1. Two oboe players can hear each other; one hovers just outside a black hole's horizon at $r = 1.01R_{\text{sch}}$ and the other is at rest far away from the black hole. (This is the George Lucas version of outer space.) Each plays an $A4$ note ($f = 440$ Hz). What is the frequency of the note each hears played by the other?

2. **Stable black hole orbits**:
   (a) Derive an expression for the orbital frequency of the innermost stable circular orbit (ISCO) around a black hole f_{ISCO} in terms of the black hole's mass.
   (b) The black hole in GRO J1655-40 is observed to have a mass of $M = 6.0M_\odot$ and emits X-rays which exhibit quasiperiodic oscillations at a frequency of 450 Hz. It is thought that this frequency indicates material in orbit. If the black hole were not spinning, how far (radially, in cm) from the ISCO would this orbit be, in the view of a distant observer?
   (c) If the black hole were not spinning, how far (radially, in cm) would this orbit be from the ISCO in the view of an observer in the vicinity of these orbits? Presume the local observer to be capable of measuring the distance instantaneously.

3. The pressure and temperature at the center of the Sun are $P_c = 2.1 \times 10^{17}$ dyne cm^{-2} and $T_c = 15.7 \times 10^6$ K. What are the central pressure and temperature of a $1.5M_\odot$ star with the same composition as the Sun?

   (*Hint*: Empirical measurements of eclipsing binaries show that stellar mass and radius are close to linearly proportional with coefficient 1 in solar units. Use this to infer the missing piece of information needed to solve this problem.)

4. **A helium star**: Consider stars with composition of a type called extreme "Population II": objects made only of hydrogen (fraction X by mass) and helium (fraction $1 - X$ by mass), with negligible quantities of everything else.
   (a) Starting with the definition $\mu = \frac{\rho}{n}$ for the average particle mass, where ρ is the mass density and n is the number of particles (electrons, protons, and helium nuclei) per unit volume, show that the average mass in an extreme Population II star is

   $$\mu = \frac{4m_p}{5X + 3}$$

   Hint: Recall that each hydrogen atom contributes two particles when ionized, and each helium atom contributes three.
   (b) Show that the luminosity in a star that is supported by ideal gas pressure scales with average particle mass, total mass, and internal temperature as

   $$L \propto \mu^7 M^5 T^{0.5}$$

   (c) A $1M_\odot$ Population II star with $X = 0.75$ is quite similar to the Sun; it has luminosity $L_\odot$ and radius $R_\odot$. Use this information and the result of part b to estimate the luminosity, radius, and surface temperature of a $1M_\odot$ pure helium star ($X = 0$).

5. **Radiation pressure and hydrostatic equilibrium in giant stars**: Recall that the momentum of a photon is $p = \frac{E}{c}$.

(a) Derive an expression for the radiation pressure, defined as the outward momentum per unit time per unit area, delivered to the outer layers of a star with luminosity L, assuming that all the photons from the interior are absorbed in those layers.

(b) Calculate the total radiation force on the outer layers of a star with $L = L_\odot$. How does the radially outward force from radiation compare to the force of gravity on the layers if the layers lie at $R = 100R_\odot$, have mass $m = 10^{-6}M_\odot$, and the rest of the star has a mass $M = 1M_\odot$?

(c) Repeat part b for a typical AGB star: $L = 7000L_\odot$, $T_e = 3000$ K, same masses. (*Hint*: You will need to first work out the star's radius.)

6. A certain star has a measured V magnitude equal to 13.54 and a measured B magnitude of 14.41. A U magnitude is measured that leads to a B-V color excess of $E(B-V) = 0.25$.

(a) Calculate the *visual extinction* A_V toward the star, the star's *extinction-corrected V magnitude* and the *extinction-corrected B-V color index*, and the extinction-corrected apparent bolometric magnitude and effective temperature of the star. Bolometric corrections and effective temperatures can be found in Table 1.

$(B-V)_0$	T_e	BC	$(B-V)_0$	T_e	BC
-0.35	40000	-4.5	0.5	6320	-0.04
-0.31	31900	-3.34	0.53	6200	-0.05
-0.3	30000	-3.17	0.6	5920	-0.06
-0.26	24200	-2.5	0.64	5780	-0.07
-0.24	22100	-2.23	0.68	5610	-0.1
-0.2	18800	-1.77	0.72	5490	-0.15
-0.16	16400	-1.39	0.81	5240	-0.19
-0.14	15400	-1.21	0.92	4780	-0.25
-0.12	14500	-1.04	0.98	4590	-0.35
-0.09	13400	-0.85	1.15	4410	-0.65
-0.06	12400	-0.66	1.3	4160	-0.9
0	10800	-0.4	1.41	3920	-1.2
0.03	10200	-0.32	1.48	3680	-1.48
0.06	9730	-0.25	1.52	3500	-1.76
0.09	9260	-0.2	1.55	3360	-2.03
0.15	8620	-0.15	1.56	3230	-2.31
0.2	8190	-0.12	1.61	3120	-2.62
0.33	7240	-0.08	1.73	3050	-3.21
0.38	6930	-0.06	1.8	2940	-3.46
0.45	6540	-0.04	1.91	2640	-4.1
0.47	6450	-0.04			

Table 1: Color index, effective temperatures, and bolometric correction for main-sequence stars.

(b) Suppose you had ignored extinction. Use the observed V magnitude and color index to infer a *bolometric magnitude* and *effective temperature* for the star, and compare your results to those of part a, assuming the star is main sequence. How large an error in luminosity is made by ignoring extinction?

(c) Estimate the *absolute bolometric magnitude* of the star, calculate its distance, and estimate its spectral type. You can use the data file `ZAMS.txt` on the course website.

7. Suppose a newly-formed O5 star ($T_e = 35000$ K, $R = 18R_\odot$) lies within a dusty shell of radius 0.2 pc. Under the assumption that the grains are small spherical blackbodies heated by light from the central

star, calculate the temperature of the grains in the dusty shell. At what wavelength do the dust grains shine brightest?

8. Suppose a spherical cloud is made of pure molecular hydrogen and has a uniform number density 10^6 cm^{-3}, uniform temperature 100 K, and mass $1M_\odot$.
   (a) Show that this cloud is gravitationally stable; that is, it could be in hydrostatic equilibrium.
   (b) To what temperature would it have to cool in order to become gravitationally unstable?
   (c) Suppose the cloud radiates like a blackbody and stays uniform in temperature and constant in size as it does so. How long, in hours, will it last before it becomes gravitationally unstable?
   Hint: Recall that the luminosity of a spherical blackbody with radius R is $L = 4\pi R^2 \sigma T^4$ and that the thermal energy of N atoms at temperature T is $E = 3/2\ NkT$. Note that the factor in the thermal energy expression is actually 5/2 for a gas of diatomic molecules like hydrogen if the temperature is high enough (but 100 K is not high enough).

9. **Extinction toward open clusters**: On the course website, you will find photometric data files on the zero-age main sequence (ZAMS) derived from observations of nearby stars, and six additional open clusters.
   (a) Choose two of the clusters, and for each cluster, determine the color excesses $E(B-V)$ and $E(U-B)$ by plotting their $U-B$ colors as functions of their $B-V$ colors on the same plot as $U-B$ vs. $B-V$ for the ZAMS, and shifting the cluster data until they line up best with the ZAMS. Estimate the uncertainties in both quantities by the range of each color excess over which the fit to the ZAMS is good.
   (b) Measurements on large collections of open clusters typically give $E(U-B)/E(B-V) = 0.72$, but with substantial scatter about this value that is partly due to observational uncertainties and partly due to real differences between the extinction in various directions. How does this value compare with your results? Are they the same within uncertainties or does it look as if the extinction toward your cluster really has a different ratio of color excesses?

10. **Measuring distances to open clusters by main-sequence fitting**:
   (a) For each of the two clusters you used in the previous problem, calculate extinction-corrected $B-V$ colors and V magnitudes for the stellar members, $(B-V)_0$, and $m_{V0} = V_0$. Plot (with Python or similar) V_0 as a function of $(B-V)_0$, the extinction-corrected H-R diagrams.
   (b) The difference along the V axis between the main sequences of your clusters and the ZAMS are the same as the distance moduli of your clusters. Plot (again, using Python or something similar) each extinction-corrected HR diagram along with that of the ZAMS and shift the cluster's points until the cluster's main sequence and the ZAMS line up well. What is the distance modulus (magnitudes) and distance (in parsecs) of each cluster?

