1. The Milky Way's rotation curve

- (a) Argue that in the frame of reference revolving around the Galactic Center with angular speed $\Omega(r_{\odot})$, the circular velocity of a gas cloud in the plane of the Milky Way at galactocentric distance r is $r [\Omega(r) \Omega(r_{\odot})]$.
- (b) Use the diagram below to justify the various angular identifications to show that the radial velocity of the gas cloud is given by

$$v_r = r \left[\Omega(r) - \Omega(r_{\odot})\right] \sin\left(\theta + \ell\right)$$

where ℓ is the Galactic longitude of the cloud.



(c) Use the law of sines to show that

$$v_r = r_{\odot} \left[\Omega(r) - \Omega(r_{\odot}) \right] \sin \ell \tag{1}$$

- (d) For a given Galactic longitude ℓ , the only thing that can vary in Equation 1 is the galactocentric distance of the gas cloud. Argue that if $\Omega(r)$ is a monotonically decreasing function of r, then v_r acquires its maximum positive value (for $0^{\circ} \leq \ell \leq 90^{\circ}$) when r corresponds to the radius of the circular orbit tangent to the line of sight.
- (e) Thus, show that the orbital speed at the tangent point (the speed we would want to use with Newton's laws or Kepler's laws, to work out masses) is given by

$$v(r) = r\Omega(r) = v_{r,\max} + r_{\odot}\Omega(r_{\odot})\sin\ell$$

where everything on the right hand side can be obtained from observations.

- 2. Galactocentric distance and the distance ambiguity: If we know r_{\odot} , $\Omega(r_{\odot})$, and the functional form of $\Omega(r)$, and we measure v_r , when we point a radio telescope in direction ℓ , the equation you derived in Problem 1 allows us to deduce a cloud's galactocentric distance r.
 - (a) Derive from this an expression for r. You can assume that the rotational tangential velocities in the disk are nearly constant with r.
 - (b) With a glance at the figure in Problem 1, show that if $r < r_{\odot}$ the line of sight generally intersects the circle of radius r at two points: a near point and a far point. This is the distance ambiguity. Show that the distance ambiguity does not arise if $r > r_{\odot}$.

$$\mathcal{L}(R) = \mathcal{L}(0)e^{-R/R_0}$$

Consider a nearly face-on spiral galaxy for which the "scale length" of the disk surface brightness is $R_0 = 2$ kpc, and for which the mass-to-light ratio is 2 at this galactocentric radius. Suppose you have shown that the rotation curve is flat over the galactocentric radii 2 - 12 kpc, so that you might suggest a spherical halo with density

$$\rho(r) = \rho_0 \left(\frac{r_0}{r}\right)^2$$

to dominate the mass in this range, where r is the usual spherical coordinate. Assume for simplicity that the core radius of the halo r_0 is equal to the scale length R_0 of the disk surface brightness.

(a) Show that the effective mass per unit area of the disk due to the spherical halo is

$$\mu(R) = \frac{\pi \rho_0 r_0^2}{R}$$

by integrating the density along the direction z perpendicular to the disk. *Hint*: Note that $r = \sqrt{R^2 + z^2}$ and use a trigonometric substitution to simplify the resulting integral.

- (b) Plot (with Python or similar) the mass-to-light ratio $\mu(R)/\mathcal{L}(R)$ for R = 2 12 kpc. What is the value of the mass-to-light at the largest radii? How does this compare to the mass-to-light ratio in the solar neighborhood?
- (c) By adjustment of the halo core radius r_0 , is it possible to make the mass-to-light ratio constant? Why or why not?

4. Cepheids in M31

- (a) Suppose you found a classical Cepheid with a period of 30 days in M31, and you measure its average V magnitude to be 18.6. Calculate its absolute magnitude and estimate the distance to M31.
- (b) W Virginis stars (Population II Cepheids) are a factor of four less luminous than classical Cepheids for the same pulsation periods. By what factor would a derived distance to M31 be in error if M31 Cepheids of one type were mistakenly identified as the other? (This mistake was unwittingly made by Hubble before a distinction between the two types was discovered.)