Astronomy 111 — Problem Set #9

Prof. Douglass

Due November 11, 2025 at the beginning of lecture

1. *Mercury and tidal heating.* The situation of Mercury is similar to that of the Jovian satellites.
   (a) The tidal heating power generated in a body during an orbit is proportional to how much it stretches and relaxes under tidal forces through the orbit. In turn, the amount by which a body is stretched is proportional to the difference between the greatest and least radial tidal accelerations, g_{tr}.
   Calculate the difference between the minimum and maximum g_{tr} experienced by Mercury as it goes through its orbit, and compare the result to that for Io. Is tidal heating as important for Mercury as it is for Io?
   (b) Mercury's orbit and rotation are an example of a **spin-orbit resonance**. What is an integer-index way to characterize this resonance?
   (c) Suppose Mercury's tidal bulge is oriented exactly along the direction toward the Sun as it passes through perihelion. At what angle will it be oriented next time it passes through perihelion?

2. *The Moon and its orbital evolution.* Last homework, you found that the vertical amplitude of the Moon-induced oceanic tide is $\delta R \cong 38.5$ cm.
   (a) Estimate the volume of the oceanic bulges and the mass ΔM of each bulge by whatever reasonable means you can invent. For simplicity, you may assume that the Earth is completely covered in pure water much deeper than 40 cm. (Hint: Ellipsoids with axes a, b, and c have volume $4\pi abc/3$.)
   (b) Because of the Earth's rapid rotation, the axis of the tidal bulges induced by the Moon leads the Moon's revolution by 16° (approximately; this turns out to be extremely difficult to measure). Calculate the magnitude of the torque by these bulges on the Moon.
   (c) Derive an expression for the rate of increase of orbital radius for a satellite on which is exerted a torque $\vec{\tau}$ in the same direction as its orbital angular momentum. (Hint: Consider taking the time derivative of the expression for the satellite's angular momentum, keeping in mind that you are trying to solve for the quantity dr/dt. Do not forget the chain rule!)
   (d) Calculate the rate in cm per year at which the torque from part b would increase the Moon's orbital radius. Compare this to the result from the NASA Lunar Laser Ranging experiment, 3.82 ± 0.07 cm/yr.

3. *Triton and tidal orbital evolution.* Triton, the largest satellite of Neptune, has a mass $m = 2.14 \times 10^{25}$ g and is in an orbit with a semimajor axis $a = 3.548 \times 10^{10}$ cm and eccentricity $\varepsilon = 0.000016$. It rotates synchronously with its revolution about Neptune, but its orbit is retrograde: it revolves in the direction opposite that of Neptune's rotation.
   (a) Calculate the orbital angular momentum $\vec{L}$ of Triton in g cm^2 s^{-1} (both magnitude and direction).
   (b) Triton raises tidal bulges on Neptune. If the angle θ of the bulge axis with respect to Triton's orbital radius is such as to maximize the magnitude of the torque, what is that angle? Express your answer in degrees, positive if it leads Triton's revolution, negative if it lags.

(c) Calculate the torque on Triton exerted by the tidal bulges in units of the bulge mass ΔM if the bulge is oriented as in part b. Is the orbital angular momentum of Triton increasing or decreasing?

(d) Detailed models indicate that Triton will stray close enough to Neptune to be destroyed in about 3.6 Gyr $= 3.6 \times 10^9$ yr. Given Triton's angular momentum (from part a) and the rate at which it is currently changing (from part c), what fraction of Neptune's mass $2\Delta M/M_N$ does the pair of bulges represent? (Hint: How long will it take to use up its present orbital angular momentum?)