Astronomy 142 — Recitation 2

Prof. Douglass

January 26, 2024

Formulas to remember

The notes from this week's lectures are full of definitions which you should eventually have at your fingertips. Here are *some* of them that you might find useful today.

Probability and averages

If x ranges from a to $b > a$ and is characterized by the probability definition $p(x)$, then

$$\int_a^b p(x)\,dx = 1 \qquad \langle x \rangle = \int_a^b x p(x)\,dx \qquad \langle f(x) \rangle = \int_a^b f(x)p(x)\,dx$$

Trigonometric parallax

1 radian = 206,265 arcsec

$$r = \frac{1\ \text{AU}}{\tan p} \cong \frac{1\ \text{AU}}{p} \qquad p \text{ in radians}$$

$$r(p = 1\ \text{arcsec}) = 3.086 \times 10^{18}\ \text{cm} = 3.62\ \text{ly} \equiv 1\ \text{pc}$$

$$r = \frac{1\ \text{pc}}{p\ [\text{arcsec}]}$$

Magnitudes and color indices

All the ms and Ms are magnitudes here.

- Flux, distance, and luminosity

$$f = \frac{L}{4\pi r^2} \tag{1}$$

- Apparent magnitude and flux

$$m_2 - m_1 = 2.5 \log\left(\frac{f_1}{f_2}\right) \tag{2}$$

  This applies to any kind of magnitude, if the fluxes are measured in the same band as the corresponding magnitudes.

- Absolute magnitude (the apparent magnitude for an object 10 pc away)

$$\begin{aligned} &\text{At any wavelength:} && M_\lambda = m_\lambda - 5\log\left(\tfrac{r}{10\ \text{pc}}\right) \\ &\text{Bolometric:} && M = m - 5\log\left(\tfrac{r}{10\ \text{pc}}\right) = 4.75 - 2.5\log\left(\tfrac{L}{L_\odot}\right) \\ &\text{Distance modulus:} && DM = m - M = 5\log\left(\tfrac{r}{10\ \text{pc}}\right) \end{aligned}$$

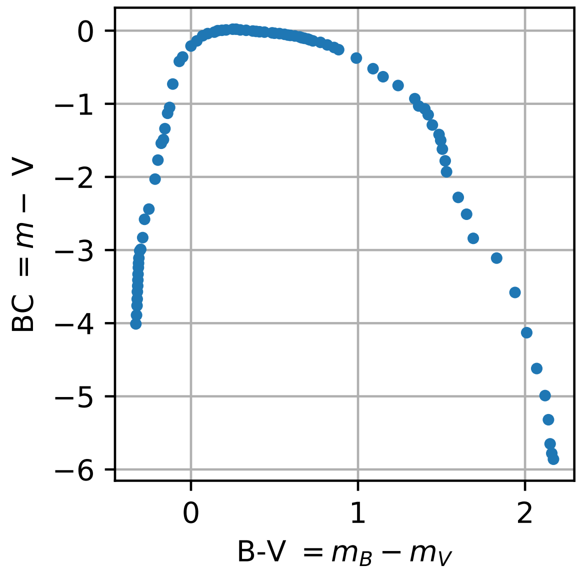


Figure 1: Bolometric correction

- Color index

$$CI(\lambda_1, \lambda_2) = m_{\lambda_1} - m_{\lambda_2} = M_{\lambda_1} - M_{\lambda_2} = 2.5 \log\left(\frac{f(\lambda_2)}{f(\lambda_1)}\right)$$
$$B - V = m_B - m_V = M_B - M_V = 2.5 \log\left(\frac{f(V)}{f(B)}\right)$$
$$B - V \cong -0.93 + \frac{9000 \text{ K}}{T_e}$$

- Bolometric correction (get bolometric magnitude from V magnitude; see Figure 1)

$$m = m_V + BC \qquad M = M_V + BC \tag{3}$$

Note that magnitudes are **dimensionless**.

Common (base 10) and natural (base e) logarithms

$$\log x = \frac{\ln x}{\ln 10} = \log e \ln x = 0.434 \ln x \tag{4}$$

By "log" we mean common logarithm ($\log_{10}$), and by "ln" natural logarithm ($\log_e$).

Planck blackbody function

Power per unit area, bandwidth, and solid angle

$$B_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} \qquad \text{peak at } \lambda_{\max} T = 0.29 \text{ cm K} \tag{5}$$

Note that Ryden's textbook calls the Planck function $I_\lambda(\lambda, T)$, and Kutner's textbook calls the Planck function $U(\nu, T)d\nu$. Be very careful converting between frequency and wavelength!

The flux from a blackbody, emitted within a small bandwidth $\Delta\lambda (\ll \lambda)$ and a small solid angle $\Delta\Omega (\ll 4\pi)$:

$$f = B_\lambda(\lambda, T) \Delta\lambda \Delta\Omega \tag{6}$$

Solid angle

$$d\Omega = \sin\theta \, d\theta \, d\phi \qquad \Omega = \iint \sin\theta \, d\theta \, d\phi$$

$\Omega \cong \pi \Delta\theta^2$ for a cone with small angular radius $\Delta\theta$

Workshop problems

Remember! The workshop problems that you do in groups in Recitation are a crucial part of the process of building up your command of the concepts important in ASTR 142 and subsequent courses. Do not, therefore, do your work on scratch paper and discard it. Better for each of you to keep your own account of each problem in some sort of bound notebook.

1. Consider a parameter x, which can take any nonnegative real value, with the probability distribution

$$p(x) = \frac{1}{a} e^{-x/a}$$

   (a) Show that $p(x)$ is properly normalized.

   (b) Calculate the average value of this parameter, $\langle x \rangle$.

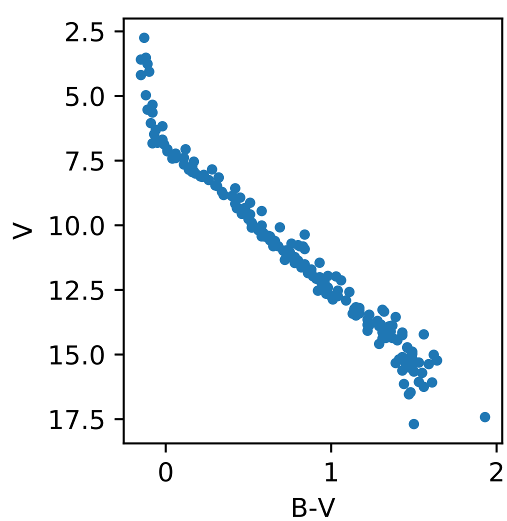
   (c) Calculate the parameter's **standard deviation**, $\sigma = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$. The standard deviation of a parameter can be considered as the width of the probability distribution about the average value, in much the same sense as an experimental uncertainty: x is very likely to lie in the range $\langle x \rangle \pm \sigma$.

2. *Magnitudes and distances*

   (a) The brightest star Vega (α Lyrae; brightest star in the northern celestial hemisphere) has an apparent magnitude of 0.0 at all the usual visible wavelengths (U,B,V). What would be its apparent magnitude at these wavelengths if Vega were moved a factor of five further away?

   (b) Vega's parallax is 0.129 arcsec. What is its absolute V magnitude?

   (c) The Pleiades are a cluster of relatively young stars in the constellation Taurus. (The stars in a cluster that occupies a very small patch of sky, like the Pleiades, can all be assumed to lie approximately the same distance away from us.) Measurements of their apparent V magnitudes and B–V color indices appear below. Given your results for Vega, how far away are the Pleiades? (You can assume that a star of the same color as Vega in the Pleiades has the same absolute magnitude as Vega.)



Apparent V magnitude as a function of color index B–V for the Pleiades (Stauffer et al. 1994).

(d) What are the absolute V magnitudes of the bluest (smallest B–V) and reddest (largest B–V) stars in the Pleiades?

(e) What are the absolute bolometric magnitudes of the bluest and reddest stars in the Pleiades? (You will need to use the chart of bolometric correction given in Figure 1.)

3. Suppose all binary stars consisted of two stars with radius $R_\odot$ orbiting each other with constant separation 1 AU, like the example shown in lecture on eclipsing binaries.

   (a) From the viewpoint of one of these systems, what is the solid angle from which an observer could see the system eclipse? (Hint: Recall the trig identity $\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$.)

   (b) What is the fraction of binary stars in which we could detect eclipses? Compare this result to that noted in the two largest unbiased surveys that could detect binaries, those by the *Kepler* and *Hipparcos* satellites: 1.2% and 0.8%, respectively (Prsa et al. 2010).

Learn your way around the sky, lesson 2. (A feature *exclusive* of ASTR 142 recitations.) You may find the lab's celestial globes and the program Stellarium useful in answering these questions about the celestial sphere and the constellations.

4. *Ursa Major*

   (a) A good illustration of how first- and second-magnitude stars look is the constellation Ursa Major. How many first-magnitude ($1 \le V = m_V < 2$) and second-magnitude ($2 \le V = m_V < 3$) stars are there in Ursa Major?

   (b) Using the first-magnitude stars in Ursa Major, describe how to find the stars Polaris (α Ursae Minoris[1], the brightest star closest to the north celestial pole) and Arcturus (α Boötis).

   (c) How far is Polaris from the north celestial pole? Give your answer in degrees, and also in units of the angular diameter of the Moon.

   (d) Calculate the angles (in degrees) between α and β UMa, and α and η UMa.

Intro to Python, lesson 2. (A feature *exclusive* of ASTR 142 recitations.)

5. *Calculations and astropy tables*

   (a) Download the data file "EMspectrum.txt" from Blackboard.

   (b) Write a python script that will import the data file using the Table class in astropy.table. Feel free to reference last week's recitation assignment for guidance on what packages need to be imported, etc. The main line for importing the file will be

   ```
   data = Table.read('EMspectrum.txt', format='ascii.commented_header')
   ```

   (c) After importing the data into the table, calculate the temperatures that blackbodies would have to peak at the wavelengths corresponding to the boundaries between the different parts of the electromagnetic spectrum. First, use a for-loop to do this element by element. A for-loop works by running through a list of elements and repeating the same task on each one. For example,

   ```
   for i in range(3):
       j = i + 3
       print(j)
   ```

   would print the following output:

   ```
   3
   4
   5
   ```

[1] Per tradition, we use the Latin names of the constellations. Unfortunately, the tradition has also come down to us to treat the names as Latin nouns and to give them their proper declension. Thus, they appear in accusative form above (e.g. Ursa Major), except when used as a Greek stellar name, in which case the genitive form is used (e.g. α Ursae Majoris = "the brightest star of Ursa Major").

You can access an element in an astropy table using the following syntax:

`data['column_name'][row_number]`

Remember, python starts counting at 0!

Next, rewrite this calculation to be all in one line (otherwise known as "vectorized code"). To access the entire column of data, you can use

`data['column_name']`

(d) Add this new data as an additional column in the table. Refer to last week's recitation for the syntax on appending columns of data to an astropy table.