1. The Planck formula for the intensity (energy per unit time, per unit area, per unit wavelength, per unit solid angle) of a blackbody is

$$B_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

Approximations to the Planck formula for very long or very short wavelengths often turn out to be useful. Derive those expressions here as an exercise in making approximations.

   (a) Carefully consider the Planck formula. To what quantity should we compare the wavelength to determine whether or not the wavelength is large or small?

   (b) To what form does the Planck function reduce for very long wavelengths? This is called the **Rayleigh-Jeans** limit of the blackbody function.

   (c) To what form does the Planck function reduce for very short wavelengths? This is called the **Wien** limit of the blackbody function.

2. The absolute bolometric magnitude of the Sun is $M_\odot = 4.75$, and the luminosity of the Sun is $L_\odot = 3.827 \times 10^{33}$ erg/s. Show that the apparent bolometric magnitude m and absolute bolometric magnitude M of a star with luminosity L that lies a distance r away from the Solar System are related by

$$M = m - 5\log\left(\frac{r}{10\text{ pc}}\right) = 4.75 - 2.5\log\left(\frac{L}{L_\odot}\right)$$

The difference between the apparent and absolute magnitudes, $m - M = 5\log(r/10\text{ pc})$, is called the *distance modulus.*

3. A variable star changes in brightness by a factor of 4. What is the change in magnitude?

4. Suppose there is a type of star we can identify independent of distance, and we know that all examples of the star have the same absolute magnitude M. Then a measurement of the apparent magnitude m of one such star can be used to infer its distance. Suppose further that the apparent magnitude can be determined to an accuracy of ± 0.01 magnitudes. What is the accuracy to which distances to the stars can be determined?

(Hint: Recall that for any function $f(x)$, a *small* interval Δx in the independent variable corresponds to an interval

$$\Delta f = \frac{df}{dx}\Delta x$$

in the value of the function.)

5. (a) Show that, for a *blackbody* which occupies a small solid angle $\Delta\Omega$, at low temperatures the relation between temperature and $B - V$ color is something like

$$T \approx \frac{7400\text{ K}}{(B - V) + 1.4}$$

   (Hint: Use the Wien approximation and assume the properties of the B and V filters given in Lecture 3.)

   (b) This relation is a very poor approximation at high temperatures. Why?