Astronomy 102 — Recitation #8

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Fall 2024

Review of lectures 14–15 and Ch. 6–7.

Spinning black holes

- Stable orbits exist closer to a spinning black hole than a non-spinning one.

Mass-energy equivalence

From Einstein's mass-energy equivalence we can see that an object with mass m has a rest-energy associated with it, given by

$$E = mc^2 \tag{1}$$

where c is the familiar speed of light ($c = 3 \times 10^{10}$ cm/s).

Units of Energy

- SI Unit (meters-kilograms-seconds): $[E] = \mathrm{J} = \mathrm{Nm} = \frac{\mathrm{kg\ m^2}}{\mathrm{s^2}}$
- CGS Unit (centimeters-grams-seconds): $[E] = \mathrm{erg} = 10^{-7}\ \mathrm{J} = \frac{\mathrm{g\ cm^2}}{\mathrm{s^2}}$
- Metabolic energy (calorie): $[E] = \mathrm{cal} = 4.184\ \mathrm{J} = 4.184 \times 10^7\ \mathrm{erg}$

Luminosity

Luminosity is the amount of power produced by a star, or the amount of energy given off by the star per unit time.

Units of Power/Luminosity

- SI Unit (meters-kilograms-seconds): $[L] = \mathrm{Watts} = \mathrm{W} = \mathrm{J/s}$
- CGS Unit (centimeters-grams-seconds): $[L] = \mathrm{erg/s} = 10^{-7}\ \mathrm{J/s}$
- Luminosity of the Sun: $1L_\odot = 3.8 \times 10^{26}\ \mathrm{W} = 3.8 \times 10^{33}\ \mathrm{erg/s}$

Doppler Shift

If an emitting object is in relative motion to an observer, the motion will produce a shift in the observed light being emitted from that object. Say an object is emitting light at a wavelength λ_0 and is moving at a velocity v relative to an observer. The observer measures the wavelength of the light being emitted to be λ, where λ and λ_0 are related by:

$$\frac{\lambda - \lambda_0}{\lambda_0} = \frac{v}{c} \tag{2}$$

This can be rearranged to produce:

$$\lambda = \lambda_0 \left(1 + \frac{v}{c}\right) \qquad \text{or} \qquad v = c\left(\frac{\lambda}{\lambda_0} - 1\right) \tag{3}$$

Observational black hole features

Black holes will exhibit at least **two** of the following:

- Gravitational deflection of light
- X-ray and/or γ-ray emission
- Orbital motion of nearby stars or gas clouds
- Motion close to the speed of light
- Extremely large luminosity
- Accretion disk

In-class problems

1. *Rest energy*
   (a) What is the rest energy (in Joules) of a 50 gram candy bar? What is the rest energy in ergs?
   (b) Unfortunately, the human body is not this efficient at acquiring energy from food. The candy bar mentioned above can only provide about 200 calories of energy if consumed. What fraction of the total rest energy stored in the candy bar is this?
2. The expected lifetime of the Sun is approximately 10 billion years. Assume the luminosity of the Sun is constant throughout its life.
   (a) What is the total energy produced by the Sun during its lifetime?
   (b) The Sun is constantly fusing Hydrogen into Helium in its core. This process requires 4 Hydrogen atoms and results in a single Helium atom, but the mass of the Helium atom is only $m_{\rm He} = 3.97 m_{\rm H}$. This "missing mass" is converted into energy through $E = mc^2$. How much energy is given off per Helium atom created (in ergs)?
   (c) Given the luminosity of the Sun and your result from the previous part, how many Helium atoms are being created each second?
   (d) How much of the Sun's mass is being converted into energy each second? If this were to continue at a constant rate, for how long would the Sun last until its entire mass was converted to energy?
3. Two black holes merge, and the mass of the resulting black hole is $3M_\odot$ smaller than the sum of the original two black holes' masses.
   (a) This $3M_\odot$ loss is converted into energy in the form of gravitational waves. Using the mass-energy relation, what energy (in Joules) does this mass-loss correspond to?
   (b) The merging of the two black holes occurs in approximately 0.01 s. What power (in Watts) does this energy release correspond to? How does this compare this to the total power output of the observable universe (assuming that the 10^{22} stars in the Universe have an average luminosity of $1L_\odot$)?
4. A star in an eclipsing binary system has an orbital velocity of 42 km/s. It orbits a much less luminous companion star.
   (a) What is the maximum and minimum wavelengths you can measure for the Doppler-shifted absorption line, which at rest would be measured to be at 500μm?

(b) Now suppose that the binary system is receding from us at a velocity of 125 km/s. In this case, what would the maximum and minimum wavelengths for the Doppler-shifted absorption line be?

(c) What are the answers to the previous two parts when the binary star system has its orbital plane perpendicular to our line of sight?

5. You measure the wavelengths of an absorption line, which at rest is 5.000×10^{-5} cm, for an eclipsing binary star system over time and find that the maximum wavelength is 5.00024×10^{-5} cm and the minimum wavelength is 4.99976×10^{-5} cm. What is the orbital speed of the star in its orbit? Assume that the system is not moving relative to you.