BIG BANG COSMOLOGY

PROBLEM SET #8 ON WEBWORK – DUE AFTER THANKSGIVING BREAK

EXAM #3 AFTER THANKSGIVING BREAK

EXAM #3 ON THURSDAY, DEC. 5

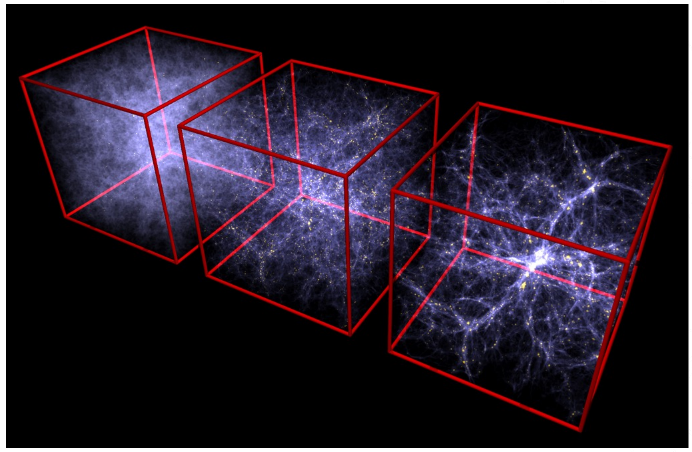
- 1 hr 15 min exam in class, open book and open notes
- Things that you should DEFINITELY bring with you:
  - Writing utensil (pencil or pen – blue or black ink)
  - Calculator
- Things that you should probably bring with you:
  - Lecture notes
  - Laptop or tablet (so that you can access the WeBWorK homework problems and the "How Big is That?" sheet on the course website)
- Practice exam is available on WeBWorK – email me when you want solutions
- REVIEW SESSION – **Wednesday, 12/4, at 7:30pm** in B&L 372

BIG BANG COSMOLOGY

Expansion rates, ages, and fates of GR universes in Big Bang cosmology

Inflation: how vacuum fluctuations might have kick-started the Universe

Matter-dominated universes, and measurements of the mass density and age of the Universe in which we live: an open Universe?



Simulation of structure in a universe dominated by cold dark matter and dark energy. (Volker Springel)

BIG BANG COSMOLOGICAL MODELS

We will not be solving the Einstein field equations for the Universe, or use it on any homework or exam, but to understand the differences between various universe models, it is useful to look more carefully at the equation.

Mass density
(mass per unit volume)

Universal expansion rate

$$\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8\pi G}{3}\rho - \frac{c^2}{3}\Lambda = -c^2\frac{k}{R^2}$$

Space (not spacetime) curvature

Typical distance between galaxies

Cosmological constant
Added by Einstein to allow static (constant in time) solutions

BIG BANG COSMOLOGICAL MODELS

$$\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8\pi G}{3}\rho - \frac{c^2}{3}\Lambda = -c^2\frac{k}{R^2}$$

This equation is a mathematical machine that can provide answers for all the terms in the equation (R, ρ, k, etc.) at all values of time (past, present, and future), if it is given "boundary" conditions: the values of these quantities at any one time during the Universe's history.

- Usually, we provide it measured values for the terms at the *present* time.

It is popular to define a **critical mass density**, $\rho_0 = \frac{3H_0^2}{8\pi G}$, a **normalized** mass density, $\Omega_M = \frac{\rho}{\rho_0}$, and a normalized cosmological constant, $\Omega_\Lambda = \frac{c^2\Lambda}{3H_0^2}$. With these quantities, the field equation is

$$\left(\frac{1}{H_0 R}\frac{dR}{dt}\right)^2 - \Omega_M - \Omega_\Lambda = -\frac{c^2}{(H_0 R)^2}k$$

BIG BANG COSMOLOGICAL MODELS

Since Ω_M is a ratio of mass densities, it may be useful to think of Ω_Λ as a ratio of densities as well. Therefore, we often define

$$\rho_\Lambda = \frac{c^2\Lambda}{8\pi G} \qquad \text{so that} \qquad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_0}$$

- Since ρ_Λ (and Ω_Λ) are expressed in the same units and terms as mass densities but are not densities of matter or radiation or anything related (like ρ, ρ_0, and Ω_M are), we need new words to name them. Currently, the most popular name for the "substance" that corresponds to ρ_Λ and Ω_Λ is **dark energy**. $\rho_\Lambda c^2$ can be thought of as a dark energy density.

BIG BANG COSMOLOGICAL MODELS

$$\left(\frac{1}{H_0 R}\frac{dR}{dt}\right)^2 - \Omega_M - \Omega_\Lambda = -\frac{c^2}{(H_0 R)^2}k$$

At first, it looks as if Ω_M and Ω_Λ should have the same effect on how R and k come out in the solutions, but they do not.

- Since mass is conserved, the normalized mass density Ω_M decreases as the Universe expands.
- Ω_Λ, related as it is to the cosmological constant, stays the same as the Universe expands.
- As we will see, this property makes positive values of Ω_Λ lead inexorably to expansion, no matter what the value of Ω_M might be. (And to inexorable collapse, for negative values.)

In these terms, we will now discuss how the GR Big Bang model applies, and how it is constrained by measurements of some of the quantities like R, k, and Ω_M:

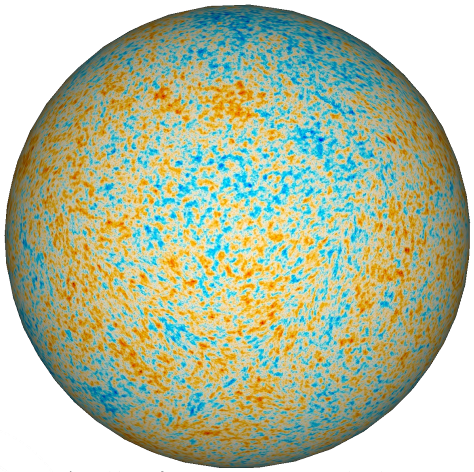
- The very early universe and **inflation**.
- Matter-dominated universes, like the one we used to think we live in, and **dark matter**.
- The new flat Universe, and **dark energy**.

INFLATION: THE CMB IS ALMOST *TOO* ISOTROPIC

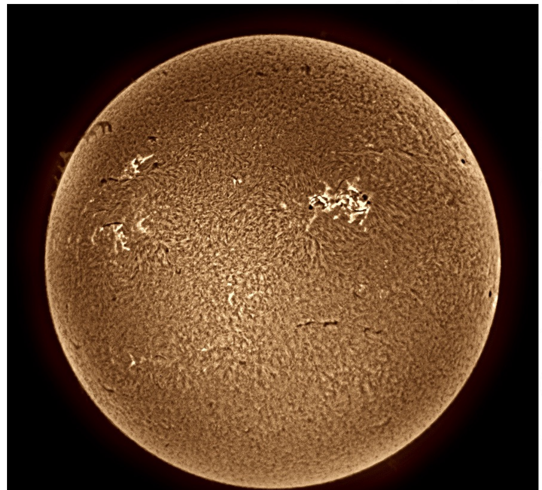
No part of the cosmic microwave background differs in brightness from the average by more than 0.001%. It is difficult to make gases, or the light they emit, *that* smooth or uniform. (Consider sunspots!)

- To do so would usually require that all parts of the gas be strongly interacting with each other, or that the gas be extremely well mixed.
- This would not seem possible for different parts of the decoupling surface. We were once part of that surface, and the parts of it that we see today have been out of contact with us (and each other) since the Big Bang, since we are only now receiving light from these parts and no signal or interaction can travel faster than light.

INFLATION



Big Bang's visible surface: temperature varies by about 0.001% of the average. (Planck/ESA)



Sun's visible surface: temperature varies by 10-20% of the average. (By Robert Gendler)

INFLATION

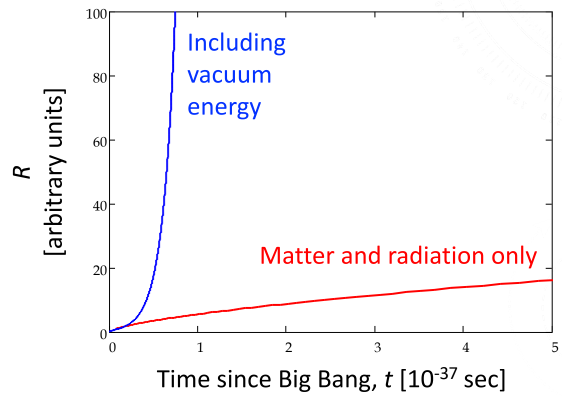
One theoretically-popular way out of this problem is to postulate a brief period of **inflation** early in the Universe's history. Briefly, this is thought to happen as follows:

- Shortly after the Big Bang, the **vacuum** *could* have had a much larger energy density, in the form of virtual pairs, than it does today. This possibility is allowed under certain theoretical models of numbers and interactions of elementary particles.
- At some time during the expansion, the vacuum underwent a **phase transition** (like freezing or condensing) to produce the lower-energy version we have today, presumably driven by the changes in spacetime curvature.
- While the vacuum was in its (possible) highest-energy-density state, it gave a large additional impulse to Universal expansion.
  - Vacuum fluctuations fill whatever volume the Universe has, independent of how much real matter it contains.
  - Thus, the high-energy vacuum acts like dark energy, i.e. like a cosmological constant.

INFLATION

- Accounting for the vacuum's influence in general relativity leads to a **much smoother and faster expansion**. During this period, spacetime's radius of curvature increases more like a bubble blowing up than like a blast wave – hence the name **inflation** for the process.

Solutions to the field equation for the early universe.



INFLATION

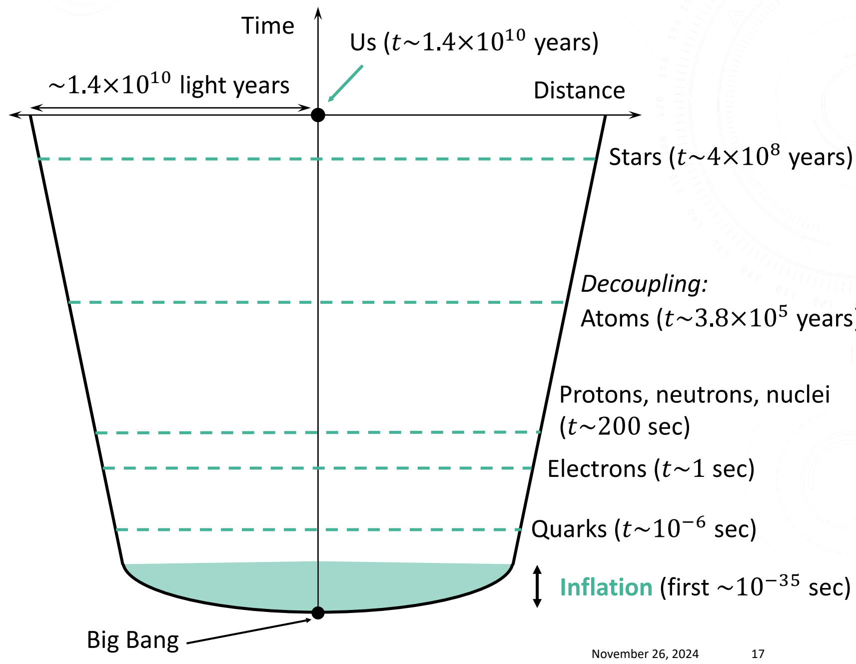
The inflationary era would have been relatively brief, much shorter than the time between Big Bang and decoupling.

- It would be enough for it to have lasted through 100 doublings of the Universe's size; this takes only about 10^{-35} seconds.
- During the remaining "normal" expansion between the end of inflation (decay of the vacuum to its lowest energy density state) and decoupling, the bumps and wiggles normally present in blast waves still would not have had enough time to develop.

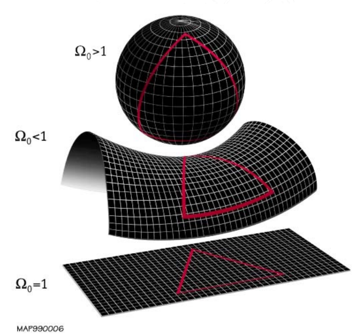
We know, of course, that the Universe has become much less smooth since decoupling. The seeds for inhomogeneities like galaxies, stars, and people were not sown before decoupling, however.

EXPANSION OF THE UNIVERSE

Not drawn to scale!



THE UNIVERSE'S CURVATURE



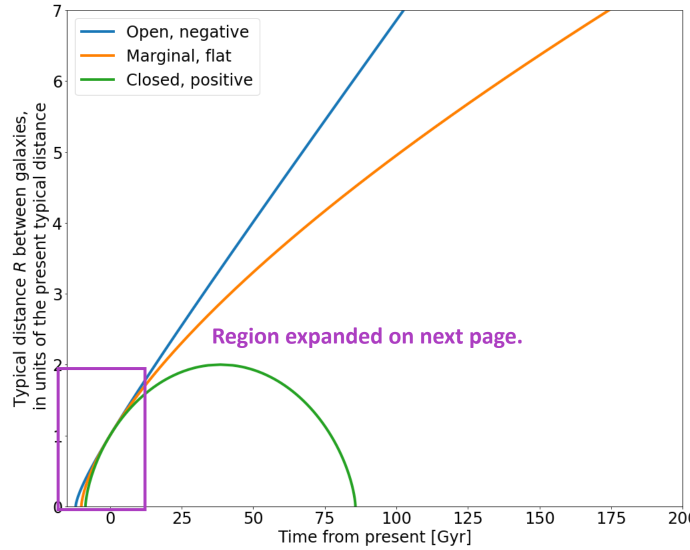
1. The total energy density, $\Omega_0 = \Omega_M + \Omega_\Lambda$, uniquely determines the value of the curvature k:
   - If $\Omega_0 > 1$, then $k = +1$ (positively-curved space).
   - If $\Omega_0 = 1$, then $k = 0$ (flat space).
   - If $\Omega_0 < 1$, then $k = -1$ (negatively-curved space).
2. The relative amounts of Ω_M and Ω_Λ uniquely determine the age and fate of the universe.
   - If $\Omega_\Lambda < 0$, or $\Omega_\Lambda = 0$ and $\Omega_M > 1$, then the universe is gravitationally bound (closed).
   - If $\Omega_\Lambda > 0$, or $\Omega_\Lambda = 0$ and $\Omega_M < 1$, then the universe is **not** gravitationally bound (open).
   - If $\Omega_\Lambda = 0$ and $\Omega_M = 1$, then the universe is called a critical, or marginal, universe.

If bound, the expansion will reverse, and the universe will re-collapse; if not, then the expansion will continue forever.

MATTER-DOMINATED UNIVERSES

After inflation has come and gone, and decoupling has already happened, the energy density of everything we know about in the Universe (so $\Omega_\Lambda = 0$) is dominated by the rest mass of matter. Such a universe is called **matter-dominated.**

Here are some results of such calculations for matter-dominated universes with three different present-day densities. Labels indicate boundedness and the sign of the space curvature k.



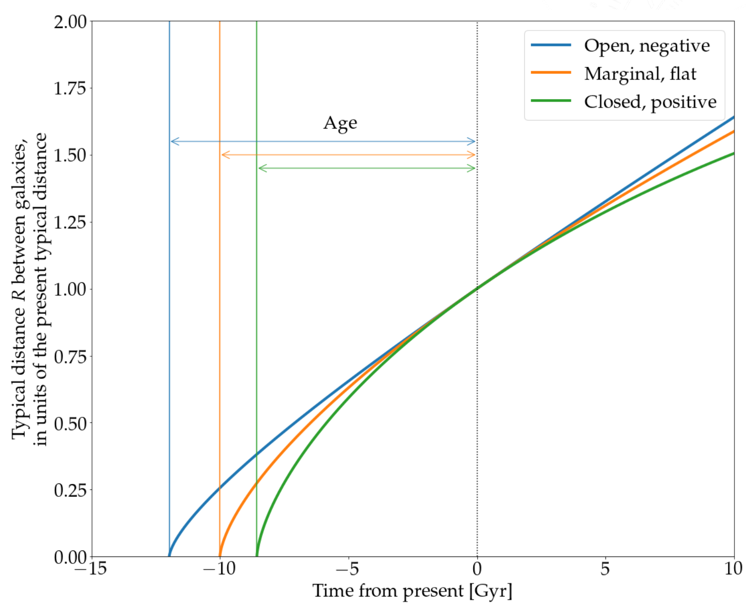
MATTER-DOMINATED UNIVERSES

Open

Marginal

Closed

All matched to observed expansion rate at the present time.



HOW CAN WE TELL WHICH "UNIVERSE" IS OURS?

There are a few "simple" measurements that we can make to determine which model applies:

1. Directly measure the **matter density**, using observations of the motions of galaxies to determine how much gravity they experience. (Much like our way of measuring black hole masses by seeing the orbital motion of companion stars.)
2. Measure the **ages of the oldest objects** in the Universe.
3. Measure the **acceleration or deceleration of galaxies**: the rate of change of the Hubble "constant."

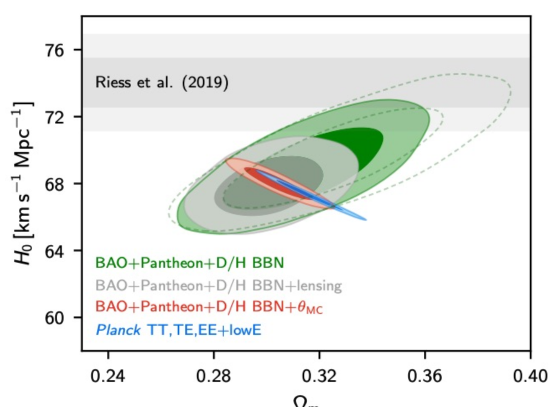
The first two ways are the least difficult and provide most of our data.

THE MASS-DENSITY OF THE UNIVERSE

Astronomers have been perfecting measurements of this quantity for decades.

- Observational bounds on Ω_M made from optically-detected galaxy clusters yield
$$\Omega_M = 0.315 \pm 0.007$$
and that **only about 15% of this is normal matter.** We will take $\Omega_M = 1/3$.

The contours indicate 68% and 95% confidence intervals on Ω_M and H_0.

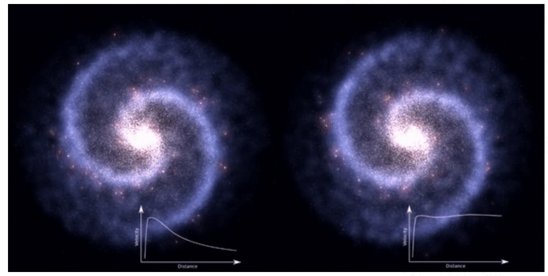


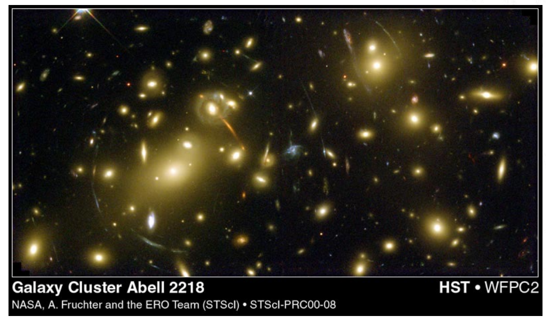
Planck Collaboration (2018)

DARK MATTER

We detect lots of matter, through that matter's gravity and its influence on the motions of galaxies – one third of the amount that it would take to close the Universe – but only a small fraction of this mass, 0.15 of it (15%) exists in the form of normal matter (i.e. atoms).

- The rest (85%) is called **dark matter** because it signals its existence only by its gravity (so far), not by emitting light.
- We do not know of what it is made; all we know is that it cannot contain protons or neutrons. (It cannot be photons, neutrinos, or electrons either!)





"SEEING" DARK MATTER

Does dark matter exist?

A. Yes – we can see the light that it emits as it interacts with regular matter.
B. Yes – we can detect its gravitational influence on regular matter.
C. Unsure – the evidence that we have for its existence is inconclusive.
D. No – all the "signatures" of dark matter are really manifestations of something else.