Astronomy 111 — Problem Set #2

Prof. Douglass

Due September 9, 2025 at the beginning of lecture

1. Consider the function
$$b(x) = \frac{h^4 c^3}{2(kT)^5} u_\lambda$$
where $u_\lambda = u_\lambda(\lambda, T)$ is the Planck blackbody function, and $x = \lambda kT/hc$.
   (a) Find an expression for $b(x)$ by substituting suitable expressions for the elimination of λ and T.
   (b) Using graphing software (Excel, Python, Maple, Mathematica, etc.), plot $b(x)$ as a function of x between 0 and 1.5. At what x is $b(x)$ maximum?
   (c) Using the results of part b, derive Wien's displacement law, $\lambda_{\max} T = \text{constant} = 0.29$ cm K.

2. Take a look at the crater data posted in the LunarPlates.pptx file on the class website.
   (a) In a few complete sentences, cite evidence that the craters are primarily impact craters. Can you find any evidence for a filled (possibly volcanic) crater?
   (b) Are the highland areas on the Moon older or younger than the maria? Justify your answer from your observations of the crater data.

3. Derive an equation for the initial relative abundance of the daughter nuclide in a two-mineral system in terms of the present-day relative abundances of radionuclide and daughter, similar to the equation derived in class for the age of a rock in terms of these relative abundances.

4. You are given a moon rock, a dental pick, and a mass spectrometer. You break the rock and use the pick to extract some crystals of plagioclase and olivine from within the rock, and measure the rubidium and strontium isotopic abundances. You get

	$^{87}\text{Rb}/^{86}\text{Sr}$	$^{87}\text{Sr}/^{86}\text{Sr}$
Plagioclase	0.0468	0.70188
Olivine	0.1829	0.70946

   Calculate the age of the rock and the initial relative abundance of the daughter nuclide. Is this rock likely to be from the highlands or the maria?

5. Suppose that, in the same zeal with which they defined new names for the months and the days of the week, the French Revolutionaries had also changed the celestial coordinate system to commemorate the storming of the Bastille, so that their new North Pole would be located at the Bastille (latitude $+48.8531°$) and their new origin for right ascension would be that of the stars that were overhead at midnight on July 14, 1789 (Bastille Day): in terms of the "old" RA (the RA that we use), this RA is $\alpha = 19.6430^{\text{h}}$.

   What would be the Revolutionary Declination of the bright star Vega on Bastille Day (the angle between the new North Pole and Vega)?