Revolution of the moons of Jupiter and Saturn, Kepler's third law, and Newton's law of gravity

Fall 2021

1 Introduction

This is an observing experiment in which we will determine the mass of two of the Solar System's giant planets by measuring the orbital properties of their moons and analyzing their motions.

This observing project will involve frequent visits to the telescope over the course of 4–6 weeks, including at least one sequence of four consecutive nights. Everyone in the class will go to the telescope — in groups of two or three — with the object of sharing all the data. Each student will perform their own analysis and write an independent report on the results.

Around the first week in October, we will derive Kepler's third law,

$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)}(a_1 + a_2)^3 \qquad (1)$$

from Newton's laws of motion and law of gravity. Here, P is the period of revolution of two bodies about each other, m_1 and m_2 are their masses, a_1 and a_2 are the semimajor axes of their elliptical orbits as measured from the center of mass of the two-body system, and $m_1/m_2 = a_2/a_1$. If one of the masses, say m_1, is much larger than the other, then the more massive member can be taken to be fixed (because its semimajor axis is small), and the smaller one can be considered to do all the orbiting. In this case, Kepler's third law simplifies to

$$P^2 = \frac{4\pi^2}{GM}a^3 \qquad (2)$$

This approximation applies to the period and orbital semimajor axis of each planet in the Solar System, taking M to be the mass of the Sun. It also applies to the moons of Jupiter and Saturn, all in orbit around a much heavier planet. In one of the first notable uses of a telescope, Galileo showed that Jupiter and its satellites look like a miniature solar system and thus provided substantial evidence that Copernicus's heliocentric model of the Solar System was viable.

In this experiment, we will monitor the motions of the satellites of Jupiter and Saturn. From the data gathered, we will be able to measure the orbital period and semimajor axis of each of these satellites. The experiments consist of a series of observations with a CCD camera on the Mees 24-inch telescope; we will take images in which we can identify the moons and determine the distance between each moon and its host planet as a function of time.

The moons of the giant planets all travel in orbits that are very nearly circular. The orbits are also very nearly coplanar; the moons appear to be lined up because we are looking almost edge-on to this plane. We can only see the projection of each moon's orbital radius in the plane perpendicular to the line of sight between the Earth and each planet, as in Figure 1. This apparent distance is $R_{\text{apparent}} = a \sin\phi$, where a is the distance from the moon to the center of its host planet and ϕ is the "azimuthal" angle of the moon's radius with respect to the line of sight toward the planet. For each moon, the determination of the maximum value R_{apparent} comprises the measurement of its orbital semimajor axis, and determination of the time between successive maxima comprises the measurement of its orbital period. But ϕ increases proportionally with time; plotting R_{apparent} as a

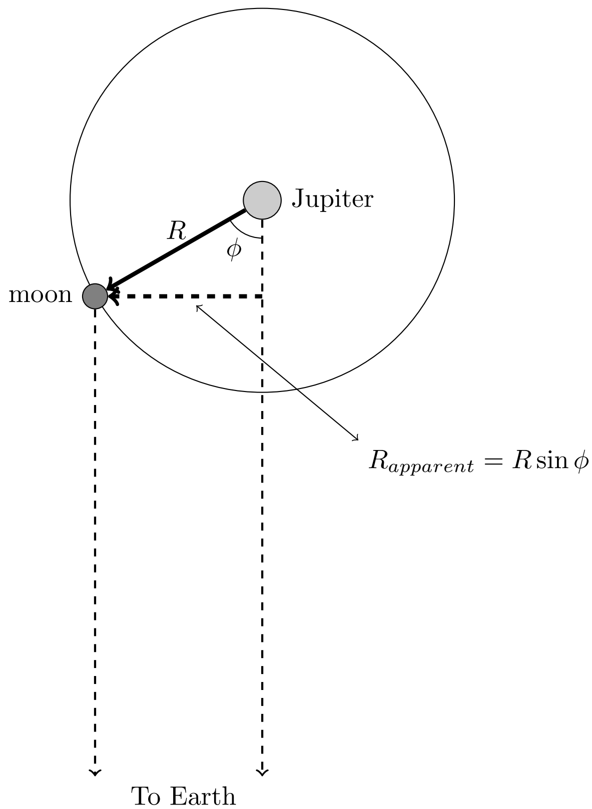

Figure 1: Geometry of observations

function of time will yield a sine curve of amplitude a and period P, the period of the moon's orbit. Once we know a and P in any units, we can validate Kepler's third law, Equation 2.

Note that this works even if there are time gaps in the data — as those that usually arise between sunrise and sunset or due to cloudy nights — as long as a sine function can be fit to the data.

2 Procedure

2.1 At the telescope

You will be personally instructed by the instructors in the start-up and shut-down procedures for the telescope and CCD camera, over the course of which you will become familiar with the checklists we maintain for these tasks. You will also learn how to use *TheSkyX*, a planetarium program that we use to control the telescope and camera at Mees. Here, we will discuss just the observing procedures you will follow *after* everything is powered up and running.

Note, throughout, that by "take an image" is meant: tell *TheSkyX* to take the image, save this image as a file with a name that helps you remember what it is (e.g. "Jupiter_moons_L_0p1s_1.fits"), and record in your observing notes a description of the image and the local date and time on which it was taken (e.g. "Jupiter_moons_L_0p1s_1.fits: Jupiter and moons in the L filter, 0.1 sec, optimized for the moons, September 5, 2021, 22:45 EDT").

1. Begin by pointing the telescope at Saturn, which is the earliest riser of our two planetary targets these days. Start by moving the diagonal mirror on the bottom of the telescope to the position that permits the eyepiece to see. Then use *TheSkyX* to move the telescope to Saturn. Bring the planet into the center of the field using the telescope paddle, and adjust the focus knobs on the eyepiece to produce a sharp image. You will see Saturn, its rights, and some of its brightest moons. Make sure everyone gets a chance to view Saturn and the moons through the main telescope and eyepiece.

   Next, return the diagonal mirror to the CCD camera's. Take an image of Saturn with the CCD camera using *TheSkyX*. Center Saturn in the CCD's field using the CenterMe.xlsx file, and Sync *TheSkyX* to Saturn.

2. Note that the plate scale is about 0.224 arcsec per CCD pixel at Mees. If the system is well focused, the apparent angular diameters of the much smaller moons are dominated by blurring from turbulent variations of the refractive index of the atmosphere, an effect often referred to as *seeing*. (That is, the moons are smaller than the image-blur from the atmosphere, and Saturn is larger in size to this blur.)

   Choose two integration times: one such that the peak signal of Saturn is as low as possible, and one that brings out the moons well enough to see their locations without overexposing Saturn. Take one image of the Saturn-moon system with each integration time. Keep track of all integration times used.

3. Once Jupiter is high enough in the sky to permit observation, point the telescope at it and repeat the procedure in step 2. Choose at least two exposure times, one of which shows the moons well but which may overexpose the planet, and one which shows the planet well. Identify the moons in your image using *TheSkyX*.

4. Repeat steps 2 and 3 about once per half-hour (for the Jovian system) or once per hour (for the Saturnalian system) until the planets set. The shorter cadence for the Jovian system is to capture the motion of those moons that are moving rapidly across the sky, which they will when they get close to their planets. Make sure you continue to note in your lab book the conditions (exposure time, object, filter, etc.) under which each image was taken and which of the targets is in each image.

5. When you have completed your work, disconnect the camera and shut down the telescope following the checklist. Copy *all* of your data files to a USB drive.

6. **For inclusion in your lab report,** you should note the conditions of observing: the weather, the operation of the telescope, and the observing procedure. Include telescope pointing and focus-position information. Describe the objects you observed and what you saw.

2.2 Data reduction

The main results we are after are the distances between the center of each planet and the center of each moon as a function of time, measured in units of the planet's radius. The planets' diameters are significantly affected by seeing, and the orbit of their satellites are not viewed exactly edge-on. To simplify this matter, we will simply record from the images how many pixels east and north of the center of the planet each moon is. You have recorded the time for each image, so all you have to do now is measure the distances.

1. Import each FITs file into SAOImage DS9 (free to download).
2. Click on the Scale button, and find the appropriate scale for which you can clearly see the planet and its moon(s). Feel free to play with the different Color options as well.
3. *Measuring the radius.* Click on the Edit button. Select Region, and click on the image at the center of the planet. A circle should appear on your image. Double-click on the circle to open its attributes window. Adjust the radius of the circle to gain the best fit for the size of the planet. The units of the radius can be seen in the drop-down menu to the right of the Radius box. Record both the circle's radius and center coordinates — these are your planet's radius and center coordinates.
4. *Measuring the distance.* Place your mouse on the center of the planet. Record the pixel coordinates shown in the boxes at the top of the screen. Repeat for each moon. For each moon, calculate the distance between its center coordinates and the planet's center coordinates, and convert to whatever units your planet's radius is in. Divide these distances by the average of all the measured planet's radii to get the position in units of planet radii.

As a summary of your results, prepare a table of UT and the apparent positions of each moon at that UT. Estimate the uncertainty in each of your measurements, and include those estimates in your table as well. Note that you will need to record whether the moon is located to the left (negative) or right (positive) of the planet, as well as its apparent distance from the planet's center in planet-radius units. Add your measurements to the class's Google spreadsheet.

3 Data analysis

3.1 Orbital radii and periods for the satellites

1. Once all of the observations are done for the season, collect all their data from the class's Google spreadsheet and make a table of UT (including day as well as time) and apparent distance from the planet's center (in planet radii) for each moon. For convenience in plotting, express the JD-UT combination in decimal form in units of days from the date of the first observation. For example: as I type these words it is September 4, 2021, 5:18 PM EDT. If tonight is the first night of observing, then midnight (00:00 AM EDT) last night is when $t = 0$, and the UT for this time (five hours later than EDT) is 10:18 PM = 22:18 = 0.929 days, so I would plot this date and time as 0.929.
2. Then, for each moon, plot their apparent distances as a function of time. From each plot, determine the amplitude (maximum apparent distance) in planetary radii and period (time between consecutive maxima) in days.
   (a) In the Projects section of the ASTR 111 class website, you can find an Excel workbook called moon.xlsx, which is designed to help you fit sine curves through the data. Download this file, and save a copy of it for each moon-planet combination.

(b) Then put the data — UT in days and apparent distance in planetary radii, in columns, in that order — into the columns on the first sheet of the workbook. On the second sheet, you will see these results plotted. You will also find places to enter three choices for amplitude, period, and phase difference (offset of the first planet-crossing from the zero of time, in days) for each direction on this sheet, that will result in three different-color sine curves being drawn on top of your data.

(c) Adjust the amplitude, period, and phase difference until you get a curve that is a good fit to your data, and save that version of each workbook; the resulting amplitude and period are the orbital radius in planetary radii, and the orbital period in days, for that particular moon.

3. Tabulate your results: name, orbital radius a (in planet radii), orbital period P (in decimal days) for each moon, and an estimate for the uncertainties in these quantities.

3.2 Kepler's third law, Newton's law of gravity, and the masses of the giant planets

1. From your table, calculate a^3 and P^2 for the Jovian and Saturnalian systems, and plot a^3 as a function of P^2 for each of these planets. Label the axes appropriately. What is the shape of the curve that results? Does the precise distance to the planetary systems matter — that is, do the values of the as in centimeters, instead of planet radii, matter?

2. In a paragraph or two, discuss these results in light of Kepler's third law (Equation 2).

3. Kepler's laws are consequences of Newton's law of gravity. In a paragraph or two, discuss the degree to which your results support Newton's law of gravity.

4. Of course, we do know how far away the planets are and can measure their radii from their angular radii: the results are $r = 7.1492$ and 6.0268×10^9 cm (Jupiter and Saturn) Using these values and Equation 2, convert each a into centimeters, each P into seconds, and deduce the mass M of each planet in grams. Use average and the spread of the four values you have that result to arrive at a "best" value and an estimate of the uncertainty in your determination of Jupiter's and Saturn's masses.