# Astronomy 142 — Recitation #2

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# Formulas to remember

## Binary stars and their motions

All the ms and Ms here are masses; a is an orbital semimajor axis length; a = r for circular orbits.

• Doppler effect

$$\frac{\lambda_{\text{observed}} - \lambda_{\text{rest}}}{\lambda_{\text{rest}}} = \frac{v_r}{c} \tag{1}$$

• Center of mass, circular orbits

$$\frac{m_1}{m_2} = \frac{r_2}{r_1}$$
(2)

• Kepler's third law, any eccentricity

$$P^{2} = \frac{4\pi^{2}}{G(m_{1} + m_{2})}(a_{1} + a_{2})^{3}$$
(3)

• Kepler's third law, circular orbits

$$P^{2} = \frac{4\pi^{2}}{G(m_{1} + m_{2})}(r_{1} + r_{2})^{3}$$
(4)

• Conservation of momentum

$$\frac{m_1}{m_2} = \frac{a_2}{a_1} = \frac{v_{2r}}{v_{1r}} = \frac{v_2}{v_1} \tag{5}$$

• Mass function

$$f(m_1, m_2) = \frac{P}{2\pi G} v_{1r}^3 < m_2 \tag{6}$$

# Hydrostatic equilibrium

Gravity v. pressure gradient — resulting in steady mass configuration

$$\begin{split} M(r) &= \int_0^r \rho(r') 4\pi r'^2 \, dr' \\ &\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2} \\ P(R) - P(r) &= \int_r^R \frac{dP}{dr'} dr' = -\int_r^R \frac{GM(r')\rho(r')}{r'^2} \, dr' \\ \text{Scaling: } \rho_C \propto \frac{M}{R^3}, P_C \propto \frac{GM^2}{R^4}, T_C \propto \frac{GM\mu}{R} \end{split}$$

#### Ideal gases

$$P = \frac{\rho kT}{\mu} \tag{7}$$

where  $\mu$  is the average mass of particles (i.e. atoms or ions or molecules) in the gas.

#### 3-D random walk

To travel a distance R in randomly-directed steps of length  $\ell$  requires going N steps, given by

$$N = \frac{3R^2}{\ell^2} \tag{8}$$

## Starlight heating by a variable-luminosity star

If the average luminosity of a star is L and the average temperature to which this heats a planet is T, then a small variation by  $\Delta L < L$  in the star's luminosity leads to a small variation  $\Delta T$  in the planet's temperature, given by

$$\frac{\Delta T}{T} = \frac{\Delta L}{4L} \tag{9}$$

#### Energy-mass equivalence

$$E = mc^2 \tag{10}$$

## Local sidereal time (LST)

LST = right ascension of celestial objects on the meridian (the meridian = arc through the poles and the zenith); therefore, LST has the same units as right ascension.

The Earth rotates by  $360^{\circ} = 2\pi$  radians in 24 hours of sidereal time; the Earth rotates by  $2\pi(1 + \frac{day}{TY})$  in one day. 1 day = 24 hours, or 86400 seconds; 1 tropical year (TY) = 365.242189 days. The latter means that the Earth's rotational angular speed is  $\Omega_{\oplus} = 7.292115855 \times 10^{-5}$  radians/s, and that a sidereal day — 24 hours of sidereal time, and the actual rotation period of the Earth — is 23.9344696 hours.

For a given time of day, the corresponding sidereal time advances by 24 hours as the date advances one year.

# Workshop problems

**Remember!** The workshop problems you will do in groups in Recitation are a crucial part of the process of building up your command of the concepts important in ASTR 142 and subsequent courses. Do not, therefore, do your work on scratch paper and discard it. Better for each of you to keep your own account of each problem in some sort of bound notebook.

1. A star with total mass M has density given by

$$\rho(r) = \rho_0 e^{-r/R} \tag{11}$$

The star does not have a sharp outer edge; R is just the characteristic radius over which the density changes by a factor of 1/e.

What is the central density,  $\rho_0$ , in terms of M and the characteristic radius R?

2. Stellar structure gets complicated quickly. The following is a demonstration as to why we will consider stars to have a density with a functional form simpler than above for the remainder of the semester.

(a) Using the results from above for a star with density

$$\rho(r) = \rho_0 e^{-r/R}$$

and the hydrostatic equilibrium equation, derive an expression for the pressure P(r) at radius r within the star. You will need to use integration by parts and the integral shown below:

$$\int_{z}^{\infty} \frac{e^{-x}}{x} dx \equiv Ei(z) \text{ the Exponential Integral})^{1}$$

- (b) Consider the behavior of this result at the center of the star. Can a star with this density be in hydrostatic equilibrium?
- 3. What is the total gravitational potential energy U for a *uniform*-density star with mass M and radius R, in terms of M and R?

Learn your way around the sky. (A feature *exclusive* of ASTR 142 recitations.) You may find the lab's celestial globes and the program Stellarium useful in answering these questions about the celestial sphere and the constellations.

4. Sidereal time and hour angle

*zenith* (the point directly overhead).

- (a) A day is measured from noon to noon: the time it takes to return the Sun to its maximum elevation in the sky. Assume that Earth's orbit is circular. Calculate the angle  $\eta$  (in radians) that Earth's orbital radius sweeps out in a day.
- (b) By what angle, in radians, does Earth turn on its axis in the course of a day?
- (c) Earth spins at a constant rate. How long does it take, in hours, minutes, and seconds, for Earth to make one complete rotation with respect to the distant stars? This duration is called the *sidereal day*; it is the time it takes to return the fixed stars to their positions in the sky. Sidereal time tells us where the stars are in the sky, in the same way as ordinary civil time tells us where the Sun is in the sky. The rules are that the sidereal time is equal to the solar time at the moment of the vernal equinox: the time the Sun passes the celestial equator going north. The vernal equinox is also the origin of right ascension, so the sidereal time is also the right ascension of the stars that lie along the meridian: the line of celestial longitude connecting the poles and the
- (d) Suppose the vernal equinox happened precisely at noon on March 21. What is the right ascension of the star directly overhead in Rochester precisely at midnight on March 21? What is its declination?
- (e) During the same year as in part d: what is the sidereal time at midnight on September 21?
- (f) The coordinates of the black hole at the center of the Milky Way Galaxy, an object called Sagittarius A\* (Sgr A\*), are  $\alpha = 17^{h}45^{m}40.0^{s}$ ,  $\delta = -29^{\circ}00'28$ ". On what date is Sgr A\* on the meridian at midnight? (Or, on what date is the sidereal time at midnight equal to the object's right ascension?) Therefore, what would be a good time of year to observe Sgr A\*?
- 5. The vernal equinox is when the Sun reaches the intersection of the celestial equator and the ecliptic, going north; this particular intersection defines the zero of the right ascension scale. This year, the vernal equinox takes place on March 20 at 5:01 AM EDT: 09:01 coordinated universal time (UTC; standard time at zero longitude) on March 20.

$$Ei(z) = \ln z + \sum_{j=1}^{\infty} \frac{z^j}{j \cdot j!}$$
 (12)

<sup>&</sup>lt;sup>1</sup>The Exponential Integral is an object that you will learn more about in Math 282. For real positive values of z, it is given by the following expression, obtainable by expanding the exponential integrated in a power series and integrating directly:

Naturally, this event will take place at different times of day for observers at different longitudes. By convention, longitude, L, is measured in degrees of arc, or degrees-minutes-seconds of arc. We define east to be the positive direction, and L = 0 to be on the prime meridian: the arc that runs through the geographic north and south poles, and the Royal Greenwich Observatory in London. Standard time on the prime meridian is the same as UTC.

Like its cousins the right ascension and azimuthal coordinate  $\phi$  in spherical and cylindrical coordinates, longitude is not an open-ended scale because it is measured on a closed surface. The longitude is therefore within the range  $0^{\circ} - 360^{\circ}$ . Negative values are allowed, as these indicate longitude differences to the west. For example: downtown Rochester has longitude L equal to  $282.384^{\circ} = 282^{\circ}23'04"$  or  $-77.616^{\circ} = -77^{\circ}36'56"$ .

As described above, the local sidereal time (LST) is closely related to longitude, except for having the same zero as right ascension and being measured in hours, minutes, and seconds of time instead of angular measures (as with right ascension). LST is always positive, and is between 0 and 24 hours (24<sup>h</sup>). The zero point, however, is not simply a fixed point in the sky — defining the zero point of LST is what you will explore in this problem.

- (a) At what longitude is the Sun on the meridian at the moment of the vernal equinox? What is the local sidereal time at this location and moment?
- (b) At the moment of the vernal equinox, what is the local sidereal time at zero longitude, and in Rochester?
- (c) Look at the result of an accurate calculation of local sidereal time:

#### https://aa.usno.navy.mil/data/siderealtime

Calculate the local sidereal time the moment of the vernal equinox, at the longitude calculated in part a, at zero longitude, and at Rochester's longitude. If all has gone well, you will find that your answers differ from the results in parts a and b by the same amount. What do you suppose is the cause of this offset? (That is, what extra offset does the calculator include that we are currently leaving out?)

## Intro to Python. (A feature exclusive of ASTR 142 recitations.)

- 6. Calculations and astropy tables
  - (a) Download the data file "EMspectrum.txt" from the course website.
  - (b) Write a python script that will import the data file using the Table class in astropy.table. Feel free to reference last week's recitation assignment for guidance on what packages need to be imported, etc. The main line for importing the file will be

```
data = Table.read('EMspectrum.txt', format='ascii.commented_header')
```

(c) After importing the data into the table, calculate the temperatures that blackbodies would have to peak at the wavelengths corresponding to the boundaries between the different parts of the electromagnetic spectrum. First, use a for-loop to do this element by element. A for-loop works by running through a list of elements and repeating the same task on each one. For example,

```
for i in range(3):
    j = i + 3
    print(j)
would print the follow
```

would print the following output:

3 ⊿

4 5

You can access an element in an astropy table using the following syntax: data['column\_name'][row\_number] Remember, python starts counting at 0!

Next, rewrite this calculation to be all in one line (otherwise known as "vectorized code"). To access the entire column of data, you can use

- data['column\_name']
- (d) Add this new data as an additional column in the table. Refer to last week's recitation for the syntax on appending columns of data to an astropy table.
- 7. Consider a star of mass  $M_* = M_{\odot}$  and radius  $R_* = R_{\odot}$ . Numerically integrate the equations of hydrostatic equilibrium and mass conservation,

$$\frac{dP}{dr} = -\frac{GM(r)\rho}{r^2} \quad \text{and} \quad \frac{dM(r)}{dr} = 4\pi r^2 \rho \tag{14}$$

to find M(r) and P(r) for three cases:

- (a) Uniform density:  $\rho(r) = M_*/(4/3 \pi R_*^3)$
- (b) Linear density:  $\rho(r) = 3M_*/(\pi R_*^3) [1 r/R_*]$
- (c) Quadratic density:  $\rho(r) = 15M_*/(8\pi R_*^3) \left[1 (r/R_*)^2\right]$

Plot the mass and pressure for these cases as a function of  $r/R_*$ , and check that the value of the central pressure  $P(r = 0) = P_c$  is sensible. (See https://matplotlib.org/stable/gallery/index.html for example plots and the corresponding code.)

Hint 1: When numerically integrating, remember the boundary conditions  $P(r) \to 0$  as  $r \to R_*$  and  $M(r) \to 0$  as  $r \to 0$ . This suggests that you may wish to integrate in opposite directions — surface to center and center to surface — to find M and P.

*Hint 2:* There are many solvers available for systems of 1D ordinary differential equations, but you can approximate the integrals as simple Riemann sums

$$\int f(r) \, dr \approx \sum_{i} f(r_i) \Delta r_i \tag{15}$$

for a suitably small choice of  $\Delta r_i$ .