1. Suppose we somehow know that the mass density with a star of mass M and radius R decreases linearly from the center to the surface of the star and vanishes at the surface, i.e.,

$$\rho(r) = \rho_c \left(1 - \frac{r}{R}\right)$$

   (a) What is the density ρ_c at the center of the star in terms of M and R?

   (b) Show that the pressure at the center of the star is

$$P_c = \frac{5}{4\pi} \frac{GM^2}{R^4}$$

2. Suppose that the mass density of a star of radius R increases quadratically with distance from the center,

$$\rho(r) = \rho_c \left[1 - \left(\frac{r}{R}\right)^2\right]$$

   (a) Find the mass M of the star in terms of ρ_c and R.

   (b) Find the average density of the star in terms of ρ_c.

   (c) Show that the central pressure of the star is

$$P_c = \frac{15}{16\pi} \frac{GM^2}{R^4}$$

3. Can gravitational energy alone supply a star's luminosity?

   (a) A star with mass M and radius R has a uniform density ρ. What is its gravitational potential energy in terms of M and R?

   (b) The star shrinks in radius at a small but constant rate $dR/dt = -v$ where v is a positive number with units of velocity. At what rate dU/dt does the star's gravitational potential energy change?

   (c) Normally, the change in potential energy would be accompanied by a change in thermal energy. But suppose that the star stays at constant temperature (constant thermal energy) during this collapse and radiates the energy away that would normally add to its thermal energy. Suppose also that this radiation is the only radiation emitted by the star. What is the star's luminosity?

   (d) Suppose the Sun derived its luminosity in this fashion. At what speed would it need to shrink to produce the presently observed luminosity? How long in years would it continue to shine? How long in years would it take for the luminosity to double? Do you think this process can be ruled out as the source of the Sun's power?

4. At the center of the Sun, the mass density is $\rho_c = 1.52 \times 10^5$ kg/m^3 and the mean opacity is $\kappa = 0.12$ m^2/kg. (The opacity is a measure of how opaque a material is — it is often a function of temperature, density, and chemical composition.) What is the mean free path for a photon at the Sun's center?

