- 1. Consider $10^{16} M_{\odot}$ of atomic hydrogen spread uniformly over a volume 10 Mpc in diameter and with a velocity dispersion $\sqrt{v^2} = 1000 \text{ km/s}$.
 - (a) Take the hydrogen to be an ideal monatomic gas, so that the thermal energy per molecule is $E = \frac{3}{2}k_BT$. What is the temperature of the big hydrogen cloud?
 - (b) What is the Jeans mass of this material in M_{\odot} ? What is the radius of a sphere with total mass equal to the Jeans mass (which we may as well call the **Jeans length**)?
 - (c) Compare your answer from part b to typical masses and sizes of galaxies and galaxy groups. If galaxy cluster-sized objects formed first in the early Universe, which formed next: galaxy-sized objects or galaxy group-sized objects?
- 2. In a few billion years, our galaxy and the Andromeda galaxy will merge. Compute the expected number of collisions between stars when this occurs. Assume that the typical star in each galaxy is an M dwarf with a radius of $0.5R_{\odot}$, there are $N = 10^{11}$ stars in the Milky Way and 10^{12} stars in Andromeda, and that the average space density of stars in the Milky Way is $n = 1 \text{ pc}^{-3}$, equal to that in the solar neighborhood.
- 3. The Lorentz transformation between two inertial reference frames with coordinate systems (x, t) and (x', t'), with the latter moving at constant speed v in the +x direction, is

$$x' = \gamma(x - vt) \qquad y' = y \qquad z' = z \qquad t' = \gamma \left(t - \frac{vx}{c^2}\right)$$
$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Suppose two events are observed by experimenters in each reference frame. The intervals between their coordinates in the "unprimed" coordinate system are $\Delta x = x_1 - x_2$, $\Delta y = y_1 - y_2$, $\Delta z = z_1 - z_2$, and $\Delta t = t_1 - t_2$. Show that the intervals between the two events in the "primed" coordinate system have different values than in the unprimed system, but that both observers agree on the value of the absolute interval

$$\Delta s^{2} = c^{2} \Delta t^{2} - \Delta x^{2} - \Delta y^{2} - \Delta z^{2} = c^{2} \Delta t^{2} - \Delta x^{2} - \Delta y^{2} - \Delta z^{2}$$

4. Light is emitted at t = 0 from an object at r = 0 and arrives later at a telescope located at $r = R(t)r_*$. Use the Robertson-Walker absolute interval,

$$ds^{2} = c^{2}dt^{2} - d\ell^{2} = c^{2}dt^{2} - R^{2}(t)\left(\frac{1}{1 - kr_{*}^{2}}dr_{*}^{2} + r_{*}^{2}d\theta^{2} + r_{*}^{2}\sin^{2}\theta d\phi^{2}\right)$$

to show that the proper distance ℓ traveled by the light is not equal to the coordinate distance r unless the Universe is flat.