1. A certain quasar has an average luminosity of $3.3 \times 10^{13} L_\odot$ and an X-ray brightness that can vary substantially in as little as three hours. Assume that the quasar's black-hole engine is accreting at the Eddington rate and show that these two findings are consistent with each other.

2. M87 has an accretion disk around its central black hole for which the rotational velocity has been measured in HST spectra. The disk extends 20 pc from the center and exhibits Doppler velocities as large as ± 500 km/s with respect to the galaxy's overall radial velocity.
   (a) Calculate the mass of the black hole in M87 to two significant digits. Comment on the assumptions under which you did your calculation (e.g., orbital plane viewed edge-on; note the appearance of the galaxy and its disk in the notes) and the effect this may have on the accuracy of your answer.
   (b) M87 is 16 Mpc away. With the mass you calculated for its central black hole, calculate the diameter of the black hole's event horizon (in pc) and the angle the event horizon subtends (in arcseconds).
   (c) Compare this result with the diffraction-limited angular resolution $\Delta\theta = 1.2\lambda/D$ (where D is telescope diameter) of the Hubble Space Telescope ($D = 2.4$ m) at a wavelength of 400 nm. Do the same for the VLBA ($D = 8611$ km) at a wavelength of 2 cm. Can we see details in images as small as the horizon? How far away would M87 have to be for the event horizon to subtend an angle equal to the best angular resolution?

3. The largest *apparent* superluminal motions seen so far in quasar jets are about $20c$.
   (a) What would this imply for the ejection speed v (in units of c to three significant figures) and jet angle with the line of sight θ (in degrees) if the jet is oriented for maximum apparent superluminal motion?
   (b) If this ejection speed applies to 3C 273, at what angle from the jet axis (in degrees) do we view this quasar?

4. Consider $10^{16}\ M_\odot$ of atomic hydrogen spread uniformly over a volume 10 Mpc in diameter and with a velocity dispersion $\sqrt{\overline{v^2}} = 1000$ km/s.
   (a) Take the hydrogen to be an ideal monatomic gas, so that the thermal energy per molecule is $E = 3k_BT/2$. What is the temperature of the big hydrogen cloud?
   (b) What is the Jeans mass of this material in $M_\odot$? What is the radius of a sphere with total mass equal to the Jeans mass (which we may as well call the **Jeans length**)?
   (c) Compare your answer from part b to typical masses and sizes of galaxies and galaxy groups. If galaxy cluster-sized objects formed first in the early Universe, which formed next: galaxy-sized objects or galaxy group-sized objects?

5. In a few billion years, our galaxy and the Andromeda galaxy will merge. Compute the expected number of collisions between stars when this occurs. Assume that the typical star in each galaxy is an M dwarf with a radius of $0.5R_\odot$, there are $N = 10^{11}$ stars in the Milky Way and 10^{12} stars in Andromeda, and that the average space density of stars in the Milky Way is $n = 1$ pc^{-3}, equal to that in the solar neighborhood.