1. Two oboe players can hear each other; one hovers just outside a black hole's horizon at $r = 1.01R_{\rm sch}$ and the other is at rest far away from the black hole. (This is the George Lucas version of outer space.) Each plays an A4 note (f = 440 Hz). What is the frequency of the note each hears played by the other?

Solution: The frequency f = 440 Hz corresponds to a period 1/f = 2.27 ms. If that is the note played by the oboist near the black hole such that $\Delta \tau = 1/f$ then the distant observer hears

$$\Delta t = \frac{\Delta \tau}{\sqrt{1 - 2GM/rc^2}} = \frac{2.27 \text{ ms}}{\sqrt{1 - 1/1.01}} = 22.8 \text{ ms} \qquad f = \frac{1}{\Delta t} = 43.9 \text{ Hz}$$

This is an F more than two octaves below middle C and barely within the range of human hearing. If it is the note played by the "inertial" oboist ($\Delta t = 1/f$) then the oboist by the black hole hears

$$\Delta \tau = \Delta t \sqrt{1 - \frac{2GM}{rc^2}} = 2.27 \text{ ms} \sqrt{1 - \frac{1}{1.01}} = 0.226 \text{ ms} \qquad f = \frac{1}{\Delta \tau} = 4.42 \text{ kHz}$$

This is a C# more than four octaves above middle C, but still in the human range of hearing.

2. Stable black hole orbits:

(a) Derive an expression for the orbital frequency of the innermost stable circular orbit (ISCO) around a black hole $f_{\rm ISCO}$ in terms of the black hole's mass.

Solution: The ISCO has a coordinate radius of $r_{\rm ISCO} = 3R_{\rm Sch} = 6GM/c^2 = 53.2$ km. The circumference of circular orbits and the orbital velocity are the same in all reference frames, so

$$f = \frac{1}{P} = \frac{v_{\phi}}{2\pi r} = \frac{\sqrt{GM/r}}{2\pi r} = \frac{1}{2\pi} \sqrt{\frac{GM}{r^3}}$$
$$f_{\rm ISCO} = \frac{1}{2\pi} \sqrt{\frac{GM}{r_{\rm ISCO}^3}} = \frac{1}{2\pi} \sqrt{GM} \left(\frac{c^2}{6GM}\right)^3 = \frac{c^3}{12\pi G\sqrt{6}} \frac{1}{M}$$

(b) The black hole in GRO J1655-40 is observed to have a mass of $M = 6.0M_{\odot}$ and emits X-rays which exhibit quasiperiodic oscillations at a frequency of 450 Hz. It is thought that this frequency indicates material in orbit. If the black hole were not spinning, how far (radially, in cm) from the ISCO would this orbit be, in the view of a distant observer?

Solution: Invert the previous result for f to estimate the coordinate radius r of the orbit corresponding to 450 Hz:

$$r = \left(\frac{GM}{4\pi^2 f^2}\right)^{1/3} = 4.64 \times 10^6 \text{ cm} = 46.4 \text{ km}$$

Therefore, the radial distance from the ISCO orbit is

$$\Delta r = 3R_{\rm Sch} - r = \frac{6GM}{c^2} - \left(\frac{GM}{4\pi^2 f^2}\right)^{1/3} = 6.8 \times 10^5 \text{ cm} = 6.8 \text{ km}$$

(c) If the black hole were not spinning, how far (radially, in cm) would this orbit be from the ISCO in the view of an observer in the vicinity of these orbits? Presume the local observer to be capable of measuring the distance instantaneously.

Solution: Inside the gravitational well we have to account for curvature effects. Using the integral from Recitation 4 we have

$$\Delta \mathcal{L} = \int_{r}^{r_{\rm ISCO}} \frac{dr'}{\sqrt{1 - R_{\rm Sch}/r'}} = R_{\rm Sch} \int_{r/R_{\rm Sch}}^{r_{\rm ISCO}/R_{\rm Sch}} \frac{du}{\sqrt{1 - 1/u}}$$
$$= R_{\rm Sch} \left[\frac{1}{2} \ln \left(\frac{1/\sqrt{1 - 1/u} + 1}{1/\sqrt{1 - 1/u} - 1} \right) + \sqrt{u(u+1)} \right]_{r/R_{\rm Sch}}^{r_{\rm ISCO}/R_{\rm Sch}}$$
$$= R_{\rm Sch} \left[\frac{1}{2} \ln \left(\frac{1 + \sqrt{1 - 1/u}}{1 - \sqrt{1 - 1/u}} \right) + \sqrt{u(u+1)} \right]_{r/R_{\rm Sch}}^{r_{\rm ISCO}/R_{\rm Sch}}$$
$$= 8.4 \times 10^5 \text{ cm} = 8.4 \text{ km}$$

3. The pressure and temperature at the center of the Sun are $P_c = 2.1 \times 10^{17}$ dyne cm⁻² and $T_c = 15.7 \times 10^6$ K. What are the central pressure and temperature of a $1.5 M_{\odot}$ star with the same composition as the Sun?

(*Hint*: Empirical measurements of eclipsing binaries show that stellar mass and radius are close to linearly proportional with coefficient 1 in solar units. Use this to infer the missing piece of information needed to solve this problem.)

Solution: According to the empirical mass-radius relation for the components of eclipsing binaries, mass and radius are close to linearly proportional with coefficient 1 in solar units. Therefore the radius of a $1.5M_{\odot}$ star is about $1.5R_{\odot}$. Using the scaling relations developed in class, we get

$$\begin{split} P_c &= 2.1 \times 10^{17} (1.5)^2 (1.5)^{-4} \text{ dyne cm}^{-2} = 9.3 \times 10^{16} \text{ dyne cm}^{-2} \\ T_c &= 15.7 \times 10^6 (1.5) \left(\frac{1}{1.5}\right) (1) \text{ K} = 15.7 \times 10^6 \text{ K} \end{split}$$

- 4. A helium star: Consider stars with composition of a type called extreme "Population II": objects made only of hydrogen (fraction X by mass) and helium (fraction 1 X by mass), with negligible quantities of everything else.
 - (a) Starting with the definition $\mu = \frac{\rho}{n}$ for the average particle mass, where ρ is the mass density and n is the number of particles (electrons, protons, and helium nuclei) per unit volume, show that the average mass in an extreme Population II star is

$$\mu = \frac{4m_p}{5X+3}$$

Hint: Recall that each hydrogen atom contributes two particles when ionized, and each helium atom contributes three.

Solution: We know that in terms of the number density of H and He, $\rho = \mu n = m_p n_{\rm H} + 4m_p n_{\rm He}$ and $n = 2n_{\rm H} + 3n_{\rm He}$, so

$$\mu = \frac{\rho}{n} = \frac{n_{\rm H} + 4n_{\rm He}}{2n_{\rm H} + 3n_{\rm He}} m_p$$

We can eliminate $n_{\rm He}$ from this expression in favor of the hydrogen mass fraction X by noting that

$$m_p n_{\rm H} = X \rho \qquad \qquad 4m_p n_{\rm He} = (1 - X) \rho$$

Dividing the two expressions gives

$$n_{\rm He} = \frac{n_{\rm H}(1-X)}{4X}$$

which we substitute back into the expression for μ :

$$\mu = \frac{n_{\rm H} + 4n_{\rm H}(1-X)/4X}{2n_{\rm H} + 3n_{\rm H}(1-X)/4X} m_p = \frac{4X + 4(1-X)}{8X + 3(1-X)} m_p = \frac{4m_p}{5X + 3}$$

(b) Show that the luminosity in a star that is supported by ideal gas pressure scales with average particle mass, total mass, and internal temperature as

$$L \propto \mu^7 M^5 T^{0.5}$$

Solution: Since the star is supported by ideal gas pressure,

$$\begin{split} P &= nkT = \frac{\rho}{\mu}kT \\ T &= \frac{P\mu}{\rho k} \propto \frac{GM^2R^{-4}}{MR^{-3}}\frac{\mu}{k} \propto \frac{\mu M}{R} \end{split}$$

The luminosity in the center of the star was given in class as

$$L \propto \frac{R^3 T^4}{R^2/\ell} \propto \ell R T^4$$

In low-mass stars $\ell \propto T^{3.5}\rho^{-2}$. Thus

$$L \propto \frac{RT^{7.5}}{\rho^2} \propto R \left(\frac{\mu M}{R}\right)^{7.5} \frac{R^6}{M^2} \propto \mu^{7.5} M^{5.5} R^{-0.5}$$

Using $T \propto \mu M/R$ one more time to eliminate R gives

 $L \propto \mu^7 M^5 T^{0.5}$

(c) A $1M_{\odot}$ Population II star with X = 0.75 is quite similar to the Sun; it has luminosity L_{\odot} and radius R_{\odot} . Use this information and the result of part b to estimate the luminosity, radius, and surface temperature of a $1M_{\odot}$ pure helium star (X = 0).

Solution: The standard $1M_{\odot}$ star has X = 0.75 so in its center the average particle mass is about $\mu = 0.59m_p$. In the He star with X = 0, $\mu = 4m_p/3$. Therefore, in terms of the normal

star, the luminosity of the $1 M_{\odot}$ He star is

$$L_{\rm He} = \frac{\left(\mu^7 M^5 T^{0.5}\right)_{\rm He}}{\left(\mu^7 M^5 T^{0.5}\right)_{\rm X=0.75}} L(X=0.75)$$
$$= \left(\frac{4m_p/3}{0.59m_p}\right)^7 \left(\frac{10^8 \text{ K}}{1.57 \times 10^7 \text{ K}}\right)^{0.5} L_{\odot}$$
$$= 760L_{\odot}$$

To get the radius we use $T \propto \mu M/R$ to express the radius in terms of the normal star:

$$R_{\rm He} = R_{\rm H} \left(\frac{(\mu M/T)_{\rm He}}{(\mu M/T)_{\rm X=0.75}} \right)$$
$$= R_{\odot} \frac{4m_p/3}{0.59m_p} \frac{1.57 \times 10^7 \text{ K}}{10^8 \text{ K}}$$
$$= 0.36R_{\odot}$$

The effective temperature is given by the usual expression $L = 4\pi R^2 \sigma T^4$, so

$$T_e \propto \left(\frac{L}{R^2}\right)^{1/4}$$
$$T_e(\text{He}) = \left(\frac{L_{\text{He}}}{R_{\text{He}}^2} \frac{R_{X=0.75}^2}{L_{X=0.75}}\right)^{1/4} T_e(X = 0.75)$$
$$= \left(\frac{760}{0.36^2}\right)^{1/4} 5800 \text{ K}$$
$$= 51000 \text{ K}$$

As shown in the figure below, this places the He star well to the left of and below the main sequence, as determined using eclipsing binary stars.



- 5. Radiation pressure and hydrostatic equilibrium in giant stars: Recall that the momentum of a photon is $p = \frac{E}{c}$.
 - (a) Derive an expression for the radiation pressure, defined as the outward momentum per unit time per unit area, delivered to the outer layers of a star with luminosity L, assuming that all the photons from the interior are absorbed in those layers.

Solution: By outer layers we mean
$$r = R$$
:

$$P_{rad} = \frac{F}{A}$$

$$= \frac{1}{4\pi R^2} \frac{dp}{dt}$$

$$= \frac{1}{4\pi R^2} \frac{1}{c} \frac{dE}{dt}$$

$$P_{rad} = \frac{L}{4\pi R^2 c}$$

(b) Calculate the total radiation force on the outer layers of a star with $L = L_{\odot}$. How does the radially outward force from radiation compare to the force of gravity on the layers if the layers lie at $R = 100R_{\odot}$, have mass $m = 10^{-6}M_{\odot}$, and the rest of the star has a mass $M = 1M_{\odot}$?

Solution:
$$F = PA$$
 so

$$F_{rad} = \frac{L}{c} = \frac{3.827 \times 10^{33} \text{ erg/s}}{3 \times 10^{10} \text{ cm/s}}$$

$$\boxed{F_{rad} = 1.3 \times 10^{23} \text{ dynes}}$$

$$F_{grav} = -\frac{GMm}{R^2} = -\frac{(6.674 \times 10^{-8} \text{ dyn cm}^2 \text{ g}^{-2})(1.989 \times 10^{33} \text{ g})^2(1 \times 10^{-6})}{((100)(6.96 \times 10^{10} \text{ cm}))^2}$$

$$= -5.5 \times 10^{27} \text{ dynes}$$

So even with the high luminosity radiation pressure is several orders of magnitude too small to hold up the star. Gas pressure holds up the star, and radiation pressure is too small to blow off the outer layers.

(c) Repeat part b for a typical AGB star: $L = 7000L_{\odot}$, $T_e = 3000$ K, same masses. (*Hint*: You will need to first work out the star's radius.)

Solution: First solve for the radius of the star:

$$L = 4\pi R^2 \sigma T_e^4$$

$$R = \sqrt{\frac{L}{4\pi\sigma T_e^4}} = \sqrt{\frac{(7000)(3.827 \times 10^{33} \text{ erg/s})}{4\pi (5.6704 \times 10^{-5} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ K}^{-4})(3000 \text{ K})^4}}$$

$$R = 2.2 \times 10^{13} \text{ cm} = 310 R_{\odot}$$

Thus

$$\begin{split} F_{\rm rad} &= \frac{L}{c} = \frac{(7000)(3.827 \times 10^{33} \ {\rm erg/s})}{3 \times 10^{10} \ {\rm cm/s}} \\ \hline F_{\rm rad} &= 8.9 \times 10^{26} \ {\rm dynes} \\ F_{\rm grav} &= -\frac{GMm}{R^2} = -\frac{(6.674 \times 10^{-8} \ {\rm dyn} \ {\rm cm}^2 \ {\rm g}^{-2})(1.989 \times 10^{33} \ {\rm g})^2(1 \times 10^{-6})}{(2.2 \times 10^{13})^2} \\ F_{\rm grav} &= -5.6 \times 10^{26} \ {\rm dynes} \\ \end{split}$$
This time radiation wins out and the outer layers will be propelled further outward.

- 6. A certain star has a measured V magnitude equal to 13.54 and a measured B magnitude of 14.41. A U magnitude is measured that leads to a B-V color excess of E(B V) = 0.25.
 - (a) Calculate the visual extinction A_V toward the star, the star's extinction-corrected V magnitude and the extinction-corrected B-V color index, and the extinction-corrected apparent bolometric magnitude and effective temperature of the star. Bolometric corrections and effective temperatures can be found in Table 1.

$(B-V)_0$	T_e	BC	(B-V) ₀	T_e	BC
-0.35	40000	-4.5	0.5	6320	-0.04
-0.31	31900	-3.34	0.53	6200	-0.05
-0.3	30000	-3.17	0.6	5920	-0.06
-0.26	24200	-2.5	0.64	5780	-0.07
-0.24	22100	-2.23	0.68	5610	-0.1
-0.2	18800	-1.77	0.72	5490	-0.15
-0.16	16400	-1.39	0.81	5240	-0.19
-0.14	15400	-1.21	0.92	4780	-0.25
-0.12	14500	-1.04	0.98	4590	-0.35
-0.09	13400	-0.85	1.15	4410	-0.65
-0.06	12400	-0.66	1.3	4160	-0.9
0	10800	-0.4	1.41	3920	-1.2
0.03	10200	-0.32	1.48	3680	-1.48
0.06	9730	-0.25	1.52	3500	-1.76
0.09	9260	-0.2	1.55	3360	-2.03
0.15	8620	-0.15	1.56	3230	-2.31
0.2	8190	-0.12	1.61	3120	-2.62
0.33	7240	-0.08	1.73	3050	-3.21
0.38	6930	-0.06	1.8	2940	-3.46
0.45	6540	-0.04	1.91	2640	-4.1
0.47	6450	-0.04			

Table 1: Color index, effective temperatures, and bolometric correction for main-sequence stars.

Solution: For dark diffuse clouds, recall from lecture that

$$R = \frac{A_V}{E(B-V)} = 3.06 \qquad \implies A_V = 3.06E(B-V) = (3.06)(0.25) = 0.77$$

Therefore the corrected magnitude and color index are

$$V_0 = V - 0.77 = 12.78$$

 $(B - V)_0 = B - V - E(B - V) = 0.87 - 0.25 = 0.62$

For this color index the bolometric correction is -0.065 and $T_e = 5850$ K. The apparent bolometric magnitude is $m = V_0 - 0.065 = 12.71$.

(b) Suppose you had ignored extinction. Use the observed V magnitude and color index to infer a *bolometric magnitude* and *effective temperature* for the star, and compare your results to those of part a, assuming the star is main sequence. How large an error in luminosity is made by ignoring extinction?

Solution: Ignoring extinction we obtain B-V = 0.87, a bolometric correction of -0.22, a $T_e = 5010$ K and an apparent magnitude of m = 13.32. Therefore,

$$\Delta m = m_{\text{ext}} - m_{\text{no ext}} = 2.5 \log\left(\frac{f_{\text{no ext}}}{f_{\text{ext}}}\right) = 2.5 \log\left(\frac{L_{\text{no ext}}}{L_{\text{ext}}}\right)$$
$$\frac{L_{\text{no ext}}}{L_{\text{ext}}} = 10^{(m_{\text{ext}} - m_{\text{no ext}})/2.5} = 0.57$$

This is almost a factor of two error in luminosity.

(c) Estimate the *absolute bolometric magnitude* of the star, calculate its distance, and estimate its spectral type. You can use the data file ZAMS.txt on the course website.

Solution: With its corrected colors the star appears to be Solar type, so its class is about G2. Therefore, taking a value M = 4.68 for its absolute bolometric magnitude, its distance r is given by

$$\begin{split} m &= M + 5 \log{(r/10 \text{ pc})} \\ r &= (10 \text{ pc}) 10^{(m-M)/5} = (10 \text{ pc}) 10^{(12.71-4.68)/5} \\ r &= 403.6 \text{ pc} \end{split}$$

7. Suppose a newly-formed O5 star ($T_e = 35000$ K, $R = 18R_{\odot}$) lies within a dusty shell of radius 0.2 pc. Under the assumption that the grains are small spherical blackbodies heated by light from the central star, calculate the temperature of the grains in the dusty shell. At what wavelength do the dust grains shine brightest?

Solution: The star emits a total power $L = 4\pi R^2 \sigma T_e^4$ and does so isotropically. At the radius r of the dust shell the stellar flux is $f = L/4\pi r^2$. If the spherical grains have radius a, their geometrical cross section is πa^2 , so they absorb a power

$$P_{\rm abs} = fA = \frac{L}{4\pi r^2} \pi a^2$$

If the grain is treated as a spherical blackbody of surface area $4\pi a^2$, then it emits $L_{\rm gr} = 4\pi a^2 \sigma T_{\rm gr}^2$.

If it is in thermal equilibrium with starlight then the input and output power (luminosity) are equal:

$$\begin{split} P_{\rm abs} &= L_{\rm gr} \\ \frac{L}{4\pi r^2} \pi a^2 &= \frac{R^2 \sigma T_e^4}{r^2} \pi a^2 = 4\pi a^2 \sigma T_{\rm gr}^4 \\ T_{\rm gr} &= T_e \left(\frac{R}{2r}\right)^{1/2} = (35000 \text{ K}) \left(\frac{(18)(7 \times 10^{10} \text{ cm})}{(2)(0.2)(3.1 \times 10^{18} \text{ cm})}\right)^{1/2} \\ T_{\rm gr} &= 35.3 \text{ K} \end{split}$$

By Wien's Law, the blackbody with this temperature has peak brightness

$$\lambda_{\max} = \frac{0.29 \text{ cm K}}{35.3 \text{ K}} = 82.2 \ \mu\text{m}$$

which lies in the far-IR range.

- 8. Suppose a spherical cloud is made of pure molecular hydrogen and has a uniform number density 10^6 cm^{-3} , uniform temperature 100 K, and mass $1M_{\odot}$.
 - (a) Show that this cloud is gravitationally stable; that is, it could be in hydrostatic equilibrium.

Solution: The Jeans mass for this density ($\rho = \mu n = 3.3 \times 10^{-18} \text{ g/cm}^3$), composition ($\mu = 3.347 \times 10^{-24} \text{ g for H}_2$) and temperature is

$$M_J = \left(\frac{kT}{\mu G}\right)^{3/2} \left(\frac{3}{4\pi\rho}\right)^{1/2} = 4.2 \times 10^{33} \text{ g} = 2.1 M_{\odot} > M_{\text{cloud}}$$

so it will not collapse under its own weight.

(b) To what temperature would it have to cool in order to become gravitationally unstable?

Solution: If the Jeans mass were less than or equal to the cloud mass then it would be unstable, so we find the critical temperature from the inequality

$$M_J = \left(\frac{kT}{\mu G}\right)^{3/2} \left(\frac{3}{4\pi\rho}\right)^{1/2} \le 1M_{\odot}$$
$$T \le \left(\frac{4\pi G^3 \mu^4 n M^2}{3k^3}\right)^{1/3} = 60.6 \text{ K}$$

(c) Suppose the cloud radiates like a blackbody and stays uniform in temperature and constant in size as it does so. How long, in hours, will it last before it becomes gravitationally unstable? *Hint*: Recall that the luminosity of a spherical blackbody with radius R is $L = 4\pi R^2 \sigma T^4$ and that the thermal energy of N atoms at temperature T is E = 3/2 NkT. Note that the factor in the thermal energy expression is actually 5/2 for a gas of diatomic molecules like hydrogen if the temperature is high enough (but 100 K is not high enough). Solution: Since

$$E = \frac{3}{2}NkT = \frac{3}{2}\frac{M}{\mu}kT \qquad \qquad L = -\frac{dE}{dt}$$

we can derive, separate, and integrate the following differential equation:

$$\begin{aligned} \frac{dE}{dt} &= \frac{3}{2} \frac{M}{\mu} k \frac{dT}{dt} = -L = -4\pi R^2 \sigma T^4 \\ &- \frac{3kM}{8\pi R^2 \sigma \mu T^4} \frac{dT}{dt} = 1 \\ &- \frac{3kM}{8\pi R^2 \sigma \mu} \int_{T_0}^{T_1} \frac{dT}{T^4} = \int_{t_0}^{t_1} dt \\ &- \frac{3kM}{8\pi R^2 \sigma \mu} \left[-\frac{1}{3T^3} \right]_{T_0}^{T_1} = t_1 - t_0 = \Delta t \\ &\Delta t = \frac{kM}{8\pi R^2 \sigma \mu} \left(\frac{1}{T_1^3} - \frac{1}{T_0^3} \right) \end{aligned}$$

Since $T_0 = 100$ K and $T_1 = 61.7$ K and the cloud radius is

$$R = (3M/4\pi\rho)^{1/3} = 5.2 \times 10^{16} \text{ cm}$$

we obtain

$$\Delta t = 7.54 \times 10^4 \text{ sec} \approx 20.9 \text{ hr}$$

This is a short time on the scale of the dimensions and internal velocities of such a cloud, which shows that radiative cooling is efficient enough that a molecular-cloud fragment can find itself suddenly unstable.

- 9. Extinction toward open clusters: On the course website, you will find photometric data files on the zero-age main sequence (ZAMS) derived from observations of nearby stars, and six additional open clusters.
 - (a) Choose two of the clusters, and for each cluster, determine the color excesses E(B-V) and E(U-B) by plotting their U-B colors as functions of their B-V colors on the same plot as U-B vs. B-V for the ZAMS, and shifting the cluster data until they line up best with the ZAMS. Estimate the uncertainties in both quantities by the range of each color excess over which the fit to the ZAMS is good.

Solution: See the left panel in the figures at the end of these solutions and the Python notebook posted on the course website. Note that none of the corrections come out as nicely as the Hyades, which you did in recitation; these clusters are much more distant and their members are much fainter, so the magnitude measurements are fewer and contain larger uncertainties. The range of E(B-V) over which the fits are about right is roughly ± 0.03 to ± 0.05 magnitudes. Results are listed in the table below.

(b) Measurements on large collections of open clusters typically give E(U-B)/E(B-V) = 0.72, but with substantial scatter about this value that is partly due to observational uncertainties and partly due to real differences between the extinction in various directions. How does this value compare with your results? Are they the same within uncertainties or does it look as if the extinction toward your cluster really has a different ratio of color excesses? **Solution:** Five of the clusters are indistinguishable from E(U-B)/E(B-V) = 0.72. Only NGC 7790 came out significantly different, with E(U-B)/E(B-V) = 0.42 giving the best fit. Thus, we would not be surprised for this cluster to demand a value of R different from 3.06.

10. Measuring distances to open clusters by main-sequence fitting:

(a) For each of the two clusters you used in the previous problem, calculate extinction-corrected B-V colors and V magnitudes for the stellar members, $(B-V)_0$, and $m_{V0} = V_0$. Plot (with Python or similar) V_0 as a function of $(B-V)_0$, the extinction-corrected H-R diagrams.

Solution: Computed by

$$(B - V)_0 = B - V - E(B - V)$$
$$V_0 = V - R \cdot E(B - V)$$

Details are available in HW6.ipynb. See also the table below and the center panels of the below figures.

(b) The difference along the V axis between the main sequences of your clusters and the ZAMS are the same as the distance moduli of your clusters. Plot (again, using Python or something similar) each extinction-corrected HR diagram along with that of the ZAMS and shift the cluster's points until the cluster's main sequence and the ZAMS line up well. What is the distance modulus (magnitudes) and distance (in parsecs) of each cluster?

Solution: Computed from

$$DM = m - M = V_0 - M_V$$

where the values of M_V are those on the ZAMS curve. Details are in HW6.ipynb. See also the table below and the rightmost panels of the below figures. To compute the distance in parsecs:

 $DM = 5 \log (r/10 \text{ pc})$ $r = 10^{DM/5+1} \text{ pc}$

Solution:

Tabulated values and plots for the previous two problems:

Cluster	E(B-V)	E(U-B)	A_v	$V_0 - M_v$	$r [\mathrm{pc}]$
NGC 129	0.55	0.4	1.7	11.1	1660
NGC 1647	0.37	0.27	1.1	8.7	550
NGC 5662	0.31	0.22	0.9	9.2	692
NGC 6067	0.38	0.27	1.2	10.8	1445
NGC 6087	0.19	0.13	0.6	9.8	912
NGC 7790	0.53	0.22	1.6	12.4	3020



