Observations of Stars

Measurable properties of stars
Stellar photometry

January 23, 2024

University of Rochester

Observations of stars

Today's topics:

- Ensemble averages
- Measurable properties of stars:
  - Parallax and the distance to the nearest stars
  - Magnitude, flux, and luminosity
- Blackbodies as approximations to stellar spectra
- Stellar properties from photometry
  - Color and temperature
  - Bolometric correction and bolometric magnitude

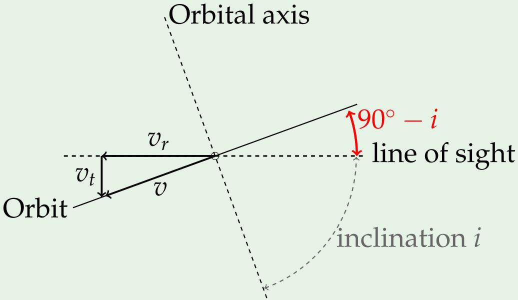
Reading: Kutner Ch. 2, Ryden Sec. 5.7 & 13.1–13.4

Open cluster M39 in Cygnus

Averages: Binary star systems

Example

- If the stars are too close together to observe their orientation, then i cannot be determined.
- Still, an estimate of $\sin i$ would be useful. Could we make a reasonable assumption?
- Let us try an average value, assuming that binary star orbits are in general uniformly distributed. That is, all orientations are equally likely:



$$p(i) = \text{constant} = C, \quad i = 0 \to \frac{\pi}{2}$$

$$1 = \int_0^{\pi/2} p(i)di = C\int_0^{\pi/2} di = \frac{\pi C}{2}$$

$$\therefore C = \frac{2}{\pi}$$

Averages: Binary star systems

Example

Now that we know $p(i)$,

$$\langle \sin i \rangle = \int_0^{\pi/2} \sin i \, p(i) \, di = \frac{2}{\pi} \int_0^{\pi/2} \sin i \, di$$
$$= \left[-\frac{2}{\pi} \cos i \right]_0^{\pi/2} = -\frac{2}{\pi} (-1 - 0) = \frac{2}{\pi}$$

Thus, a reasonable *estimate* of the true orbital speed v given the *measured* line-of-sight speed v_r is

$$v = \frac{v_r}{\langle \sin i \rangle} = \frac{\pi}{2} v_r$$

Note that a more sophisticated treatment with *axes randomly oriented* yields $\langle \sin i \rangle = \pi/4 \ldots$

Averages: Binary star systems

Example

Allowing all *orientations* requires us to average over the full sky (Ω) defined by θ, ϕ:

$$p(\theta,\phi) = \text{constant} = C$$

$$1 = \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta \, p(\theta,\phi) = 2\pi \cdot 2 \cdot C \qquad \therefore p(\theta,\phi) = \frac{1}{4\pi}$$

$$\begin{aligned}
\langle \sin i \rangle &= \int_\Omega d\Omega \, p(\theta,\phi) \sin\theta = \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_0^{\pi} \sin^2\theta d\theta \\
&= \frac{1}{2} \int_0^{\pi} \frac{1-\cos 2\theta}{2} d\theta \\
&= \frac{1}{4} \left[\theta - \frac{1}{2} \sin 2\theta \right]_0^{\pi} = \frac{\pi}{4}
\end{aligned}$$

What do we want to know about stars?

As a function of mass and age:

- Structure of their interiors
- Structure of their atmospheres
- Chemical composition
- Power output and spectrum
- Spectrum of oscillations
- All the ways they can form
- All the ways they can die
- The details of their lives in between
- The nature of their end states
- Their role in the dynamics and evolution of their host galaxy

For visible stars, measure…

- Flux at all wavelengths, at high resolution (probes atmosphere)
- Magnetic fields, rotation rate
- Oscillations in power output (probes interior)
- Distance from us and Galactic position
- Orbital parameters (for binaries)

…and match up the measurements with theory.

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…and match up the measurements with theory. We can do what is highlighted this semester.

Flux and luminosity

- Primary observable quantity: **flux**, f, or *power per unit area* within some range of wavelengths.
- **Surface brightness**: flux at the *surface* of an emitting object.
- **Luminosity**: L, total power output.

Like most astronomical objects, stars emit light isotropically (same in all directions).

- Since the same total power L must pass through all spheres centered on a star and is uniformly distributed on those spheres, the flux a distance r away is

$$f = \frac{L}{4\pi r^2}$$

- To obtain luminosity from measured flux, we must (1) add up flux measurements over all wavelengths, and (2) measure the distance.

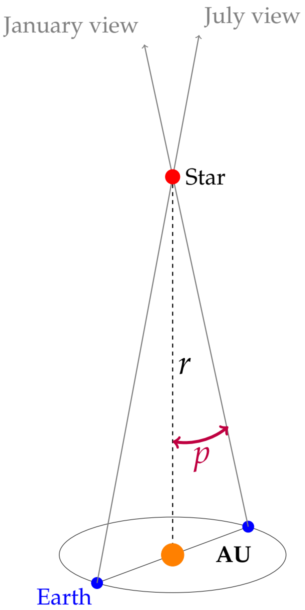
Distance from trigonometric parallax

The best distance measurement is the trigonometric parallax:

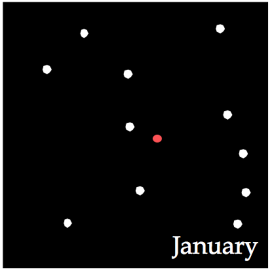
- Nearby stars *appear* to move back and forth with respect to more distant stars as Earth travels in its orbit.
- Since Earth's orbit is nearly circular, this works no matter what the direction is to the star.
- Since the size of Earth's orbit is know very accurately from radar measurements ($\pm$ meters), measurement of the **parallax** p determines the distance to the star precisely and accurately.

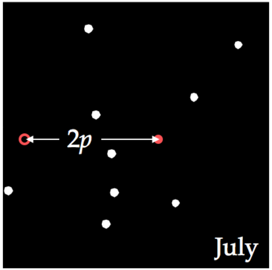
Astronomical Unit (AU) is the standard distance:

$$1 \text{ AU} = 149,597,870.7 \text{ km}$$



Trigonometric parallax





How small is the typical stellar parallax? Very small ($< 1''$)

Example

Suppose stars were typically 1 ly (light year) away — an extreme underestimate, as we will see. Then

$$\tan p = p + \frac{p^3}{3} + \frac{2p^5}{15} + \ldots$$

$$= \frac{1\ \text{AU}}{r} \approx \frac{1.5 \times 10^{13}\ \text{cm}}{9.5 \times 10^{17}\ \text{cm}}$$

$$= 1.6 \times 10^{-5}$$

$$\tan p \ll 1$$

Therefore, $p \approx 1\ \text{AU}/r$ is an excellent approximation.

Trigonometric parallax

- Note that in expressions such as

$$\tan p \approx p \qquad \text{and} \qquad r = \frac{1\ \text{AU}}{p}$$

  p is understood to be measured in radians.
- For convenience, parallaxes of nearby stars are usually reported in *arcseconds* or fractions thereof:

$$\pi \text{ radians} = 180^\circ = 180^\circ \times \frac{60 \text{ arcmin}}{1^\circ} \times \frac{60 \text{ arcsec}}{1 \text{ arcmin}} = 6.48 \times 10^5 \text{ arcsec}$$

- The distance to an object with a parallax of 1 arcsecond is called a **parsec (pc)**:

$$1 \text{ pc} = 3.0857 \times 10^{18} \text{ cm} = 3.2616 \text{ ly}$$

- Useful to remember: 206265 arcsec/rad and 206265 AU/pc.
- Smallest parallax measurements possible: about 1×10^{-3} arcsec from the ground, and about 1×10^{-5} arcsec (currently) from Gaia.

Parallax: Mapping the galaxy

What is the largest distance that can be measured with ground-based and space-based telescopes using trigonometric parallax?

We can write the distance-parallax relation as

$$r = \frac{1\ \text{AU}}{p\ [\text{rad}]} = \frac{1\ \text{pc}}{p\ [\text{arcsec}]}$$

- Ground-based: $r_{\text{max}} = \frac{1\ \text{pc}}{10^{-3\prime\prime}} = 1000\ \text{pc} = 1\ \text{kpc}$
- Gaia: $r_{\text{max}} = \frac{1\ \text{pc}}{10^{-5\prime\prime}} = 100{,}000\ \text{pc} = 100\ \text{kpc}$

Note: We live 8 kpc from the center of our Galaxy.

Parallax from Mars

If we lived on Mars, what would be the numerical value of the parsec?

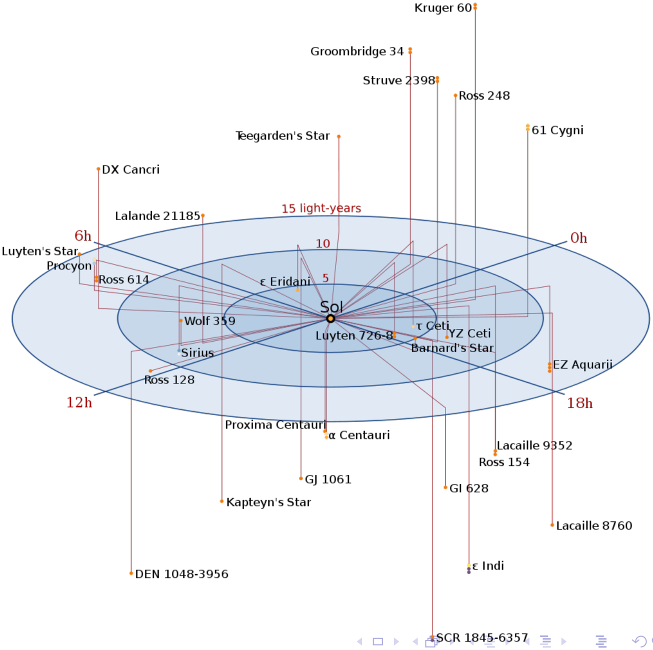
Mars's orbital semimajor axis is 1.524 AU, so the Martian "parsec" would be larger by a factor of 1.524:

$$
\begin{aligned}
r = 1\ \text{parsec} &= \frac{1.524\ \text{AU}}{1\ \text{arcsec}} \\
&= \frac{(1.524\ \text{AU})(1.496 \times 10^{13}\ \text{cm/AU})}{1\ \text{arcsec}} \left(\frac{3600\ \text{arcsec}}{1^\circ}\right)\left(\frac{180^\circ}{\pi}\right) \\
&= 4.703 \times 10^{18}\ \text{cm} = 4.971\ \text{ly}
\end{aligned}
$$

Note: Mars's orbit is twice as eccentric as Earth's, so be cautious about trusting those last decimal places.

The thirty nearest stars

- There are 35 stars within 12 ly of Earth.
- Nearest: Proxima Centauri, 1.3 pc (4.2 ly), part of a triple system.
- 79% of stars are low-mass red dwarfs.



Astronomical units: Magnitudes

Magnitude m, Brightness f

Ancient Greek system: first magnitude (and less) are the brightest stars. Sixth magnitude is the faintest the naked eye can see. (Magnitude ↑, brightness ↓)

The eye is approximately **logarithmic** in its response: perceived brightness is proportional to the logarithm of flux f. To match up with the Greek scale,

$$5 \text{ mag} \Leftrightarrow \text{a factor of } 100 \text{ in flux}$$

$$1 \text{ mag} \Leftrightarrow \text{a factor of } 100^{1/5} \approx 2.5 \text{ in flux}$$

So, for two stars with **apparent magnitudes** m_1 and m_2,

$$\boxed{m_2 - m_1 = 2.5 \log\left(\frac{f_1}{f_2}\right)}$$

Note: $\log \equiv \log_{10}$ from now on.

Magnitudes

Most people find magnitudes to be confusing at first, but they turn out to be very useful once you get used to the convention:

- Magnitudes are dimensionless quantities. They are related to the **ratios** of fluxes or distances.
- Another legacy from Greek astronomy: magnitudes run **backwards** from the intuitive sense of brightness.
  - Brighter objects have smaller magnitudes. Fainter objects have larger magnitudes. Think of them like a "rank."
- Fluxes from a combination of objects measured all at once add up algebraically as usual. The magnitudes of combinations of objects do not (examples in a few slides).

Magnitudes

The reference point of the apparent magnitude scale is a matter of arbitrary convention.

- As usual! Same as for the meter, second, kilogram, etc.

The practical definition of **zero apparent magnitude** is Vega (α Lyrae) which has $m = 0.0$ at a wide range of wavelengths (UV, optical, IR).

- This reference is hard to lose; it is the brightest star in the northern hemisphere.
- The apparent magnitude m of a star from which we measure a flux is

$$m = m - m_{\mathrm{Vega}} = 2.5 \log \left(\frac{f_{\mathrm{Vega}}}{f} \right)$$

- Fluxes from Vega have been measured with exquisite care and are tabulated.

Magnitudes

Two other magnitude definitions are necessary:

Absolute magnitude is the apparent magnitude of a star if it were placed at a standard distance of 10 pc.

Bolometric magnitude is the magnitude calculated from the flux from *all* wavelengths, rather than from a small range of wavelengths.

In your homework, you will show that the absolute and apparent bolometric magnitudes M and m of a star are related by

$$\boxed{M = m - 5\log\left(\frac{r}{10\ \mathrm{pc}}\right) = 4.74 - 2.5\log\left(\frac{L}{L_\odot}\right)}$$

Magnitude calculations

The *absolute* bolometric magnitude of the Sun is 4.74. What is its apparent bolometric magnitude?

The Sun's absolute magnitude is 4.74 if seen from 10 pc, but we see it from 1 AU. Thus,

$$
\begin{aligned}
m_1 &= m_2 + 2.5\log\left(\frac{f_2}{f_1}\right) = m_2 + 2.5\log\left(\frac{L_\odot}{4\pi r_2^2}\frac{4\pi r_1^2}{L_\odot}\right) \\
&= 4.74 + 2.5\log\left(\frac{(1\ \mathrm{AU})^2}{(10\ \mathrm{pc})^2}\right) = 4.74 + 5\log\left(\frac{1\ \mathrm{AU}}{10\ \mathrm{pc}}\right) \\
m_1 &= -26.83
\end{aligned}
$$

Magnitude calculations

Two objects are measured to have fluxes f and $f + \Delta f$, with $\Delta f \ll f$. What is the difference Δm between their magnitudes?

$$\Delta m = m_1 - m_2 = 2.5 \log \left(\frac{f + \Delta f}{f} \right) = 2.5 \log \left(1 + \frac{\Delta f}{f} \right)$$

$$= \frac{2.5}{\ln 10} \ln \left(1 + \frac{\Delta f}{f} \right)$$

$$\Delta m \approx 1.09 \frac{\Delta f}{f} \text{ to first order in } \Delta f$$

Thus, if two objects differ in flux by 1%, then they differ in magnitude by 0.011. It is very difficult to measure fluxes from astronomical objects more accurately than 1%, so magnitudes are not typically known more accurately than ± 0.01.