1. The CNO Bi-Cycle: The complete CNO cycle of nuclear reactions is

- (a) Fill in the blanks in the table of reactions with the missing names of the reacting particles.
- (b) One part of the bi-cycle is indicated in the reaction table. Find another catalytic cycle among the reactions and label it like the first part.
- (c) What are the *overall* reactions associated with each of the two catalytic cycles? How does the energy released in each overall reaction compare with that released in the pp chain?

**Solution:** The solution is given below. The new cycle, which we may call the "NO cycle," uses  $_{7}^{14}$ N as a catalyst. Both cycles add up to the same overall reaction:

$$4 \begin{pmatrix} 1\\ 1 \end{pmatrix} \rightarrow {}^{4}_{2} \text{He} + 2 \begin{pmatrix} 0\\ 1 \end{pmatrix} + 2 \begin{pmatrix} 0\\ 0 \end{pmatrix} + 3 \begin{pmatrix} 0\\ 0 \end{pmatrix}$$

Apart from the number of (massless) photons this is also the same as the pp chain, and hence involves the same energy release.

- 2. Suppose we have Z protons and we have to distribute them into two nuclei, one with  $Z_1$  protons and the other with  $Z_2$  protons, such that  $Z = Z_1 + Z_2$ .
  - (a) What arrangements give the maximum and minimum Coulomb repulsion between the two nuclei?

**Solution:** For a given distance between the nuclei the repulsive force is proportional to the product of the two nuclear charges  $Z_1Z_2q_p^2 = Z_1(Z - Z_1)q_p^2$ , so the extrema in the force correspond to the extrema of  $Z_1Z_2 = Z_1(Z - Z_1)$ . If we pretend the atomic number is a continuous variable then this function of  $Z_1$  is a parabola that opens downward. We can then determine the extremum in the usual way:

$$\frac{d}{dZ_1} \left( Z_1(Z - Z_1) \right) = Z - 2Z_1 = 0 \text{ at the extremal values of } Z_1;$$
$$Z_{1,\text{ext}} = \frac{Z}{2}$$
But 
$$\frac{d^2}{dZ_1^2} \left( Z_1(Z - Z_1) \right) = -2 < 0$$

which is the condition for a maximum. Thus the maximum involves equally charged nuclei with  $Z_1 = Z_2 = Z/2$  for  $Z_1Z_2 = Z^2/4$ . The function  $Z_1Z_2$  decreases as  $Z_1$  changes in either direction from Z/2. Formally, there is no mimimum except for the trivial case where  $Z_1 = 0$ . The smallest the product gets is that produced by  $Z_1 = 1$ ,  $Z_2 = Z - 1 = Z_1Z_2$ . The product for Z = 24 is plotted below.



(b) What does this tell you about the types of fusion most likely to take place in stars?

**Solution:** It suggests that fusion reactions involving heavy nuclei and light nuclei (or single protons) are faster than those involving two heavy nuclei, and thus that heavy elements tend to be produced by building up one baryon at a time rather than via direct fusion of two multi-baryon nuclei.