm of 1, which is zero.

Extremely orderly: 20 toys on 1 tile

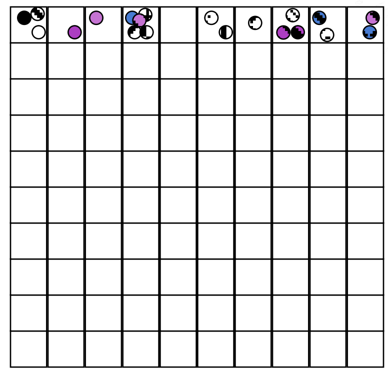


Number of equivalent arrangements = 1
"Entropy" = 0

ENTROPY EXAMPLE: TOYS IN A PLAYROOM

Parents might even accept this somewhat less orderly configuration: 20 different toys on 10 specific tiles. But there are many different equivalent arrangements (e.g. swapping the positions of two toys on different tiles produces a different arrangement that is still acceptable): 10^{20} of them, in fact, for an entropy value of $\log 10^{20} = 20$.

Orderly: 20 toys on 10 tiles



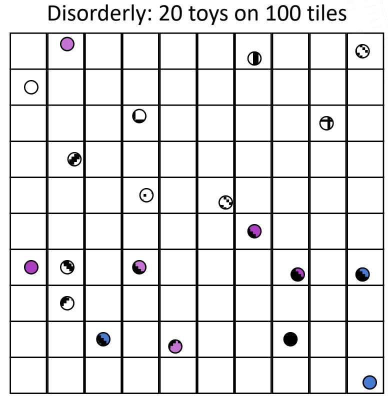
Number of equivalent rearrangements $= 10^{20}$
"Entropy" = 20

ENTROPY EXAMPLE: TOYS IN A PLAYROOM

When the children are done playing, the floor looks something like this: 20 different toys spread randomly over 100 tiles. There are $100^{20} = (10^2)^{20} = 10^{40}$ different ways to do this; the entropy is 40.

And children are like natural physical processes. Through their agency, the room will get more random; the entropy of the room full of toys and children never decreases by itself.

Disorderly: 20 toys on 100 tiles



Number of equivalent rearrangements $= 10^{40}$
"Entropy" = 40

2ND LAW OF THERMODYNAMICS – LEAST LIKELY TO BE BROKEN

Many consider the second law of thermodynamics to be the physical law least likely to ever be broken. This is because its consequences are so easily tested experimentally that you can see it verified all around you every day. Among the important corollaries to the second law:

- Heat never flows by itself from a lower temperature to a higher temperature.
- Decreasing the temperature of one part of a closed system requires raising the temperature in other parts.
- The mechanical work (*organized* energy) that can be done by a heat engine is always less than the available heat (*disorganized* energy).
- **Irreversible processes:** two sequences energetically allowed, but only one sequence ever happens.

IRREVERSIBLE PROCESSES & THE ARROW OF TIME

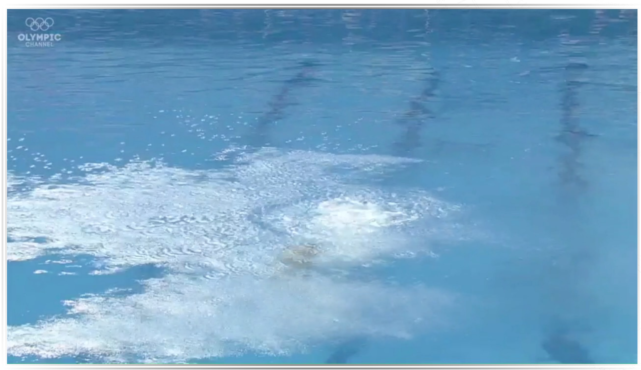
The laws of conservation of energy, momentum, and spin are indifferent to the order of time, and if they had the only say, all mechanical processes would be time-reversible.

- For instance, a diver can bound from a springboard, fall 5 meters, and hit the water with kinetic energy about 4×10^{10} erg.
- He is brought to a stop in the water as his kinetic energy is converted to heat: this raises the temperature of the water (assumed uniform) by about 4.5×10^{-7}°C.

Yuan Cao, 3m springboard, 2016 Summer Olympics

IRREVERSIBLE PROCESSES & THE ARROW OF TIME

- In principle, it is possible for the pool to cool down by a tiny amount, transfer that energy to the diver in the form of kinetic energy, and spit him back out of the pool.
- As you know, that never happens: this process is **irreversible**.
- The difference: the diver's kinetic energy is **organized**, the heat in the pool is **disorganized**. That divers are never flung back out of the pool is an expression of the fact that closed systems never spontaneously become more organized.



Yuan Cao, 3m springboard, 2016 Summer Olympics

> "From the standpoint of formal physics there is only one concept which is asymmetric in time, namely entropy. But this makes it reasonable to assume that the second law of thermodynamics can be used to ascertain the sense of time independently in any frame of reference; that is, we shall take the positive direction of time to be that of increasing entropy."

(Panofsky & Phillips, 1962)

IRREVERSIBLE PROCESSES & THE ARROW OF TIME

A deep connection exists between the second law of thermodynamics and the direction of time, which brings us back to relativity.

HORIZON AREA & ENTROPY

Black holes form from large collections of atoms and molecules with extremely large numbers of equivalent rearrangements (large entropy).

What happens to the entropy of this matter when it falls into a black hole? This was a burning issue in 1972.

- **Hawking** (and the rest of the relativists in 1972): The entropy vanishes. Black holes and their horizons are extremely simple objects with only one possible configuration ("equivalent rearrangement") each: that means zero entropy. Black holes therefore violate the second law of thermodynamics. That does not matter very much, because you cannot get accreted material back out of the black hole anyway.

HORIZON AREA & ENTROPY

- **Jacob Bekenstein** (a Princeton graduate student in 1972, all alone on this side of the argument): The second law of thermodynamics has not been violated in any other physical situation; why give up so soon? **The entropy of the ingredients may be preserved in a form proportional to the horizon area.** If a hole has entropy, it must also have a temperature, which he finds to be proportional to the strength of gravity at the horizon.
- **Hawking et al. (1972)**: But that would mean that the horizon is a **black body** at non-zero temperature that obeys the laws of thermodynamics. Any such body must radiate light – as the hot filament in a lightbulb does, for instance. *Nothing can escape from a black hole horizon*; how can it radiate? This contradiction implies that black holes cannot have entropy or temperature, and that they must violate the second law of thermodynamics.
- **Bekenstein (1972)**: He cannot think of any way for light, or anything else, to escape from a black hole; He admits that black holes cannot radiate. But there must be something wrong with Hawking's viewpoint, because it must be possible for black holes to obey the laws of thermodynamics.

HORIZON AREA & ENTROPY

- **Hawking (1974)**: Oops.

  There is a way for black holes to emit radiation: it involves quantum-mechanical processes near the black hole's horizon.

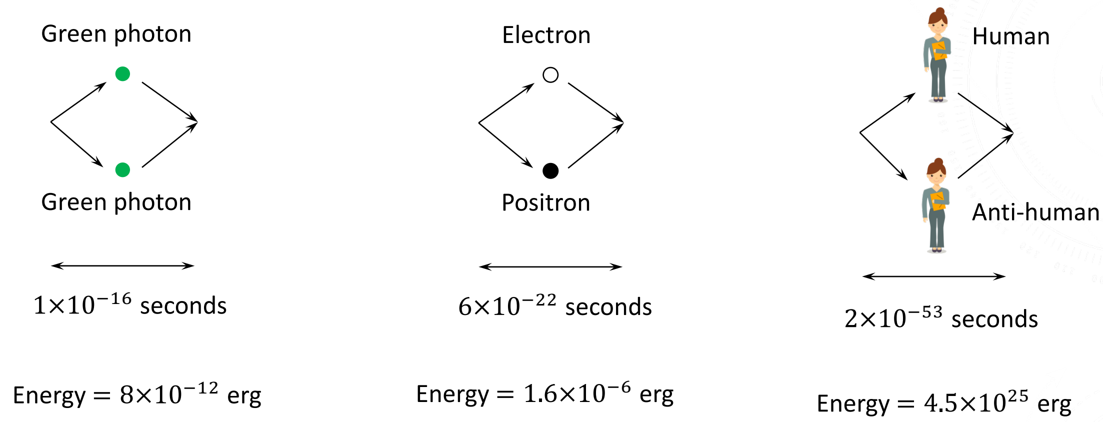
  - The emission of light is exactly as one would expect from a black body with temperature that increases as the strength of gravity at the horizon increases.
  - Therefore, the black hole has entropy, which increases as the area of the horizon increases.
  - Therefore, black holes obey the laws of thermodynamics. **Bekenstein is correct after all.**

HOW BLACK HOLES EMIT LIGHT

- In subatomic matter, we see that energy conservation can be violated, though only temporarily and very briefly. This is expressed in one of Heisenberg's uncertainty principles:

$$\Delta E \times \Delta t \approx h \approx 10^{-27} \text{ erg s}$$

- **Vacuum fluctuations:** the shorter the time interval Δt, the larger the energy ΔE that can be **temporarily** produced. For extremely short time intervals, enough energy can be borrowed from the vacuum (i.e., nothingness) to produce photons, or even massive particles.
  - These particles are called **virtual particles.** They vanish again at the end of the time interval Δt.
  - Virtual particles are produced as **particle-antiparticle pairs.**



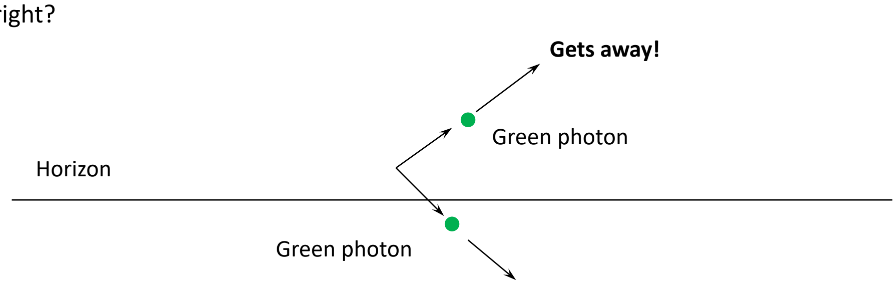
PARTICLE-ANTIPARTICLE PAIRS

Examples of particle-antiparticle pairs made from the vacuum.

VIRTUAL PARTICLES NEAR A HORIZON

Normally, virtual pairs vanish too quickly to be noticed or to interact much with anything else.

Near a black hole horizon: what if one of the pair falls in, and the other does not and is aimed just right?



HAWKING RADIATION

Details of black hole emission, nowadays called **Hawking radiation**:

- Virtual pairs, produced by vacuum fluctuations, can be split up by the strong gravity near a horizon. Both particles can fall in, but it is possible for one to fall in while the other escapes.
- The escaping particle is seen by a distant observer as emission by the black hole horizon: **black holes emit light** (and other particles), though only in this weird way.
- The energy conservation "debt" involved in the vacuum fluctuation is paid by the black hole itself: the black hole's mass decreases by the energy of the escaping particle divided by c^2. The emission of light (or any other particle) costs the black hole mass and energy.

EXOTIC MATTER

Part of this story should strike you as weird: Why is it that the black hole can *consume* a particle and wind up *decreasing* in mass and energy?

- In the strongly warped spacetime near the horizon, virtual particles made from vacuum fluctuations turn out to have negative energy density.
  - **Energy density** = energy per unit volume
- These particles have positive mass – look at the one that escaped! – but their mass is distributed very strangely over spacetime. (Quantum-mechanically speaking, particles have nonzero volume; this is an aspect of the wave-particle duality.)
- Matter with negative energy density is generally called **exotic matter**.

BLACK HOLE EVAPORATION

Hawking radiation is emitted more efficiently if the gravity at the horizon is stronger (i.e., its temperature is higher).

- Recall: horizon gravity is stronger for **smaller**-mass black holes.

Thus, an isolated black hole will eventually evaporate as it radiates away all its mass-energy. The smaller the black hole mass is, the larger the evaporation rate. The time it takes to evaporate:

- $10^9 M_\odot$ black hole: 10^{94} years
- $2M_\odot$ black hole: 10^{68} years
- 10^8 gram black hole: 1 second

BLACK HOLE EVAPORATION TIME

It is relatively simple to derive the equation for the time it takes a black hole to evaporate via Hawking radiation. The answer, for a black hole that starts off with mass M, is

$$\boxed{t = \frac{10240\pi^2 G^2}{hc^4} M^3 = (8.407\times10^{-26}\ \mathrm{s/g^3}) M^3}$$

where $h = 6.626\times10^{-27}$ erg s is Planck's constant, and you already know all the others.

You need to understand and learn how to use this formula (but not for Thursday's exam).

HOW LONG IS THAT?

The Large Hadron Collider (LHC) in Switzerland is ***theoretically*** able to produce tiny black holes with mass $2.50{\times}10^{-20}$ g. How long would they last before they evaporate?