Astronomy 102 — Recitation 3

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Fall 2024

Review of lectures 4 & 5 and Ch. 1.

Reference frames

- A reference frame is a setting that can hold an observer, measurement tools, etc. that are all stationary with respect to each other.
- An inertial reference frame is one that is not accelerating (there is no net force acting on it).
- Absolute quantities are those which have the same magnitude in any reference frame. The speed of light is an example of an absolute quantity.
- Relative quantities are those which have different magnitudes depending on the observer's reference frame.

Space-time diagrams

A diagram with space (x) on the horizontal axis and time (t) on the vertical axis.

Light and the aether

The aether was a theoretical medium through which light could propagate. If it existed, then the speed of light would be a relative quantity and would change depending on an observer's motion through the aether. The detailed experiments performed by Michelson and Morley helped Einstein to realize that the aether did not exist.

Einstein's special theory of relativity

1. The laws of physics have the same appearance within all inertial reference frames, independent of their motion.
2. The speed of light in vacuum is the same in all directions, independent of the motion of the observer who measures it.

Predictions and Consequences of Special Relativity

- Length contraction
- Time dialation
- Velocities are relative with the exception of the constant, maximum limit of the speed of light
- Spacetime warping — distance in a given reference frame is a mixture of distance and time from other reference frames
- Simultaneity is relative

- Mass is relative
- There is no frame of reference in which light appears to be at rest
- Mass and energy are equivalent

Limitations of Special Relativity

- Only applies to **inertial** reference frames, i.e. it can only explain motion in a straight path at constant speed
- Cannot be applied to situations involving forces

Adding velocities

Classically, two velocities v_1 and v_2 can combine like

$$v_1 = v_2 + V \tag{1}$$

In relativity, we add velocities in the following manner:

$$v_1 = \frac{v_2 + V}{1 + \frac{v_2 V}{c^2}} \tag{2}$$

The form of this equation keeps v_1 from ever having a value greater than $c = 3 \times 10^{10}$ cm/s, the speed of light (nothing can move faster than c). Since $v = \frac{\Delta x}{\Delta t}$, then the distances and times must change to ensure that this speed is never surpassed (the source of length contraction and time dilation).

Gamma

$$\gamma = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}} \tag{3}$$

Length contraction

$$\Delta x_1 = \frac{\Delta x_2}{\gamma} \tag{4}$$

for a moving object, where Δx_2 is the rest length as measured by an observer moving with the object, and Δx_1 is the contracted length as measured by a stationary observer.

Time dilation

$$\Delta t_1 = \gamma \Delta t_2 \tag{5}$$

for a moving clock, where Δt_2 is the time span measured by an observer moving with the clock, and Δt_1 is the time span measured by a stationary observer.

In-class problems

1. You are at rest in a space station. You have a friend who, according to you, is flying to the right in a spaceship at $0.5c$. According to your friend, his spaceship is 2 light-seconds long.

   (a) Draw a space-time diagram in your frame showing a radio communication between you and your friend. You should each send two messages (send-receive-send-receive). You send your first message when the back of his spaceship passes you. Make sure to include both you and your friend in the diagram.

(b) Draw the same sequence of events in your friend's frame.

2. A cannonball is fired south at a speed of 60 mph from a pickup truck that is traveling north at 60 mph. To an observer on the side of the road, what velocity is measured for the cannonball? What is the velocity of the cannonball as measured by the driver of the truck?

3. Same situation as above, but now the truck is traveling north at $0.5c$ and our cannonball is fired south at $0.4c$. What is the velocity of the cannonball as measured by the driver of the truck? What velocity would be measured by a stationary observer on the side of the road?

4. Same situation as above, but now the cannonball is fired in the direction of the truck's motion. Using the relativistic velocity addition equation, what would a stationary observer measure

for the cannonball's velocity? What would you expect the velocity to be classically?

5. Calculate the length of a starship as measured by an observer on the Earth when the ship is traveling by at the following speeds. The rest length of the starship is 100 m.
   (a) $0.01c$
   (b) $0.1c$
   (c) $0.5c$
   (d) At what speed does the starship need to be traveling so that the stationary observer measures its length to be 10 m?

6. Bill is on a train, waving to Julie (who is standing on the train platform) every 4 seconds according to his watch. What time interval between Bill's waves does Julie measure on her watch if the train is moving at
   (a) $0.01c$
   (b) $0.1c$
   (c) $0.5c$

7. Astronaut Cooper (age 40) takes a 25 year journey (as measured on his clock), during which he travels at 86% of the speed of light. When he left home, his daughter Murph was 10 years old. How old will Murph be when he returns home? How old is Cooper?