Astronomy 111 — Practice Final Exam

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Name: ______________________________

If this were a real exam, you would be reminded of the **Exam rules** here: "You may consult *only* one page of formulas and constants and a calculator while taking this test. You may *not* consult any books, digital resources, or each other. All of your work must be written on the attached pages, using the reverse sides if necessary. The final answers, and any formulas you use or derive, must be indicated clearly (answers must be circled or boxed). You will have two hours and thirty minutes to complete the exam. Good luck!"

Your results will improve if you take this practice test under realistic test-like conditions: in one sitting, with your already-prepared cheat sheet at hand, and with the will to resist peeking at the solutions until you are finished. Also, as usual:

- First, work on the problems you find the easiest. Come back later to the more difficult or less familiar material. Do not get stuck.
- The amount of space left for each problem is not necessarily an indication of the amount of writing it takes to solve it.
- You must show your work or explain your answer to receive full credit.
- Numerical answers are incomplete without units and should not be written with more significant figures than they deserve.
- Remember, you can earn partial credit for being on the right track. Be sure to show enough of your reasoning that we can figure out what you are thinking.

$R_\odot = 6.96 \times 10^{10}$ cm	$R_\oplus = 6.378 \times 10^{8}$ cm
$M_\odot = 1.989 \times 10^{33}$ g	$M_\oplus = 5.972 \times 10^{27}$ g
$L_\odot = 3.827 \times 10^{33}$ erg/s	$G = 6.674 \times 10^{-8}$ dyn cm^2 g^{-2}
$T_\odot = 5772$ K	$c = 3 \times 10^{10}$ cm/s
1 AU $= 1.496 \times 10^{13}$ cm	$k = 1.38 \times 10^{-16}$ erg/K
1 pc $= 206,265$ AU	$\sigma = 5.6704 \times 10^{-5}$ erg s^{-1} cm^{-2} K^{-4}

1. Please write in complete sentences, and feel free to use equations and/or sketches to help explain your thoughts.

   (a) (5 points) On Venus, the dominant high-altitude wind systems blow straight from the equatorial regions to the polar regions, and the low-altitude winds blow in the opposite direction. Briefly explain why this is.

   (b) (5 points) Within ten minutes or so, what will be the sidereal time at midnight tonight?

(c) (5 points) Suppose that Saturn's rings were made only of very small particles and that Saturn had no moons at all. Describe the differences this would make in the appearance of the rings.

(d) (5 points) Rank these bodies' surfaces in order of age, and briefly explain your answer: the Moon, Io, and Enceladus.

2. *The habitable zone*

   (a) (10 points) Choose a suitable Bond albedo and emissivity for a habitable planet and calculate the radii (in AU) of the inner and outer edge of the habitable zone for rapid rotators around a star with luminosity $L = 1L_{\odot}$.

   (b) (5 points) According to your results from part a, what is the maximum orbital eccentricity for a planet which always lies in the habitable zone?

(c) **(5 points extra credit)** Show that this maximum orbital eccentricity for the habitable zone depends only upon the temperatures at the inner and outer boundary of the habitable zone, not upon whether the body is uniform in temperature, slowly rotating, or rapidly rotating.

3. A meteorite from Antarctica is analyzed for the relative abundances of ^{87}Rb and ^{87}Sr, with results for a couple of different silicate minerals within the meteorite as follows:

Mineral	^{87}Rb/^{86}Sr	^{87}Sr/^{86}Sr
A	0.03653	0.70113
B	0.14560	0.70799

The decay rate of ^{87}Rb is $\lambda = 1.39 \times 10^{-11}$ yr^{-1}.

(a) (5 points) How old is the meteorite (in years)?

(b) (5 points) Is this meteorite typical? Briefly discuss its origin.

4. Consider a planet with radius R in which the density decreases linearly from the center to the edge:

$$\rho(r) = \rho_0 \left(1 - \frac{r}{R}\right)$$

(a) (15 points) Show that its total mass is given by

$$M = \frac{\pi \rho_0 R^3}{3}$$

(b) (15 points) Show that its moment of inertia is given by

$$I = \frac{4}{15} M R^2$$

(c) (15 points) Suppose that the planet has a surface temperature T_s, and that its contents have radioactive heating power per unit mass Λ and thermal conductivity κ_T. Derive a formula for the temperature as a function of radius for the interior of the planet.

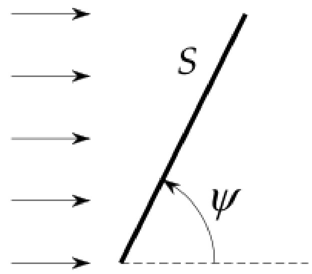
5. Over the course of many years, Jupiter (period $P_J = 11.856523$ yrs) perturbs the orbit of a main-belt asteroid that was originally in a 2:1 mean-motion resonance with the giant planet. Miraculously, it avoids collisions as its orbital eccentricity grows, until its aphelion is on its original circular orbit and its perihelion is on Earth's orbit.

   (a) (10 points) What is the orbit's eccentricity?

(b) (10 points) The asteroid has mass 10^{15} g. How much momentum (in g cm s^{-1}) did Jupiter transfer to (or from) the asteroid to change it to its new orbit?

(c) (10 points) How fast (in km/s) would the asteroid and Earth be going with respect to each other if they collide?

6. A wafer-like dust grain lies a distance r from a star with luminosity L. Its two flat surfaces have area S, and each surface has the same Bond albedo A_b and emissivity ε. The plane of one surface lies at an angle Ψ with respect to the incident starlight. It is thin enough and high enough in conductivity to be uniform in temperature.



(a) (10 points) Derive a formula for its temperature.

(b) (5 points) At what angle does the wafer have to be tilted for its temperature to be half of its value for normally-incident starlight ($\Psi = 90^\circ$)?

7. Venus and Mars both have atmospheres made mostly of CO_2, which has molecular weight $\mu = 3.71 \times 10^{-23}$ g and adiabatic index $\gamma = \frac{9}{7}$. Other properties of these two planets include:

	Venus	Mars
Mass [g]	4.87×10^{27}	6.42×10^{26}
Radius [cm]	6.05×10^{8}	3.40×10^{8}
Surface temperature [K]	373	210

(a) (5 points) What is the gravitational acceleration (in cm/s^2) at the surface of each of these two planets?

(b) (5 points) If the atmospheres were isothermal, what would be the pressure scale height (in cm) near the surface of each planet?

(c) (10 points) If the atmospheres were adiabatic, how much cooler would it be at one isothermal pressure scale-height above the surface? (Give your answer in K.)

8. In a young, sedimented protoplanetary disk, a KBO-size object ($R_0 = 100$ km $= 10^7$ cm) has just formed in a region where the disk's gas density is $\rho_g = 8.7 \times 10^{-12}$ g/cm^3, the dust density is $\rho_d = 7.0 \times 10^{-11}$ g/cm^3, and the headwind speed is $v_{HW} = 1.0 \times 10^4$ cm/s. (These are the values you would get in our usual model of the Solar nebula at a radius of 1 AU.)

   (a) (10 points) What is the minimum time (in millions of years) for this planetesimal to grow to Earth size, $R(t) = R_\oplus$, by dust accretion?

   (b) (5 points) What is the minimum time (in millions of years) for the resulting Earth-size planetesimal to grow to gas-giant planetary size by accretion of gas? (Assume that the disk's dust is all used up in the first phase.)