Astronomy 102 — Recitation #5

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Review of lectures 9–10 and Ch. 3.

Einstein's field equation

"Spacetime's curvature tells mass how to move; masses tell spacetime how to curve."

Electromagnetic radiation — light

- Light consists of electric and magnetic waves which vary sinusoidally
- Varying electromagnetic fields exert forces on charged particles, causing them to accelerate
- Accelerating charged particles produce varying electromagnetic waves (light)
- Light represents the transport of electromagnetic energy through empty space without involving the transport of electric charges or current

Gravitational radiation

- Einstein's theory of general relativity indicated that there should exist a gravitational analog of light
- Gravitational waves propagate through empty spacetime at the speed of light
- Spacetime will warp (masses will accelerate) in response to a passing gravitational wave
- Gravitational radiation represents the transport of gravitational energy through empty space without involving the transport of mass
- We have successfully detected gravitational waves using laser interferometers (like LIGO)

Experimental tests of general relativity

All reproducible experiments to date have confirmed the predictions of Einstein's theory of general relativity.

- Precession of Mercury's orbit
- Gravitational lensing (bending of light from a distance galaxy around a massive galaxy cluster)
- Detection of gravitational waves (Hulse & Taylor, LIGO)

Schwarzschild Radius

What happens if you condense an object so much that the escape velocity at the surface is equal to the speed of light?

$$v_{\text{escape}} = \sqrt{\frac{2GM}{R}} = c \tag{1}$$

We can rearrange this equation for the expression for the **Schwarzschild Radius**, which is the radius of the event horizon for a black hole of mass M.

$$R_{\text{Sch}} = \frac{2GM}{c^2} \tag{2}$$

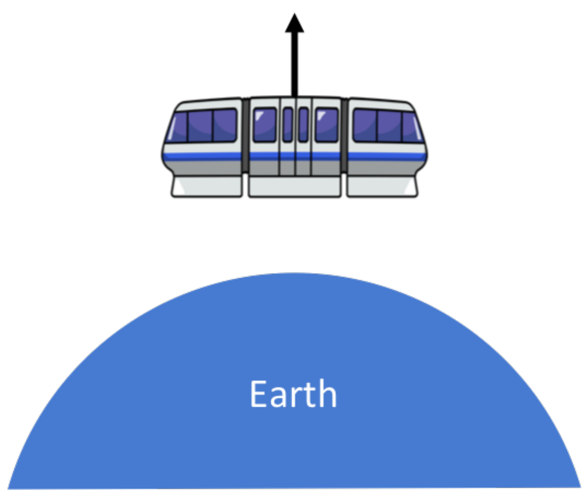
Since we know Circumference is given by $C = 2\pi R$, we can find the **Schwarzschild Circumference**:

$$C_{\text{Sch}} = \frac{4\pi GM}{c^2} \tag{3}$$

This is the event horizon circumference for a black hole of mass M.

In-class problems

1. You are launched upward inside a railway coach in a horizontal position with respect to the surface of Earth. After the launch, but while the coach is still rising, you release two balls at opposite ends of the train and at rest with respect to the train.



   (a) Riding inside the coach, will you observe the distance between the balls to increase or decrease with time?

   (b) Now you ride in a second railway coach launched upward in a vertical position with respect to the surface of Earth. Again, you release two balls at opposite ends of the coach and at rest with respect to the coach. Will you observe these balls to move together or apart?

   (c) In either of the cases described above, can you, the rider in the railway coach, distinguish whether the coach is rising or falling with respect to the surface of Earth solely by observing the ball bearings from inside the coach? What do you observe at the moment the coach stops rising with respect to Earth and begins to fall?

2. What is the Schwarzschild circumference for an object with the mass of Mars (6.42×10^{26} g)? (This is how small we would need to compress Mars to turn it into a black hole.)

3. How massive of an object would we need to create a black hole with a horizon circumference equal to the circumference of the Moon ($R_{\mathrm{Moon}} = 1.74 \times 10^8$ cm)?

4. What is the Schwarzschild circumference for the supermassive black hole at the center of the Milky Way, which has a mass $M = 4.1 \times 10^6 M_\odot$? What Schwarzschild radius does this equate to?

5. Say we collide two black holes, one with a mass $M_1 = 29 M_\odot$ and $M_2 = 36 M_\odot$. The resulting black hole has a horizon circumference of $C = 1160$ km. What are the horizon circumferences of the initial two black holes, and what is the mass of the resulting black hole?