Astronomy 102 — Recitation 4

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Review of lectures 6 & 7 and Ch. 2.

Minkowski Absolute Interval

$$(\text{Absolute Interval})^2 = \Delta x_1^2 - c^2 \Delta t_1^2 = \Delta x_2^2 - c^2 \Delta t_2^2 \tag{1}$$

where Δx_1 and Δt_1 are lengths of distance and time measured in observer 1's frame of reference, and Δx_2 and Δt_2 are measured in observer 2's frame of reference.

Principle of Equivalence

A free-falling frame of reference (e.g. a skydiver) feels weightless, and even though the frame is non-inertial, the frame acts as if it is inertial. Thus, a frame of reference freely-falling under the influence of gravity (and in vacuum) is **equivalent** to an inertial frame of reference. Special relativity and all the usual laws of physics apply to this frame the same way they apply to inertial frames. **The significance of the free-falling frame is that Einstein discovered a class of non-inertial frames whose physics he could describe.**

In-class problems

1. In 2200 A.D., the fastest available interstellar rocket moves at $v = 0.75c$. James Abbott is sent in this rocket at full speed to Sirius, the Dog Star (the brightest star in the sky as seen from Earth), a distance $D = 8.7$ ly as measured in the Earth frame. James stays there for a time $T = 7$ yr as recorded on his clock and then returns to Earth with the same speed $v = 0.75c$. Assume Sirius is at rest relative to Earth. Let the departure from Earth be the reference event (the zero of time and space for all observers).

   (a) Draw a space-time diagram in the Earth's frame that includes both the Earth and James (and his rocket). At what time does the rocket arrive at Sirius? At what time does the rocket leave Sirius? At what time does the rocket arrive back at Earth?

(b) Now draw a space-time diagram in James's reference frame, again including both the Earth and James (and his rocket). At what time does James arrive at Sirius? At what time does he leave Sirius? At what time does he arrive back at Earth?

2. The Death Star, 100 km in diameter, is moving at 50% the speed of light between two planets, planet A and planet B, which are both at rest. When the spacecraft is equidistant from the two planets it simultaneously fires a laser beam directly in front and directly behind the ship. From the perspective of the inhabitants of the planets, are the two planets destroyed at the same time? If not, which planet is destroyed first?

3. Suppose you are on a train traveling at $V = 0.8c$ which has a train car with length $l = 10$ ls. In this train car there is a light bulb at the center and two world leaders on either end of the

train car. Your friend is standing on the train station platform, and as the train passes the platform the light flashes. When the two world leaders in the train car see the flash, they both sign a declaration of war against the other. From the perspective of the observers on the platform, who declares war first?

4. A person rides in an elevator that is shot upward out of a cannon. Think of the elevator after it leaves the cannon and is moving freely in the gravitational field of Earth. Neglect air resistance.

   (a) While the elevator is still on the way up, the person inside jumps from the "floor" of the elevator. Will the person fall back to the "floor" of the elevator, hit the "ceiling" of the elevator, or do something else? If so, what?

   (b) The person waits to jump until after the elevator has passed the top of its trajectory and is falling back toward Earth. Will your answers to part (a) be different in this case?

   (c) How can the person riding in the elevator tell when the elevator reaches the top of its trajectory?

5. Test your skill as an acrobat and contortionist! Fasten a weight-measuring bathroom scale under your feet and bounce up and down on a trampoline while reading the scale. Describe readings on the scale at different times during the bounces. During what part of each jump will the scale have zero reading? Neglecting air resistance, what is the longest part of the cycle during which you might consider yourself to be in a free-falling frame?