Astronomy 102 — Recitation #10

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Review of lectures 18–20 and chapters 9 and 11–13.

Black hole evaporation

Through **Hawking radiation**, black holes can emit radiation which, over time, decreases the mass of the black hole. Given the initial mass of the black hole, the time it takes to completely evaporate is given by

$$t = \frac{10240\pi^2 G^2}{hc^4} M^3 = (8.407 \times 10^{-26}\ \mathrm{s/g^3}) M^3 \tag{1}$$

where M is the mass of the black hole, G is the Newtonian gravitational constant, c is the speed of light, and $h = 6.626 \times 10^{-27}$ erg s is Planck's constant.

Inside a black hole

- The "center" of a black hole is its mass-density singularity.
- Spacetime ends at the mass-density singularity.
- Time becomes a fourth dimension at the mass-density singularity, so there is no such thing as time at the mass-density singularity.
- Quantum foam is a physical analog for the randomly-connected four-dimensional space at the mass-density singularity.
- The mass-density singularity can both expand and collapse, depending on its interactions with mass and energy within the black hole's horizon.
- "Naked" singularities (mass-density singularities without an associated horizon) might exist.

In-class problems

1. If the Earth were compressed into a black hole, how quickly would it evaporate through Hawking radiation?
2. The supermassive black hole at the center of the Milky Way has a mass of around 4 million solar masses. How long would it take this black hole to evaporate through Hawking radiation? How does this compare to the age of the universe?
3. How massive of a black hole would we need in order for it to survive for more than 1 second before it evaporates?
4. If we turned the Moon into a black hole, how many orbits could it complete around the Earth before evaporating? What about the Earth around the Sun, if the Earth were a black hole?
5. Hawking radiation is the solution devised to give black holes nonzero entropy, so that they satisfy the second law of thermodynamics.

(a) How does Hawking radiation give a black hole entropy?

(b) In class, we saw that the luminosity due to Hawking radiation is dwarfed by a black hole's luminosity due to accretion. Is Hawking radiation enough to satisfy the second law of thermodynamics?