1. Consider a star of radius R where the temperature T and mean particle mass μ are uniform inside, except for a tiny layer on the surface in which the temperature drops from T to a much lower value.
   (a) Derive an expression for the period of the fundamental radial oscillation of this star.
   (b) Suppose a star like this were observed to have a radius $R = 1.5R_\odot$ and we could tell from its spectrum that it has the same mean particle mass and specific-heat ratio as the Sun. Suppose furthermore it oscillates with a period of 2 hours. What is its interior temperature T?

2. Clarinets can play more than one note, even without pushing on the keys. Presumably stars can too.
   (a) What is the next-lowest pitch (frequency) that can be played by the clarinet considered in the class notes? This mode is called the **first overtone**; the lowest-frequency mode is called the **fundamental**. Give your answer in Hz and/or in musical notation.
   (b) What is the next-shortest period of oscillation (i.e., the first overtone) of the uniform-density star considered in class?
   (c) The brightest classical Cepheid variable in the sky is Polaris, the North Star (α Ursae Minoris). Its pulsation period is 3.97 days and its amplitude is 0.03 mag. Its spectrum and color show that it has an effective temperature of 7200 K. It has a couple of companion stars from which its mass can be determined to be $4.3M_\odot$ and its distance has been measured with trigonometric parallax, yielding from its flux a luminosity of $2200L_\odot$. Estimate the periods of Polaris' fundamental and first-overtone pulsations. In which mode is Polaris likely to be pulsating?

3. **Brown Dwarfs**: Consider a star of such very low mass as to be only marginally capable of thermonuclear heat production. Under the assumptions that the star is all hydrogen ($Z = A = 1$), that gravity is balanced by *nonrelativistic electron degeneracy pressure*, and that protons, at the same temperature and pressure as the electrons, *act as an ideal gas*, derive the equation relating the star's central temperature T_c to its total mass M. If $T_c \geq 3 \times 10^6$ K is required to sustain the pp chain fusion reactions, what is the minimum mass of a luminous star? Express your answer in solar masses ($M_\odot$) and compare it to the mass of Jupiter ($1\ M_{\rm Jup} = 2 \times 10^{30}$ g).

   "Stars" with mass less than this minimum never undergo hydrogen fusion energy generation. These are the brown dwarfs.

4. Begin with the relativistic form of electron degeneracy pressure and the expressions for central pressure from weight and central density in a relativistic-degenerate equation of state:

$$P_e = 0.123\ hcn_e^{4/3} \qquad P_c = 11\frac{GM^2}{R^4} \qquad \rho_c = 12.9\frac{M}{R^3}$$

   Substitute $n_e = Z\rho/Am_p$ and manipulate to obtain both an expression for the electron degeneracy pressure in a star made of material with nuclear charge Z and mass number A, and an expression for the (Stoner-Andersen-Chandrasekhar) maximum mass of such a star. You will thus fill in the steps left out in arriving at the results in the lecture on white dwarfs.

   Calculate the maximum mass of a carbon white dwarf, expressing your answer in solar masses.

5. Two oboe players can hear each other; one hovers just outside a black hole's horizon at $r = 1.01R_{\rm sch}$ and the other is at rest far away from the black hole. (This is the George Lucas version of outer space.) Each plays an $A4$ note ($f = 440$ Hz). What is the frequency of the note each hears played by the other?

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