1. **Cepheids in M31**

   (a) Suppose you found a classical Cepheid with a period of 30 days in M31, and you measure its average V magnitude to be 18.6. Calculate its absolute magnitude and estimate the distance to M31.

   Solution: Use the relations given in lecture:

   $$\overline{M_V} = -2.77 \log \Pi - 1.69 = -5.78$$

   $$\begin{aligned}\overline{m_V} - \overline{M_V} &= 5 \log (d/10 \text{ pc}) \\ d &= (10 \text{ pc}) 10^{(\overline{m_V} - \overline{M_V})/5} \\ &= 0.752 \text{ Mpc}\end{aligned}$$

   (b) W Virginis stars (Population II Cepheids) are a factor of four less luminous than classical Cepheids for the same pulsation periods. By what factor would a derived distance to M31 be in error if M31 Cepheids of one type were mistakenly identified as the other? (This mistake was unwittingly made by Hubble before a distinction between the two types was discovered.)

   Solution: The flux measured is the same in either case, but the above equations used to turn the flux and luminosity into a distance are equivalent to

   $$f = \frac{L}{4\pi r^2} = \frac{L'}{4\pi r'^2},$$

   where we can take L to be the Cepheid I luminosity and L' the W Virginis luminosity (which is a factor of 4 smaller). Thus, the ratio between the distance r' inferred under the assumption that the star is a Pop. II Cepheid and the distance r if it were a Pop. I Cepheid is

   $$\frac{r'}{r} = \sqrt{\frac{L'}{L}} = \frac{1}{2},$$

   i.e., the distance would be underestimated by a factor of 2.

2. Suppose that a quasar is as bright as a solar-type star (they share similar *apparent* magnitudes) but the quasar is a factor of a million further away than the star. What is the quasar's luminosity?

   Solution: The luminosity ratio is

   $$\frac{L'}{L} = \left(\frac{r'}{r}\right)^2$$

   and taking $L = L_\odot$ and $r' = 10^6 r$, we get $L' = 10^{12} L_\odot$ for the quasar. That is quite a large luminosity — our Galaxy only puts out a total of about $2 \times 10^{10} L_\odot$.