Astronomy 111 — Practice Midterm #2

Prof. Douglass

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Name: ____

If this were a real exam, you would be reminded of the **Exam rules** here: "You may consult *only* one page of formulas and constants and a calculator while taking this test. You may *not* consult any books, digital resources, or each other. All of your work must be written on the attached pages, using the reverse sides if necessary. The final answers, and any formulas you use or derive, must be indicated clearly (answers must be circled or boxed). You will have one hour and fifteen minutes to complete the exam. Good luck!"

Your results will improve if you take this practice test under realistic test-like conditions: in one sitting, with your already-prepared cheat sheet at hand, and with the will to resist peeking at the solutions until you are finished. Also, as usual:

- First, work on the problems you find the easiest. Come back later to the more difficult or less familiar material. Do not get stuck.
- The amount of space left for each problem is not necessarily an indication of the amount of writing it takes to solve it.
- You must show your work to receive full credit.
- Numerical answers are incomplete without units and should not be written with more significant figures than they deserve.
- If you need a physical or astronomical constant that is not on your equation sheet, do not give up: estimate its value. If your estimate is reasonable, you will not lose any credit.
- Remember, you can earn partial credit for being on the right track. Be sure to show enough of your reasoning that we can figure out what you are thinking.

$$
\begin{aligned}
R_\odot &= 6.96 \times 10^{10}\ \text{cm} & R_\oplus &= 6.378 \times 10^{8}\ \text{cm} \\
M_\odot &= 1.989 \times 10^{33}\ \text{g} & M_\oplus &= 5.972 \times 10^{27}\ \text{g} \\
L_\odot &= 3.827 \times 10^{33}\ \text{erg/s} & G &= 6.674 \times 10^{-8}\ \text{dyn cm}^2\ \text{g}^{-2} \\
T_\odot &= 5772\ \text{K} & c &= 3 \times 10^{10}\ \text{cm/s} \\
1\ \text{AU} &= 1.496 \times 10^{13}\ \text{cm} & k &= 1.38 \times 10^{-16}\ \text{erg/K} \\
1\ \text{pc} &= 206{,}265\ \text{AU} & \sigma &= 5.6704 \times 10^{-5}\ \text{erg s}^{-1}\ \text{cm}^{-2}\ \text{K}^{-4}
\end{aligned}
$$

Planet	**Mass [g]**	**Radius [cm]**
Mercury	3.301×10^{26}	2.441×10^{8}
Venus	4.867×10^{27}	6.052×10^{8}
Mars	6.417×10^{26}	3.396×10^{8}
Jupiter	1.898×10^{30}	7.149×10^{9}
Saturn	5.683×10^{29}	6.027×10^{9}
Uranus	8.681×10^{28}	2.556×10^{9}
Neptune	1.024×10^{29}	2.476×10^{9}

1. Please write in complete sentences, and feel free to use equations and/or sketches to help explain your thoughts.
   (a) (5 points) Earth is not a blackbody; its UV-visible albedo is 0.37, and its infrared emissivity is 1.0. As a result, by what factor is its temperature different from that of a blackbody?

   (b) (5 points) Why do we know Europa has liquid-water oceans under its smooth ice surface?

(c) (5 points) A small main-belt asteroid has a mass $M_A = 10^{18}$ g and a radius $R_A = 6 \times 10^5$ cm. Calculate the asteroid's bulk density, describe the sorts of rocks and minerals that are likely to make it up, and describe its structure.

(d) (5 points) Besides mass and distance from the Sun, what is the biggest difference between Uranus and Neptune on one hand, and Jupiter and Saturn on the other?

2. A Hot Jupiter is a planet with a mass the same as Jupiter's ($M_J = 1.8986 \times 10^{30}$ g) and a radius 1.22 times that of Jupiter's ($R_J = 71,492$ km) that orbits extremely close to its star. Suppose one of these is in orbit around a Sun-like star.

   (a) (10 points) What is the minimum orbital radius of this Hot Jupiter? Give your answer in units of AU.

   (b) (5 points) What would happen to a typical Hot Jupiter if it were placed in an orbit smaller than this?

3. (10 points) Show that in a thin, uniform-temperature atmosphere on a terrestrial planet, the pressure depends upon the altitude z according to

$$P(z) = P_0 e^{-z/H} \tag{1}$$

where P_0 is the pressure at the surface ($z = 0$). Be sure to define the expression for the scale height H in terms of the planet's surface properties (temperature, gravitational acceleration, etc.).

4. *Limit of differentiation*

   (a) (10 points) Derive a formula for the temperature T as a function of radius r inside a uniform, spherical Solar System body which has a radius R, surface temperature T_s, density ρ, radioactive heating power per unit mass Λ, and thermal conductivity κ_T.

(b) (5 points) Suppose that $\rho = 3.35$ g/cm^3 and $\kappa_T = 4.79 \times 10^5$ erg s^{-1} g^{-1} K^{-1}, and that the radioactive heating rate is as it was when the Solar System was brand new, $\Lambda = 2.233 \times 10^{-3}$ erg s^{-1} g^{-1}. What is the maximum radius for a Solar System object with a core temperature that never exceeds that on the surface by more than 2000 K so that it does not differentiate?

5. (10 points) A cylindrical "planetoid" with radius R and length H rotates rapidly with its axis perpendicular to the Solar System and revolves at an orbital radius r. Its thermal conductivity is very poor, so its surface temperature is determined just by Solar heating and blackbody radiation. Take its albedo to be A and its emissivity to be ε.

   Derive a formula for the temperature over the planetoid's entire surface.

6. **Late giant-planet formation and the ultimate fate of Earth.** Someday, a couple billion years from now, the Sun will become a red giant star, and stay that way for about 2 Myr before shedding its outer layers and giving rise to a white dwarf from the inner ones.

   Suppose, plausibly, that when this happens, half of the Sun's mass expands into a uniform-density, pressure-supported, nonrotating sphere mostly made of atomic hydrogen gas that is just barely large enough to engulf the Earth in its orbit at $r = 1$ AU. The rest of the Sun's mass stays at the center, in a much smaller core.

   (a) (5 points) What is the density of the gas in which the Earth is engulfed?

   (b) (10 points) What is our Earth's Keplerian orbital speed v, what is its orbital angular momentum L, and what headwind speed does our Earth, with mass $M_{\oplus}$, face in the Sun's red-giant atmosphere?

(c) (10 points) Show that the path of our Earth's Hill sphere through the Solar atmosphere has volume $V = 2\pi^2 r^3 \left(\frac{M_\oplus}{3M_\odot}\right)^{2/3}$, and calculate the mass M that Earth would have if it were suddenly to accrete all this mass. Express your answer in Jupiter masses.

(d) (10 points) Does the Earth stay in orbit, or does it drift into the Sun's core? Calculate the rate at which the headwind robs the planet of angular momentum as the planet accretes the gas that falls within its Hill sphere, and estimate the time it would take for the headwind to drag Earth into the center of the red-giant Sun.