WORMHOLES, RELATIVITY & THE UNIVERSE

PROBLEM SET #7 ON WEBWORK – DUE TOMORROW AT MIDNIGHT

FIRST MOVIE NIGHT TOMORROW!

Wormhole time machines
General relativity and the Universe
Hubble: the Universe is observed to be homogenous, isotropic, and expanding
Redshift and distance: Hubble’s Law

Visible-light spectrum of the quasar 3C 273 (Maarten Schmidt, 1963)
WORMHOLES AS TIME MACHINES

How does time hook up inside a wormhole? Imagine a wormhole with \textit{constant length in hyperspace}, but with the \textit{two mouths moving with respect to each other in physical space}, with one of them experiencing acceleration.

- Time dilation: clocks just outside the two mouths would appear to a distant observer to run at different speeds; the rates of time flow are different.
- From the inside, though, the mouths appear at rest with respect to each other; the rates of time flow are the same.
- This effect, the difference in time flows at the two mouths and the joining in the middle, could enable the use of a wormhole as a time machine...
HOW TO BUILD A WORMHOLE TIME MACHINE

1. Start with a wormhole whose two mouths (called mouths A and B) are close together in space. Fix things up so that they stay the same distance apart in hyperspace.
   
   In Thorne’s description in the textbook, this is illustrated by two people reaching into opposite mouths and holding hands.

2. Take mouth B on a trip at high speeds (approaching light speed) out a great distance, and then return it to its former spot without ever changing the distance between the two mouths in hyperspace.

   Because of time dilation, the trip will take a short time according to an observer traveling with mouth B and a much longer time according to an observer who stays with the “stationary” mouth A.

   While B is gone, the observer at A can travel into the future (to the time when B returns) by passing through mouth A.

   After B returns, an observer at B can travel into the past (to the time when B left) by passing through mouth B.

   Therefore, the length of time travel is the time lag between clocks fixed at A and B during B’s trip and is therefore adjustable by modifying the details of the trip.

   Travel between arbitrary times is not provided!
STABLE WORMHOLE TIME MACHINES MAY BE IMPOSSIBLE

Geroch, Wald, and Hawking on self-destruction of wormhole time machines:

- Light leaving the origin during B’s trip and entering mouth B as it is returning, can travel backwards in time, emerge from A, and meet itself in the act of leaving.
- It can do this as many times as it likes, even an infinite number of times.
- Since light can constructively interfere (all the peaks and troughs of the light wave lining up), a large positive energy density could be generated in the wormhole, causing it to collapse.
- This process could take as little as $10^{-95}$ seconds in the reference frame of mouth A.

A RECIPE FOR WORMHOLE TIME-MACHINE DESTRUCTION

Aim a laser at mouth B; orient the mirror so that light emerging from mouth A joins up with the beam aimed at mouth B.

A huge number of photons immediately emerges from mouth A, and the vast increase of energy inside it collapses the wormhole.
STABLE WORMHOLE TIME MACHINES MAY BE IMPOSSIBLE

• It is also possible for this to happen with light created by vacuum fluctuations!
  • Since light has wave properties as well, the probability that virtual photons from near A travel to B
    and re-emerge from A pointed at B is nonzero, even if there is nothing to aim the photons that
    way.
    This would render all wormhole time machines unstable.
  • The interference might not be constructive, though, because the wormhole tends to defocus
    the light in the manner of a negative lens; therefore, we do not know whether this is a fatal
    objection.

GENERAL RELATIVITY & THE UNIVERSE

It was recognized soon after Einstein’s development of the general theory of relativity in 1915
that this theory provides the best framework in which to study the large-scale structure of the
Universe:
  • Gravity is the only force known (then or now) that is long-ranged enough to influence objects
    on scales much larger than typical interstellar distances. All the other forces (electricity,
    magnetism, and the nuclear forces) are “shielded” in large accumulations of material or are
    naturally short-ranged.
  • And there is much more matter around to serve as gravity’s source.
Einstein himself worked mostly on this application of GR, rather than on “stellar” applications
like black holes. The results wind up having a lot in common with black holes, though.
BASIC STRUCTURE OF THE UNIVERSE

The Universe contains:

- Planets (Earth’s diameter $= 1.3 \times 10^4 \text{ km}$)
- Stars (Sun’s diameter $= 1.4 \times 10^6 \text{ km}$)
- Planetary systems (Solar System diameter $= 1.2 \times 10^{10} \text{ km}$)
- Star clusters, interstellar clouds (Typical distance between stars $= \text{a few light years} = 3 \times 10^{13} \text{ km}$)
- Galaxies (Diameter of typical galaxy $= \text{a hundred thousand light years} = 10^{18} \text{ km}; \text{typical distance between galaxies} = \text{a million light years} = 10^{19} \text{ km}$)

For a long time, it was thought by many astronomers that the nebulae we now call galaxies were simply parts of the Milky Way. In the early 1920s, Edwin Hubble measured their distances and proved otherwise.

THE UNIVERSE IS FULL OF GALAXIES

To observe these galaxies is to probe the structure of the Universe.

*This is the HST Ultradeep Field (Beckwith et al., NASA/STScI). Only a few stars are present; virtually every dot is a galaxy. If the entire sky were imaged with the same sensitivity and the galaxies counted, we would get several hundred billion.*
GENERAL RELATIVITY & THE STRUCTURE OF THE UNIVERSE

The Universe is not an isolated, distinct object like those we have dealt with so far. In its description, we are interested in the large-scale patterns and trends in gravity (or spacetime curvature). Why?

• These trends tell us how galaxies and groups of galaxies – the most distant and massive things we can see – move around in the Universe, and why we see groups of the size that we do: this study is called cosmology.

• They can also tell us about the Universe’s origins and fate: this is called cosmogony.

• By large-scale, we mean sizes and distances large compared to the typical distances between galaxies.

To solve the Einstein field equation for the Universe, you need to apply what is known observationally about the Universe as “initial conditions” or “boundary conditions.” The solutions will tell us the conditions for other times.

In the early 1920s, observations (by Hubble) began to suggest that the distribution of galaxies is isotropic and homogenous on large scales, at least in the local Universe.

• Isotropic = looks the same in any direction from our viewpoint.

• Homogeneous = looks the same from any viewpoint within the Universe.

These facts serve as useful “boundary conditions” for the field equation.
ISOTROPY OF THE UNIVERSE ON LARGE SCALES
Each dot in the image is a galaxy (SDSS DR12). The galaxies are essentially randomly, and uniformly, distributed.

ISOTROPY OF THE UNIVERSE ON LARGE SCALES
Isotropy on the scale represented by these circles’ diameters means that approximately the same number of galaxies are contained within them, no matter where on the sky we put them, which is evidently true in this picture.
HOMOGENEITY OF THE UNIVERSE ON LARGE SCALES

This is a 2.5°-thick slice in declination of the SDSS survey along the celestial equator. Distances (in redshift) are indicated on the horizontal axis.

Again, the galaxies and their larger groupings tend to be randomly distributed through volume on large scales.

HOMOGENEITY OF THE UNIVERSE ON LARGE SCALES

Homogeneity on the scale of these circles’ diameters means that approximately the same numbers of galaxies are contained within them, no matter where we put them within the volume of the Universe.

Hubble demonstrated this slightly differently: he observed that fainter galaxies (same as brighter ones, but further away) appear in greater numbers than brighter galaxies by the amount that would be expected if the galaxies are distributed uniformly in space.
GENERAL RELATIVITY & THE UNIVERSE

Einstein and de Sitter (late 1910s and early 1920s, Germany), Friedmann (1922, USSR), Lemaître (1927, Belgium), and Robertson and Walker (1935-7, US/UK) produced the first solutions of the field equations for an isotropic and homogeneous universe.

The types of solutions they found:

1. Collapse, ending in a mass-density singularity
2. Expansion from a mass-density singularity, gradually slowing and reversing under the influence of gravity, ending in a collapse to a mass-density singularity.

These outcomes are for universes with a total kinetic energy (energy stored in the motions of galaxies) less than the gravitational binding energy. They are called closed universes.

Model 1 is very similar to what we now call black hole formation, since it ends in a mass-density singularity.

Note that models 2-4 all involve expansion from a mass-density singularity, so the creation and development of the Universe must be rather like black hole formation running in reverse.
THE EXPANDING UNIVERSES (2-4)
All three expanding solutions predict that the matter density in the Universe was singular at earlier times, and that the expansion started as an explosion of this singularity. This is the Big Bang.

SOLUTIONS FOR THE UNIVERSE

All the solutions have these features in common:
• Mass-density singularities
• Dynamic behavior: the structure given by the solutions is different at different times between singularities (at which time does not exist, of course).

Einstein was not happy with this. He thought that singularities such as these indicated that there were important physical effects not accounted for in the field equation. Einstein had a hunch that the correct answer would involve static behavior: large-scale structure should not change with time.
• He also saw how he could “fix” the field equation to eliminate singular and dynamic solutions: introduce an additional constant term, which became known as the cosmological constant, to represent the missing, unknown physical facts.
THE FIELD EQUATION & THE COSMOLOGICAL CONSTANT

(Not on the exam or homework)

The field equation under a particularly simple set of assumptions and conditions for a homogeneous and isotropic universe:

\[
\left(\frac{1}{R} \frac{dR}{dt}\right)^2 - \frac{8\pi G}{3} \rho = -c^2 \frac{k}{R^2}
\]

The same equation modified by Einstein (1917):

\[
\left(\frac{1}{R} \frac{dR}{dt}\right)^2 - \frac{8\pi G}{3} \rho - \frac{c^2}{3} \Lambda = -c^2 \frac{k}{R^2}
\]

A certain positive value of \(\Lambda\) leads to a static solution.

HUBBLE OBSERVES THE UNIVERSE EXPANDING

In 1929, Hubble made his third great contribution to cosmology; he observed that

- Distant galaxies are always seen to have redshifted spectra. Therefore, they all recede from us.
- The magnitude of this Doppler shift for any given distant galaxy is in direct proportion to the distance to this galaxy: with \(V = \text{velocity}\) and \(D = \text{distance to galaxy}\),

\[
V = H_0 D \quad \text{Hubble's Law}
\]

where \(H_0 = 20 \text{ km/s/Mly}\) (the Hubble constant).

In other words, we see that the Universe is expanding.
WHY GALAXIES RECEDE FROM AN OBSERVER IN AN EXPANDING UNIVERSE, NO MATTER WHERE THEY STAND

All intergalaxy distances increase: $A' > A, B' > B$. (The galaxies themselves do not expand, though.)

Galaxies recede from one another, and recede faster the further apart they are: $B' - B > A' - A$. Because the galaxies recede, the Doppler shifts are all redshifts.
THE EXPANDING UNIVERSE

HUBBLE’S LAW

Here are Hubble’s and Humason’s original data and results (1929). Clearly, the straight line is a good fit, so

\[ V = H_0 D \]

though the slope (the Hubble constant) came out much larger than the recent measurements as a result of then-unforeseen systematic distance errors.
HUBBLE’S LAW

Nowadays, better distance measurements and a better fit:

\[ H_0 = 22.52 \pm 0.509 \text{ km/s/Mly} \]

In ASTR 102, we will use a round number:

\[ H_0 = 20 \text{ km/s/Mly} \]

DIMENSIONS OF THE HUBBLE CONSTANT

Again: the Hubble constant is

\[ H_0 = 20 \text{ km s}^{-1} \text{ Mly}^{-1} \]

What are the dimensions of \( H_0 \)?
A. Time
B. 1/Time
C. Length
D. 1/Length
E. None of the above.
MAGNITUDE OF THE HUBBLE CONSTANT

\[ H_0 = 20 \text{ km s}^{-1} \text{ Mly}^{-1} \]

1 year = 3.16 × 10⁷ s

What is \(1/H_0\) in years?

SIMPLE USES OF HUBBLE’S LAW

Example: The redshift of 3C 273 corresponds to a speed of 48,000 km/s. How far away is 3C 273?

\[ D = \frac{V}{H_0} = \frac{48000 \text{ km/s}}{20 \text{ km/s/Mly}} = 2.4 \times 10^3 \text{ Mly} \]

Example: The center of the nearest cluster of galaxies, the Virgo Cluster, is 70 Mly away. What is the recession speed we expect for galaxies near the center of this cluster?

\[ V = H_0D = (20 \text{ km/s/Mly})(70 \text{ Mly}) = 1400 \text{ km/s} \]