ASTEROIDS & METEORITES

Midterm #1 on Thursday

MIDTERM EXAM #1

Takes place here next Thursday

Make sure to bring a writing instrument, a calculator, and one 8.5"x11" sheet on which you have written all the formulae and constants that you want to have at hand.

- No computers, no access to the internet or to electronic notes or stored constants in your calculator

The best way to study is to work through problems like those on the homework and recitations, understand the solutions distributed, refer to the lecture notes when you get stuck, and make up your equation sheet as you go along.

Try the practice exam on the website. Ideally, under realistic conditions (especially the time!).

Review session: Wednesday, Oct. 8 at 7:30pm in B&L 203H

ASTEROIDS & METEORITES

Restricted three-body problem (part I): The two-body potential without the Coriolis force, and the Lagrange points

Classification of asteroids by composition

The interiors of asteroids

Asteroid densities, and rubble piles

Special features

- Orbit families and collisional fragmentation
- Near-Earth-orbiters

Meteorites

- Composition, classification, and age
- Origins in planets and the asteroid belt

243 Ida and its satellite, Dactyl (Galileo/JPL/NASA)

ASTEROIDS EXAMPLE

An asteroid initially in a circular orbit at 2.5 AU suffers a collision that knocks it into an orbit tangent to the Earth's.

a) *Describe the change in its speed caused by the collision.*

Before:

$$a_1 = 2.5\text{ AU} \qquad \varepsilon_1 = 0 \qquad v_1 = \sqrt{\frac{GM_\odot}{a_1}} = 18.8\text{ km/s}$$

After:

$$r_p = 1\text{ AU} = a_2(1-\varepsilon_2) \qquad r_a = 2.5\text{ AU} = a_2(1+\varepsilon_2)$$

$$\Rightarrow a_2 = \frac{r_p + r_a}{2} = 1.75\text{ AU} \qquad v_2 = \sqrt{GM_\odot\left(\frac{2}{r_a} - \frac{1}{a_2}\right)} = 14.2\text{ km/s}$$

So the collision slows the asteroid down by 4.6 km/s.

ASTEROID EXAMPLE

b) *What sort of collision would cause this change? Could a small orbital change by a small body in a nearby orbit do this?*

Suppose that it is a completely inelastic collision with a much smaller body:

$$mv_1 + m_c v_c = (m + m_c)v_2 \approx mv_2$$

Then

$$v_c \approx \frac{m}{m_c}(v_2 - v_1) < 0$$

No, it cannot. A small body would have to be revolving in the opposite direction of everything else to cause such an orbital change in our original asteroid. There are not many such asteroids (anymore). This is why original changes by asteroid encounters are usually small and need to add up over a long time to amount to much (such as perturbations of asteroids in mean-motion orbital resonances with Jupiter, which, as mentioned above, leads to the Kirkwood gaps).

THE TWO-BODY GRAVITATIONAL POTENTIAL: INTRODUCTION TO THE RESTRICTED THREE-BODY PROBLEM

Consider a three-body orbiting system in which one body has mass m that is negligibly small compared to the other two (which have masses M_1 and M_2).

It is reasonable to construct the orbital energy of the system out of the energies of the two massive components, and consider the third body to simply follow along.

- Define the **gravitational potential, Φ,** as the potential energy per unit mass for the smaller body.

It is also reasonable to do this in a coordinate system that rotates with the revolution of the two massive bodies.

- This is called a **corotating** frame of reference. Our usual view is an **inertial** frame of reference.
- In this coordinate system, the potential energy will contain terms corresponding to "fictitious," inertial, forces such as the centrifugal force and the Coriolis force.

Let's derive a formula for the gravitational potential, Φ, of the two large masses, in the corotating frame.

THE TWO-BODY GRAVITATIONAL POTENTIAL

In the co-rotating reference frame, neglecting the Coriolis force:

$$r_1 + r_2 = a \qquad M_1 r_1 = M_2 r_2$$

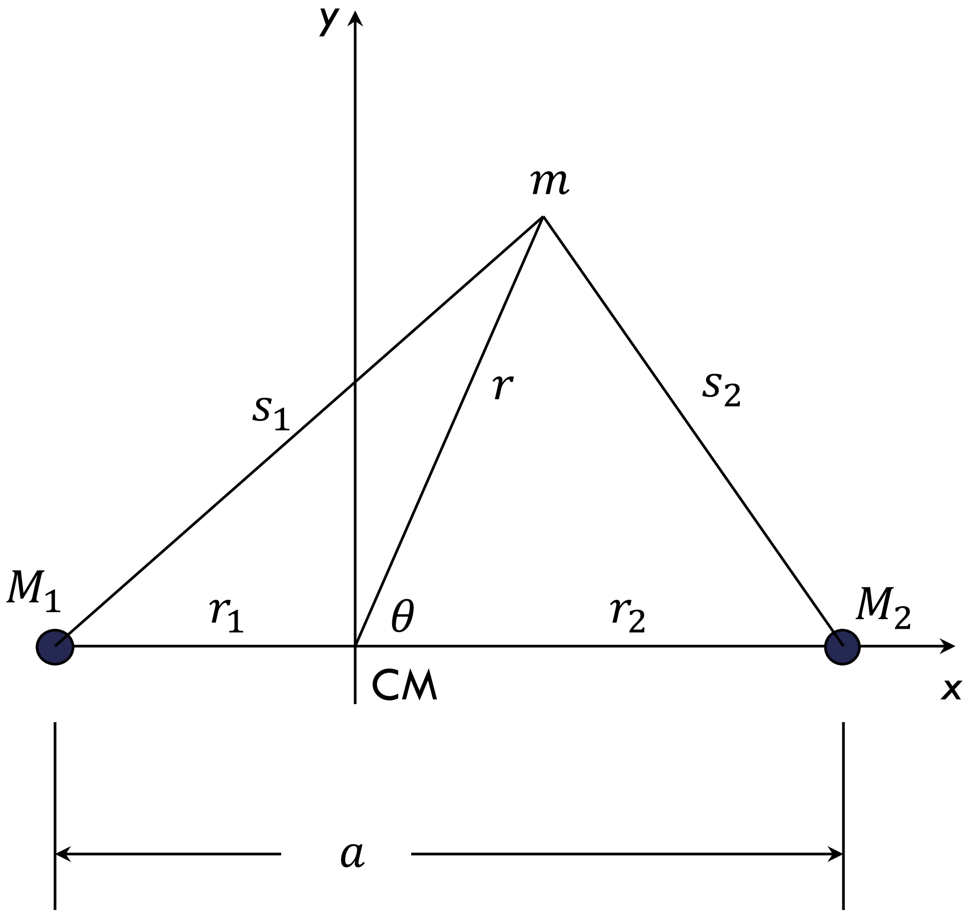
$$s_1^2 = r_1^2 + r^2 + 2 r_1 r \cos\theta$$

$$s_2^2 = r_2^2 + r^2 - 2 r_2 r \cos\theta$$

the gravitational potential is:

$$\Phi = -G\left(\frac{M_1}{s_1} + \frac{M_2}{s_2}\right) + \Phi_{\mathrm{center}}$$

where Φ_{center} is the potential due to centrifugal forces felt at r.



THE TWO-BODY GRAVITATIONAL POTENTIAL

Gravitational potential energy is the work done against the force of gravity when a body is moved in a gravitational field.

Analogously, the centrifugal potential Φ_{center} is the work done per unit mass against the centrifugal force when a body is moved around with respect to a rotating coordinate system.

In the rotating coordinate system, **the (fictitious) centrifugal force is equal and opposite to the (real) centripetal force** which, in the inertial reference frame, is necessary to keep the revolving body moving in its orbit.

As we saw earlier, centripetal acceleration has a magnitude $a = \frac{v^2}{\omega} = \omega^2 r$ and is directed at the orbit's center or focus. So

$$m\Phi_{\text{center}} = W = -\int_{\boldsymbol{r}_i}^{\boldsymbol{r}_f} \vec{F}_{\text{center}} \cdot d\vec{r} = -\int_{\boldsymbol{r}_i}^{\boldsymbol{r}_f} m\omega^2 r\, dr$$

$$\Phi_{\text{center}} = -\frac{1}{2}\omega^2 r^2$$

THE TWO-BODY GRAVITATIONAL POTENTIAL

By Kepler's third law,

$$\omega^2 = \left(\frac{2\pi}{P}\right)^2 = \frac{G(M_1 + M_2)}{a^3}$$

So the gravitational potential becomes

$$\Phi = -G\left(\frac{M_1}{s_1} + \frac{M_2}{s_2}\right) - \frac{1}{2}\omega^2 r^2$$

$$= -G\left(\frac{M_1}{s_1} + \frac{M_2}{s_2}\right) - \frac{G(M_1 + M_2)}{2a^3} r^2$$

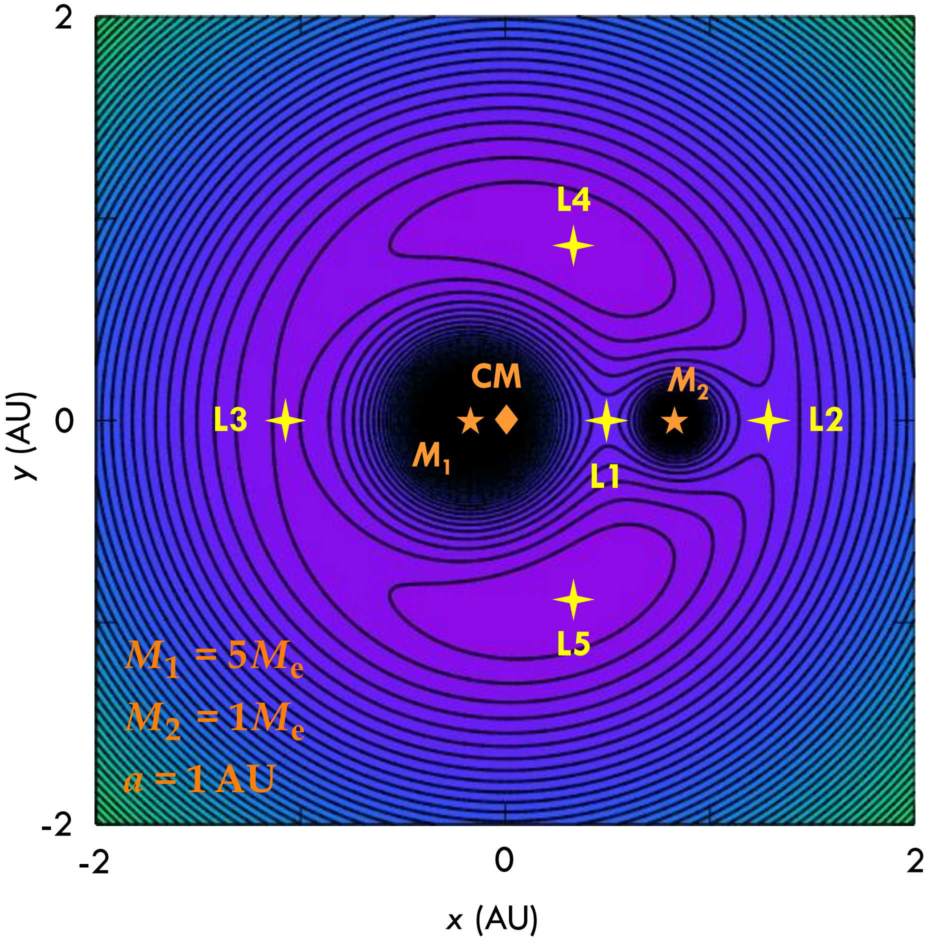
THE TWO-BODY GRAVITATIONAL POTENTIAL

Here's a contour plot of

$$\Phi = -G\left(\frac{M_1}{s_1} + \frac{M_2}{s_2}\right) - \frac{G(M_1+M_2)}{2a^3} r^2$$

as a function of $x = r\cos\theta$ and $y = r\sin\theta$, for an illustrative separation and pair of masses.

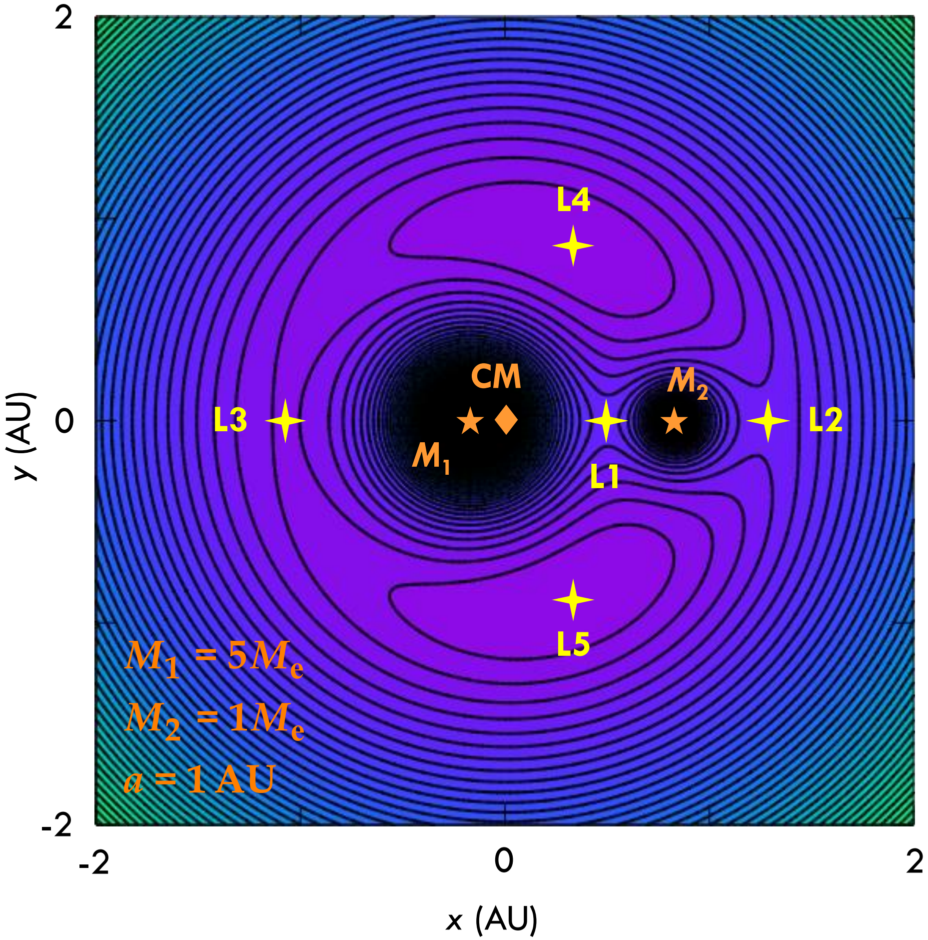
- Contours are lines of constant gravitational potential.
- Purple colors show relative maxima of potential.



LAGRANGE POINTS

Extrema of the gravitational potential, of which there are five in this plot besides the massive bodies, are called **Lagrange points.**

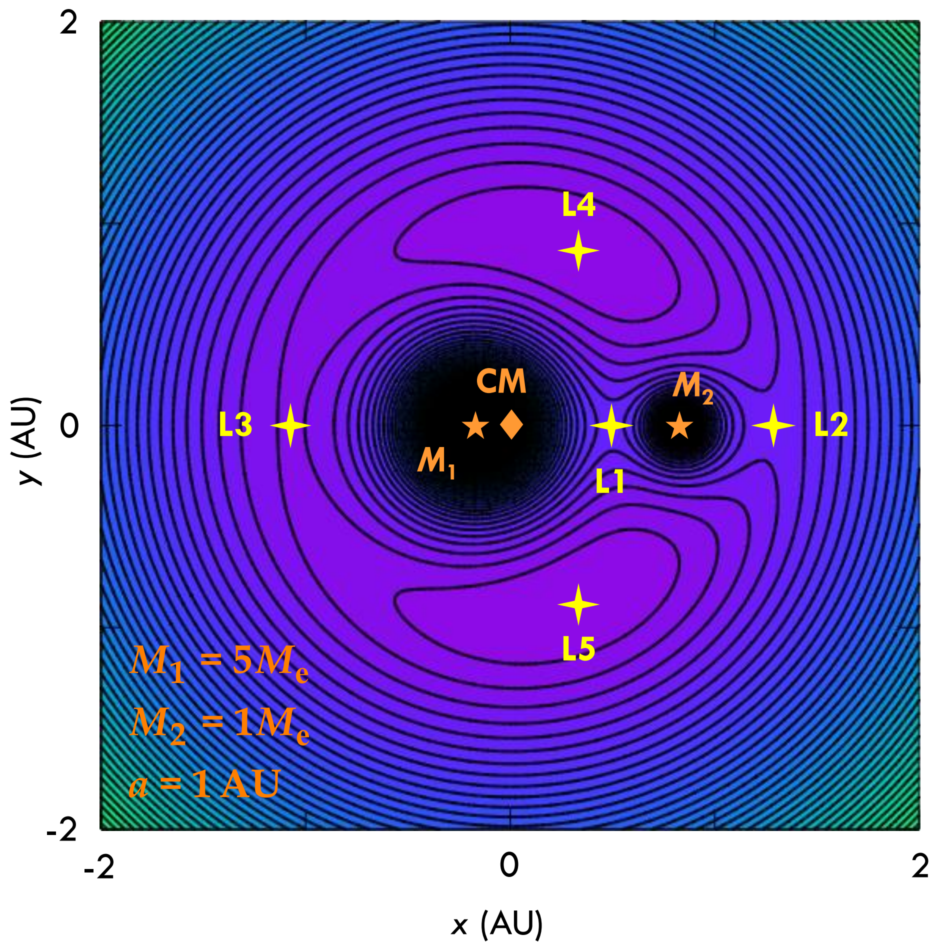
- The gravitational force vanishes at these points.
- So an object placed on one will stay there and orbit the CM along with the stars unless it suffers a perturbation: Lagrange points are *maxima* (unstable equilibria) of Φ.
- Spacecraft which are placed at Lagrange points need thrust to stay there.
- Some famous inhabitants of the Earth-Sun L points: JWST, Gaia, Herschel, Planck, WMAP (L2); SOHO, ACS, DISCOVR (L1).



LAGRANGE POINTS

Exceptions: **if Coriolis forces are included**, and **one of the massive bodies outweighs the other by a factor of 25 or more,** the small body can stably orbit the fourth or fifth Lagrange points.

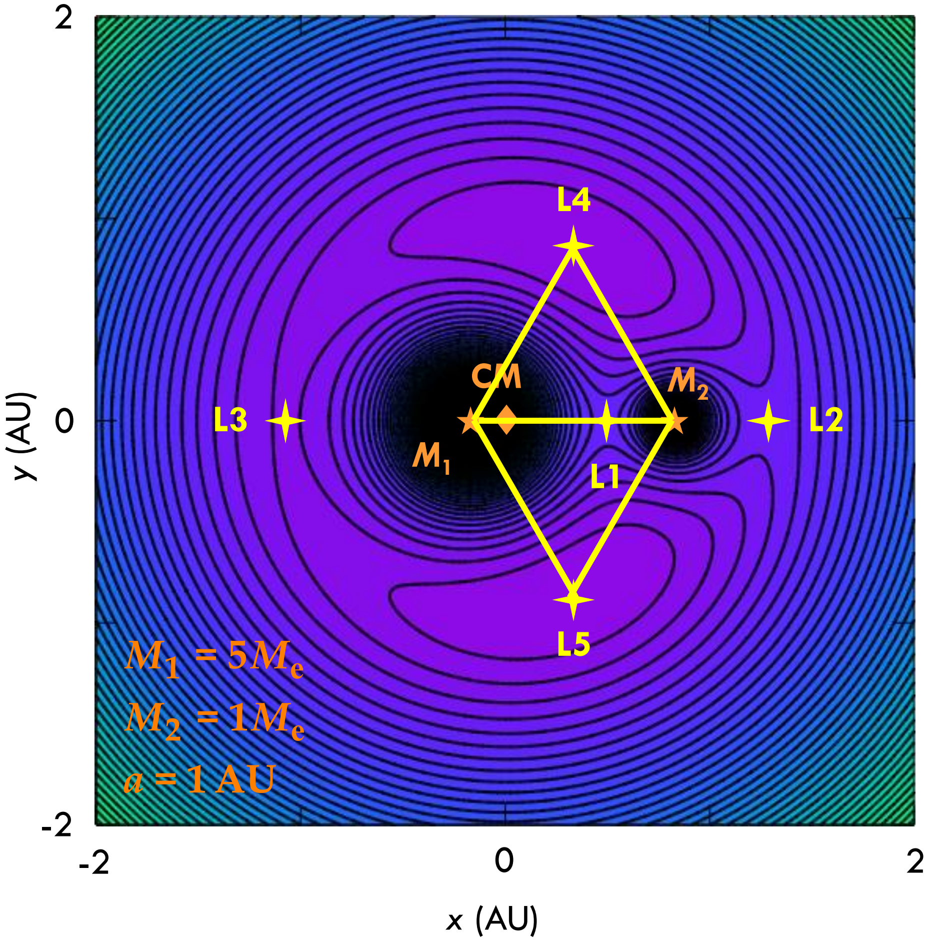
- It must *orbit* L4 or L5; it cannot sit stably exactly at L4 or L5. But stable, naturally-occurring collections of particles can orbit there.
- Such orbits are called **halo orbits**.
- **Trojans** are collections of asteroids trapped in halo orbits about the fourth and fifth Lagrange points in Jupiter's orbit.



L4 AND L5

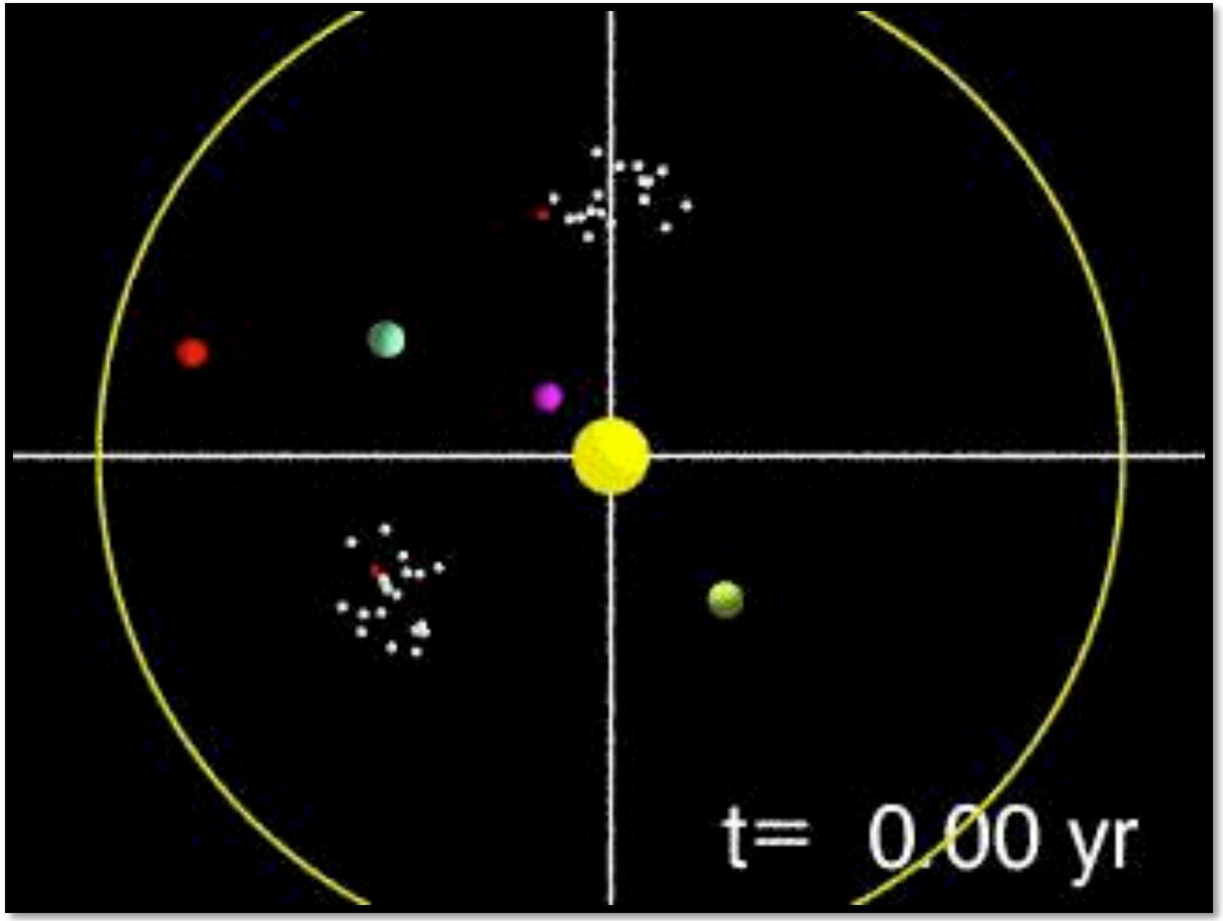
These points turn out to lie at the apices of the two equilateral triangles which can be drawn in the orbital plane using the line between the two massive objects as one side.

Since the Sun is more than 25 times as massive as any of the planets, stable halo orbits about L4 and L5 could accompany any planet's orbit. (At least the nearly circular ones.)



HALO ORBITS ABOUT L4 AND L5

Simulation of asteroids locked around L4 and L5 in a 1:1 mean-motion resonance with Earth (Paul Wiegert, U.W. Ontario, and colleagues)



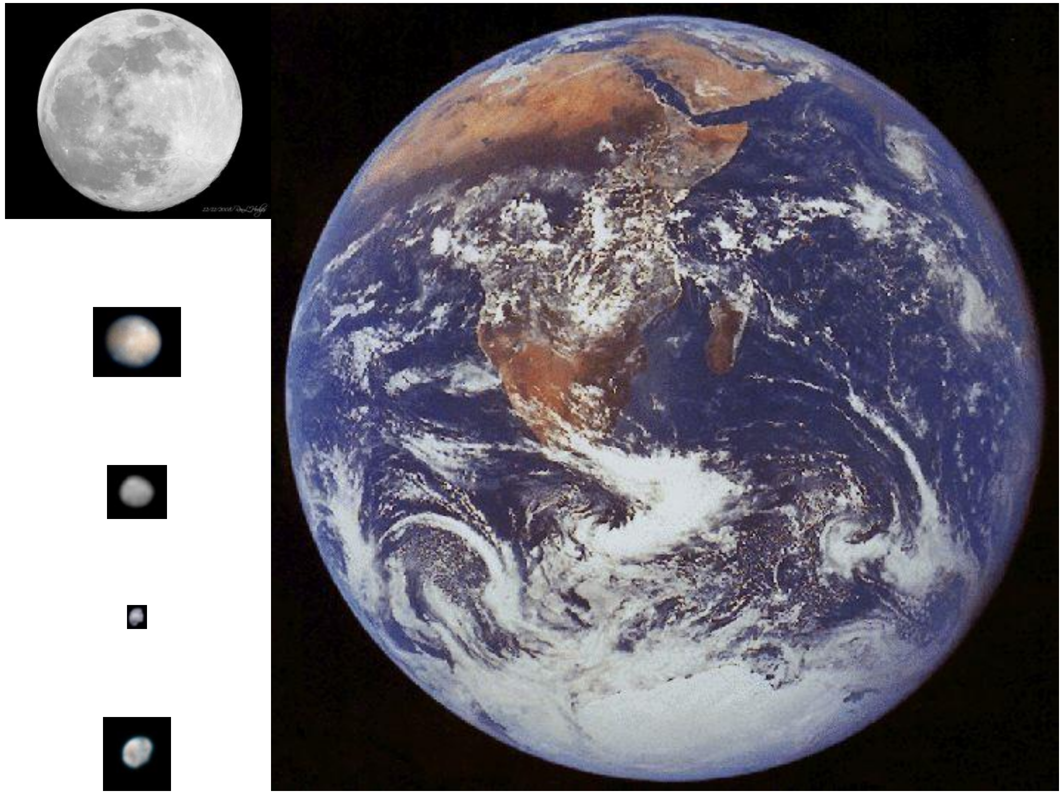
ASTEROID TAXONOMY

The composition of the surface of an asteroid can be determined by reflectance spectroscopy at ultraviolet, visible, and infrared wavelengths.

Broad classes (Bus & Binzel 2002):

- **C group** – carbonaceous, low albedo (< 0.1)
- **S group** – siliceous (stony), moderate albedo (0.1 – 0.25)
- **X group** – metallic, usually moderate to large albedo

And several "assorted" groups

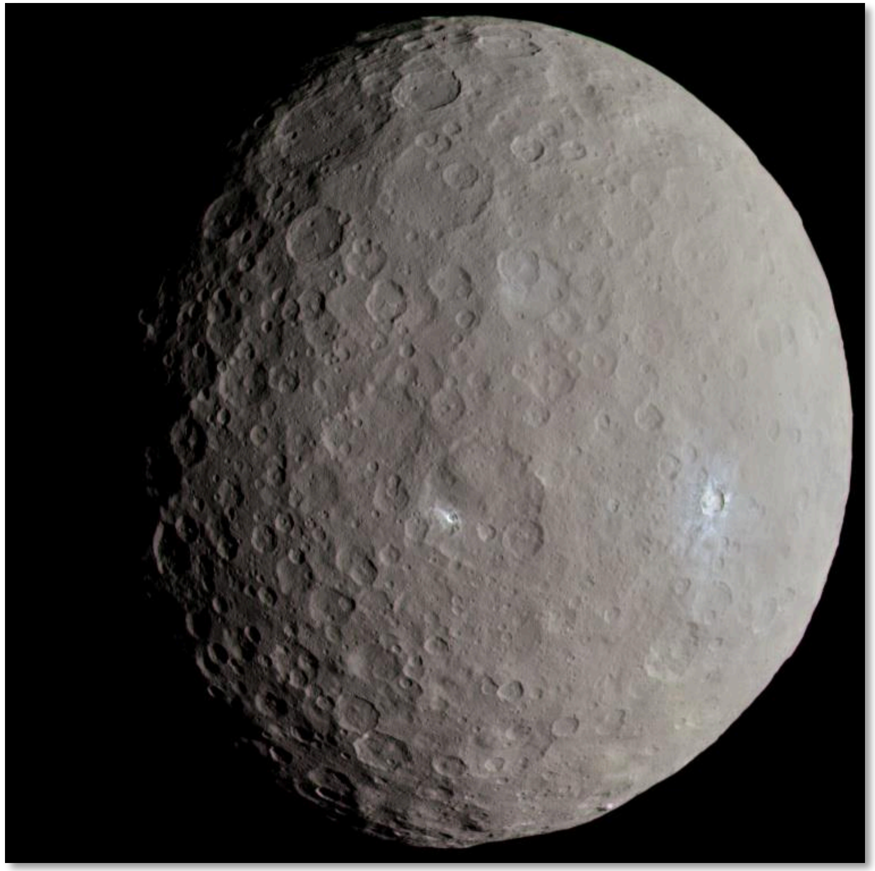


The first four asteroids discovered, shown on the same scale as Earth and the Moon (NASA). Together, they comprise 2/3 of the mass of the asteroid belt.

C-GROUP ASTEROIDS: C-TYPES

C-type asteroids are the largest population: at least 40% of all asteroids. They lie toward the outer part of the main belt.

- Dark, with albedo ~ 0.05; flat spectrum at red visible wavelengths
- Reflectance spectra generally similar to carbonaceous chondrite meteorites
- A few show additional absorption at UV wavelengths and are sometimes given the classification **G-type**.



1 Ceres, a C- (or G-) type asteroid (HST/STScI/NASA), the largest and third brightest of the asteroids

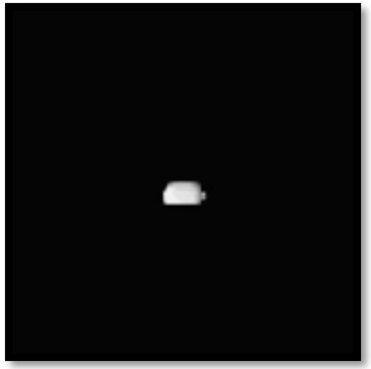
C-GROUP ASTEROIDS: D-TYPES AND B-TYPES

D-type asteroids currently appear to comprise about 5% of the total.

- Like Cs, they are concentrated in the outer main belt but are seen further out as well; e.g. among Jupiter's Trojan asteroids
- Ds are very dark – on average, even darker than Cs – and red, with featureless spectra: hard to identify their composition
- Distant + dark = hard to detect small ones. Thus, we may currently underestimate the size of this population.
- A meteoritic analog of the Ds has been found, with the result that they appear even more primitive than Cs.

B-type asteroids are much rarer, until recently counting only 2 Pallas as a member (then called "**U-type**").

- Though carbonaceous, Bs have higher albedo and bluer color than Cs and Ds.

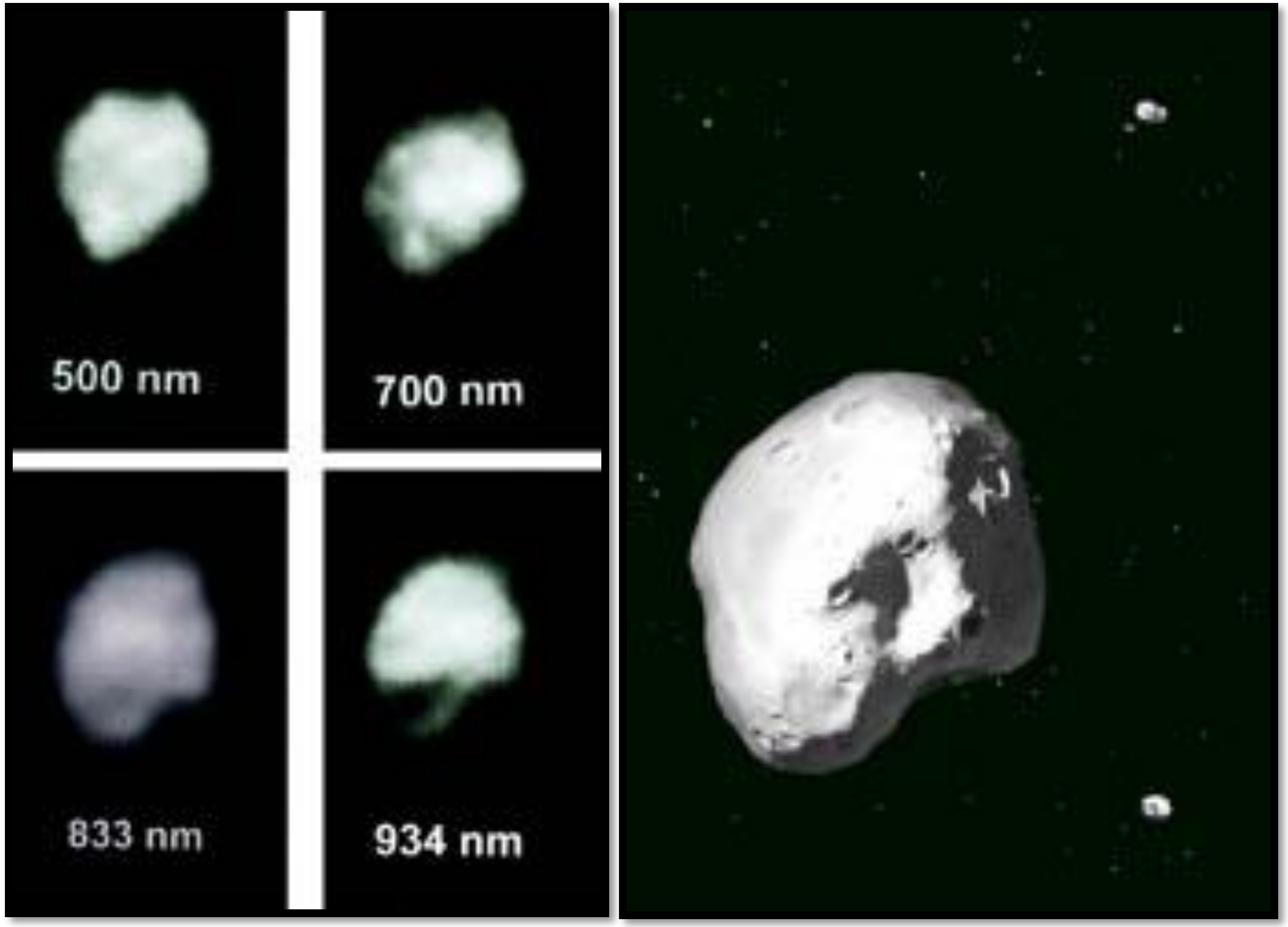


624 Hektor, perhaps the best-known D-type asteroid (HST image by Storrs et al 2005)

S-GROUP ASTEROIDS: S-TYPES

S-type (stony) asteroids are the second-most numerous type: about 30% of all asteroids.

- Concentrated toward the inner part of the main belt, with large albedos (~ 0.20).
  - We may be overestimating their fraction of the total, because they are brighter than the other types.
- Their reflection spectra in the infrared are similar to those from pyroxenes and olivines.
- The asteroid surfaces are either thermally processed and crystallized (like igneous rocks) or have been "space weathered" by impacts and UV.



Adaptive-optical images and artist's conception of 3 Juno, the second-largest S-type asteroid (CfA)

S-GROUP ASTEROIDS – OTHER TYPES

Other S-**group** asteroids are rare, but some are still notable. They differ from S-**type** by having much stronger mineral absorption features near 1 μm wavelength.

- **A-type**: olivine
- **Q-type**: pyroxene and olivine
- **R-type**: pyroxene, olivine, and plagioclase
- **V-type**: pyroxene; relative mineral abundances closely resemble those of basaltic lavas
  - Until recently, the only member of the V-type was its eponym, 4 Vesta, which was more conventionally accounted under the "**U-type**" (unclassifiable, or unique), even though 2 Pallas and 4 Vesta have little in common.
  - Now there are a few more, but all are tiny and are members of 4 Vesta's orbital family and are probably fragments of 4 Vesta shed during collisions.