1. A certain star has a measured V magnitude equal to 13.54 and a measured B magnitude of 14.41. A U magnitude is measured that leads to a B-V color excess of $E(B-V) = 0.25$.

   (a) Calculate the *visual extinction* A_V toward the star, the star's *extinction-corrected V magnitude* and the *extinction-corrected B-V color index*, and the extinction-corrected apparent bolometric magnitude and effective temperature of the star. Bolometric corrections and effective temperatures can be found in Table **??**.

$(B\text{-}V)_0$	T_e	BC	$(B\text{-}V)_0$	T_e	BC
-0.35	40000	-4.5	0.5	6320	-0.04
-0.31	31900	-3.34	0.53	6200	-0.05
-0.3	30000	-3.17	0.6	5920	-0.06
-0.26	24200	-2.5	0.64	5780	-0.07
-0.24	22100	-2.23	0.68	5610	-0.1
-0.2	18800	-1.77	0.72	5490	-0.15
-0.16	16400	-1.39	0.81	5240	-0.19
-0.14	15400	-1.21	0.92	4780	-0.25
-0.12	14500	-1.04	0.98	4590	-0.35
-0.09	13400	-0.85	1.15	4410	-0.65
-0.06	12400	-0.66	1.3	4160	-0.9
0	10800	-0.4	1.41	3920	-1.2
0.03	10200	-0.32	1.48	3680	-1.48
0.06	9730	-0.25	1.52	3500	-1.76
0.09	9260	-0.2	1.55	3360	-2.03
0.15	8620	-0.15	1.56	3230	-2.31
0.2	8190	-0.12	1.61	3120	-2.62
0.33	7240	-0.08	1.73	3050	-3.21
0.38	6930	-0.06	1.8	2940	-3.46
0.45	6540	-0.04	1.91	2640	-4.1
0.47	6450	-0.04			

Table 1: Color index, effective temperatures, and bolometric correction for main-sequence stars.

   (b) Suppose you had ignored extinction. Use the observed V magnitude and color index to infer a *bolometric magnitude* and *effective temperature* for the star, and compare your results to those of part a, assuming the star is main sequence. How large an error in luminosity is made by ignoring extinction?

   (c) Estimate the *absolute bolometric magnitude* of the star, calculate its distance, and estimate its spectral type. You can use the data file `ZAMS.txt` on the course website.

2. Suppose a newly-formed O5 star ($T_e = 35000$ K, $R = 18R_\odot$) lies within a dusty shell of radius 0.2 pc. Under the assumption that the grains are small spherical blackbodies heated by light from the central star, calculate the temperature of the grains in the dusty shell. At what wavelength do the dust grains shine brightest?

3. Suppose a spherical cloud is made of pure molecular hydrogen and has a uniform number density 10^6 cm^{-3}, uniform temperature 100 K, and mass $1M_\odot$.

   (a) Show that this cloud is gravitationally stable; that is, it could be in hydrostatic equilibrium.

   (b) To what temperature would it have to cool in order to become gravitationally unstable?

(c) Suppose the cloud radiates like a blackbody and stays uniform in temperature and constant in size as it does so. How long, in hours, will it last before it becomes gravitationally unstable?

Hint: Recall that the luminosity of a spherical blackbody with radius R is $L = 4\pi R^2 \sigma T^4$ and that the thermal energy of N atoms at temperature T is $E = 3/2\ NkT$. Note that the factor in the thermal energy expression is actually 5/2 for a gas of diatomic molecules like hydrogen if the temperature is high enough (but 100 K is not high enough).

4. **Extinction toward open clusters**: On the course website, you will find photometric data files on the zero-age main sequence (ZAMS) derived from observations of nearby stars, and six additional open clusters.

(a) Choose two of the clusters, and for each cluster, determine the color excesses $E(B-V)$ and $E(U-B)$ by plotting their $U-B$ colors as functions of their $B-V$ colors on the same plot as $U-B$ vs. $B-V$ for the ZAMS, and shifting the cluster data until they line up best with the ZAMS. Estimate the uncertainties in both quantities by the range of each color excess over which the fit to the ZAMS is good.

(b) Measurements on large collections of open clusters typically give $E(U-B)/E(B-V) = 0.72$, but with substantial scatter about this value that is partly due to observational uncertainties and partly due to real differences between the extinction in various directions. How does this value compare with your results? Are they the same within uncertainties or does it look as if the extinction toward your cluster really has a different ratio of color excesses?

5. **Measuring distances to open clusters by main-sequence fitting**:

(a) For each of the two clusters you used in the previous problem, calculate extinction-corrected $B-V$ colors and V magnitudes for the stellar members, $(B-V)_0$, and $m_{V0} = V_0$. Plot (with Python or similar) V_0 as a function of $(B-V)_0$, the extinction-corrected H-R diagrams.

(b) The difference along the V axis between the main sequences of your clusters and the ZAMS are the same as the distance moduli of your clusters. Plot (again, using Python or something similar) each extinction-corrected HR diagram along with that of the ZAMS and shift the cluster's points until the cluster's main sequence and the ZAMS line up well. What is the distance modulus (magnitudes) and distance (in parsecs) of each cluster?

6. Disks generally decrease strongly in density from the inside out. Eventually they cease to be self-gravitating, changing their structure. Consider the outer reaches of a galactic disk at a distance at which the gravitational forces are dominated by the central part of the galaxy (mass M), which you can assume to be spherical. The disk is still supported centrifugally in the radial direction and hydrostatically in the vertical direction, but now the weight of a test particle is determined by the vertical component of the force from the galactic center (like the situation of a protoplanetary disk around a young star).

(a) Under the assumptions $r \gg z$ and that the vertical component of random stellar velocities v_z is independent of z, solve the equation of hydrostatic equilibrium at radius r for the mass density ρ as a function of z. What is the density scale height?

(b) Under the assumption that the vertical component of random stellar velocities v_z is also independent of r, show that the disk is flared, i.e., the scale height increases with increasing radius.

(c) Look online for an image of the galaxy NGC 3628. Might this be a good example of a flared galactic disk? Why or why not? Discuss briefly.

