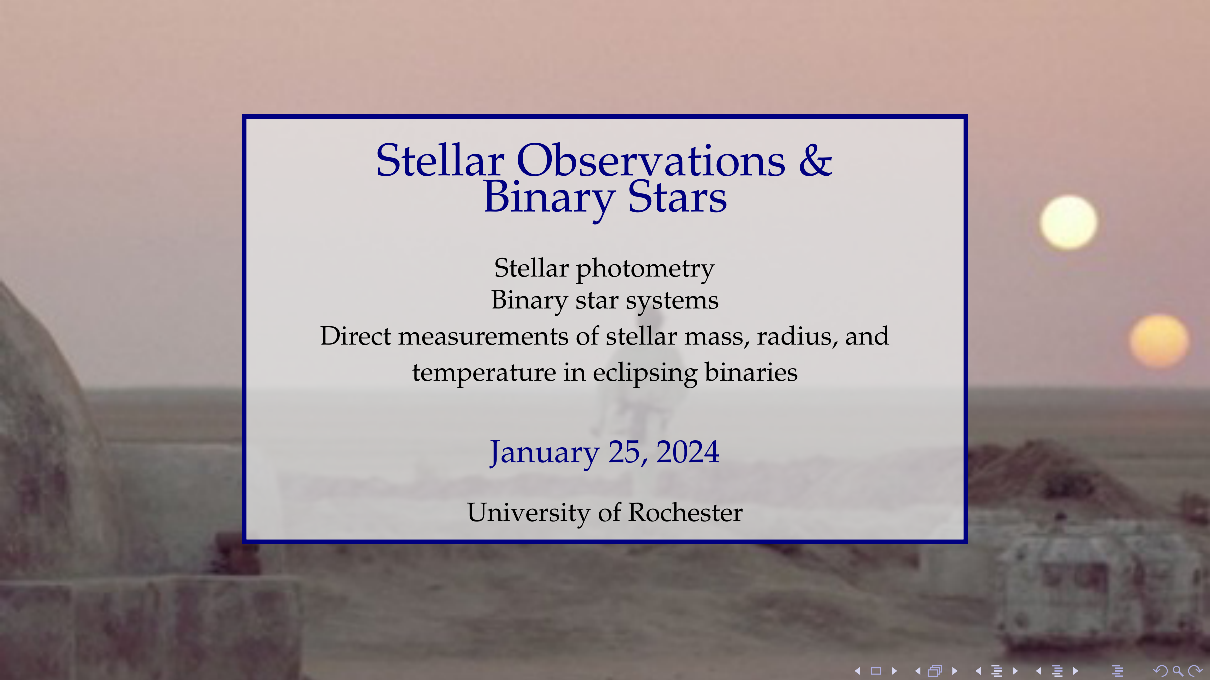


Stellar Observations & Binary Stars

Stellar photometry
Binary star systems
Direct measurements of stellar mass, radius, and temperature in eclipsing binaries

January 25, 2024

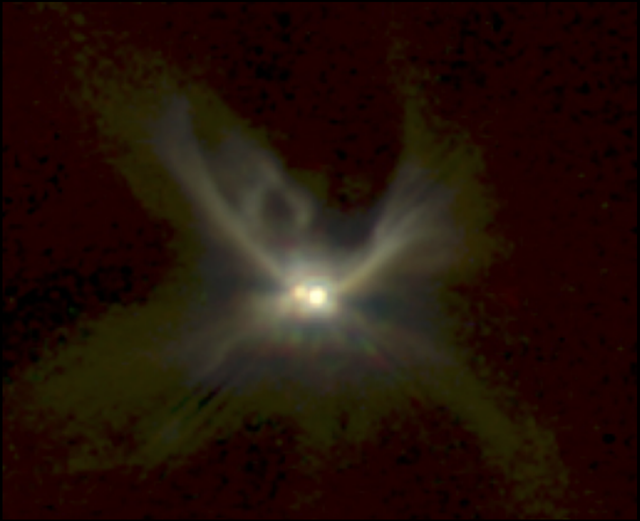
University of Rochester

Stellar observations & Binary stars

Today's topics:

- Blackbodies as approximations to stellar spectra
- Stellar properties from photometry
  - Color and temperature
  - Bolometric correction and bolometric magnitude
- Binary star systems
- Eclipsing binaries
- Direct measurements of stellar mass, radius, and temperature

Reading: Kutner Ch. 5, Ryden Sec. 13.5–13.6



CoKu Tau/1, a young binary system in the Taurus star-forming region. D. Padgett and K. Stapelfeldt, HST (STScI/NASA).

High opacity and the appearance of stars

Blackbody emission

As we will see, stars are **opaque** at essentially all wavelengths. Because of this, they emit light much like ideal **blackbodies**.

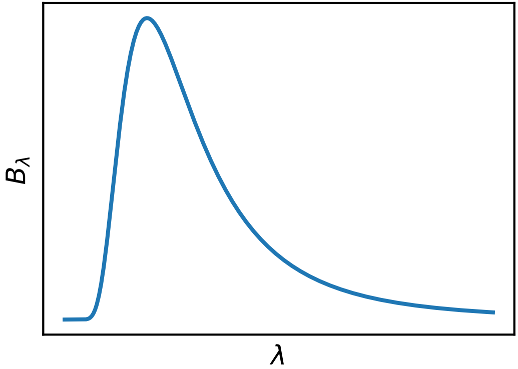
Blackbody radiation from a perfect absorber/emitter is described by the Planck function:

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT} - 1}$$

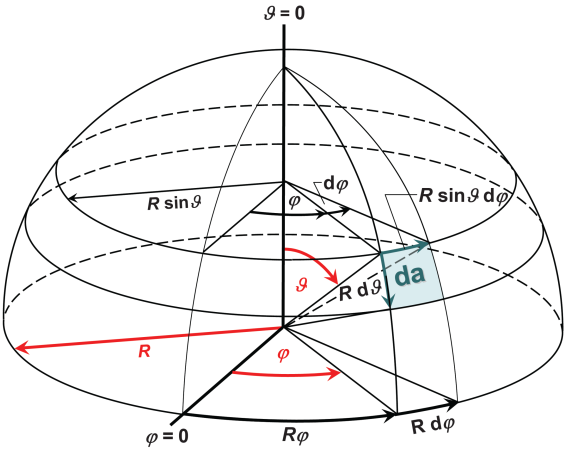
with dimensions of power per unit area per **bandwidth** (wavelength interval $\Delta\lambda$) per solid angle Ω.

For small bandwidth $\Delta\lambda \ll \lambda$ and solid angle $\Delta\Omega \ll 4\pi$, the flux f emitted by a blackbody is

$$f = B_\lambda(T)\,\Delta\lambda\,\Delta\Omega$$



Notes on solid angle



- Solid angle is the 2D angle in 3D space subtended by a cone or the intersection of planes meeting at a point. It is used to calculate area on the unit sphere.
- Units: the **steradian**.
- Differential element of solid angle in spherical coordinates:

$$d\Omega = \sin\theta d\theta \, d\phi$$

- Finite solid angle Ω:

$$\Omega = \int_0^{\phi_0} d\phi \int_0^{\theta_0} \sin\theta d\theta$$

Notes on solid angle

For a cone subtending an angle $\Delta\theta$,

$$\Omega = \int_0^{2\pi} d\phi \int_0^{\Delta\theta} \sin\theta d\theta = 2\pi\left(1 - \cos\Delta\theta\right)$$

Example

Cone with $\Delta\theta \ll 1$:

$$\Omega \approx 2\pi\left(1 - \left[1 - \frac{\Delta\theta^2}{2}\right]\right) = \boxed{\pi\Delta\theta^2}$$

For the full sky,

$$\Omega = \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta = 4\pi$$

For one hemisphere,

$$\Omega = \int_0^{2\pi} d\phi \int_0^{\pi/2} \sin\theta d\theta = 2\pi$$

Blackbody Emission

The total flux emitted *isotropically* into all directions at all wavelengths by a blackbody is

$$f = \int d\lambda \int d\Omega \cos\theta \, B_\lambda(T) = 2\pi \int_0^\infty d\lambda \int_0^{\pi/2} \sin\theta \, d\theta \;\cos\theta \, B_\lambda(T)$$
$$f = \sigma T^4$$

where

$$\sigma = 5.67 \times 10^{-5} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ K}^{-4}$$

is the Stefan-Boltzmann constant. The luminosity of a spherical blackbody is

$$L = 4\pi R^2 \sigma T^4$$

The Sun's luminosity indicates a $T_{\odot e} = 5772$ K blackbody. In fact, this is the star's **effective temperature** as defined by L and R.

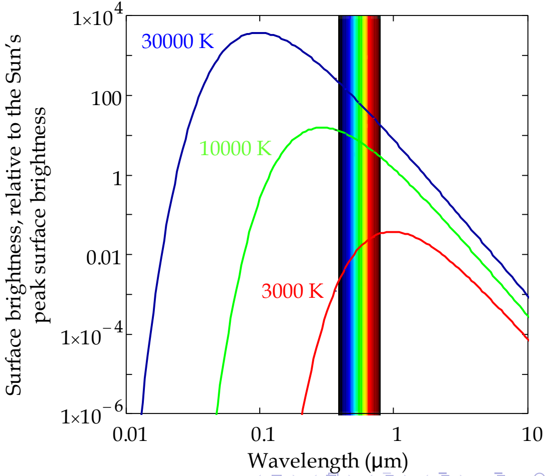
Blackbody emission

Wien's Law

The wavelength defining the maximum of the Planck function changes with temperature according to

$$\lambda_{\max} T = 0.29 \text{ cm K}$$

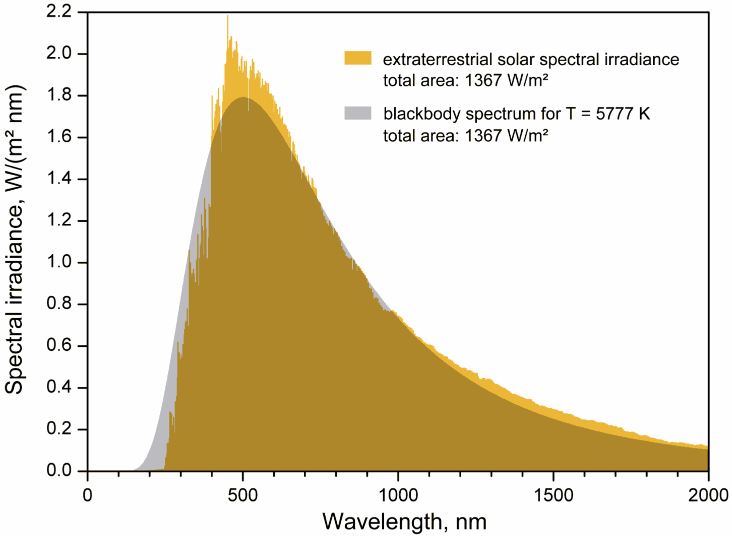
This is **Wien's Law**, which you have likely seen in previous courses. It describes how increasing the temperature of the blackbody decreases its peak wavelength. E.g., blue stars are hotter than red ones.



What is the peak wavelength of the solar spectrum?

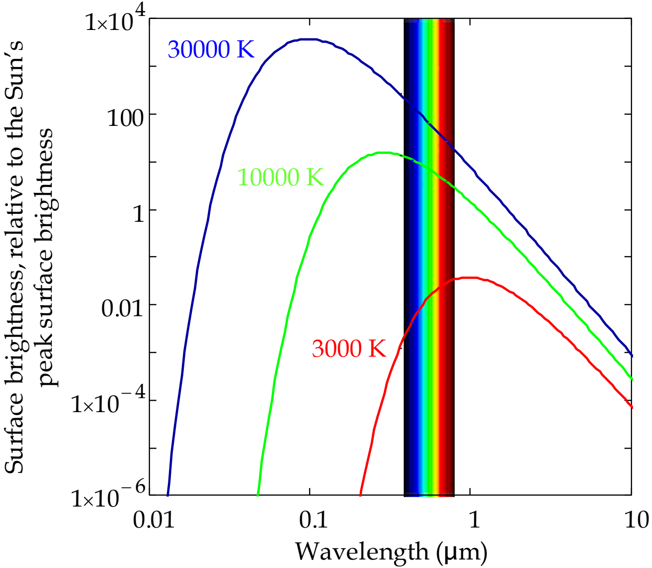
$$\lambda_{\max} \approx \frac{0.3 \text{ cm K}}{5772 \text{ K}} \approx 5 \times 10^{-5} \text{ cm} \approx 500 \text{ nm}$$

Stars are pretty good approximate blackbodies



Blackbody emission

- Although stars like the Sun are brightest in the visible band, the Planck function for star-like temperatures is much wider than the visible spectrum.
- Due to the tails in the Planck function, most of a star's luminosity actually radiates at ultraviolet and/or infrared wavelengths.



Stellar photometry

To the extent that stars emit approximately as blackbodies, it does not take very many measurements over a broad spectrum to determine a star's temperature and luminosity.

- If stars were perfect blackbodies, just two accurate measurements of flux within bands at different wavelengths would be sufficient to determine both T and L.
- Since the peak wavelength λ_{max} moves through the visible spectrum as temperature ranges over values common for stellar surfaces ($T \sim$ 2000–50,000 K), even the relatively narrow visible spectrum can be used to determine T and f (or bolometric magnitudes) for stars.

Stellar photometry

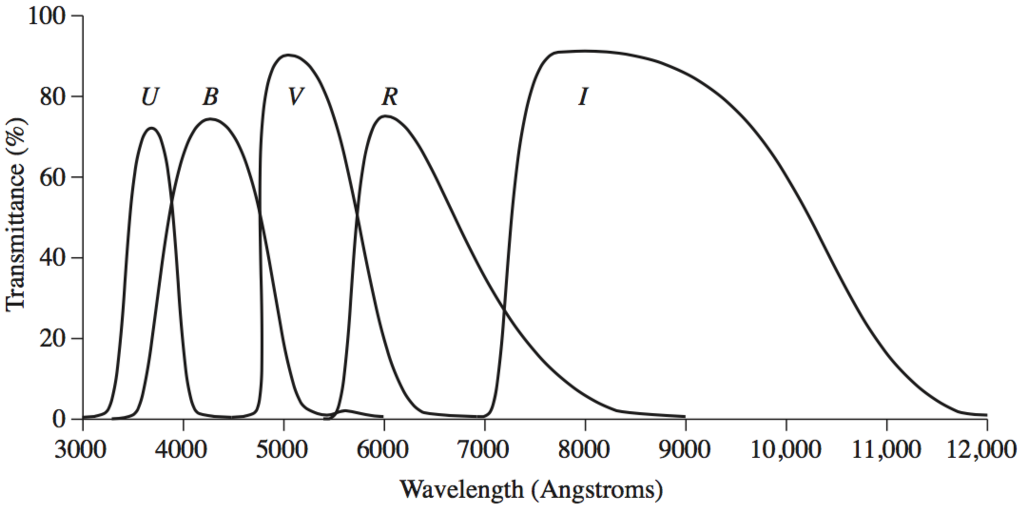
To facilitate the comparison of measurements by different people, astronomers have defined standard **bands** for observing stars. Each band is defined by a central wavelength and a bandwidth.

At visible wavelengths, the bands used most often belong to the **UVB** or **Johnson photometric system** (Johnson & Morgan 1953):

Band	Wavelength [Å]	Bandwidth [Å]
U	3600	700
B	4300	1000
V	5400	900
R	7000	2200

Measurement of starlight fluxes (or magnitudes) within such bands is called **photometry**.

Stellar Photometry



Spectral sensitivity of the colored filters used in the Johnson-Cousins UBVRI system.

Color and effective temperature

Color index, or simply **color**, is the difference between the magnitudes of an object in different photometric bands, or 2.5 times the logarithm of the ratio of the fluxes in the object in the two bands.

- An oft-used color index involves the B and V bands:

$$B - V = m_B - m_V = M_B - M_V$$
$$= 2.5 \log \left(\frac{f(V)}{f(B)} \right)$$

- Note that if $B - V$ is large and positive, it means the object's B magnitude is much larger than its V magnitude, so it is much brighter at V than B.
  - Large $B - V$ means a redder color.
  - Small $B - V$ means a bluer color.

Color and effective temperature

In the absence of extinction (the scattering/absorption of starlight by dust and gas), a star's **effective temperature** T_e can be determined from its color index. T_e is the surface temperature of a blackbody of the same size as a star giving the same luminosity as the star:

$$T_e = \left(\frac{L}{4\pi\sigma R^2} \right)^{1/4}$$

- Typically, low-order polynomial fits are calibrated to (color, T_e) for well-studied stars. The fits can then be applied to estimate T_e for many stars using only colors.

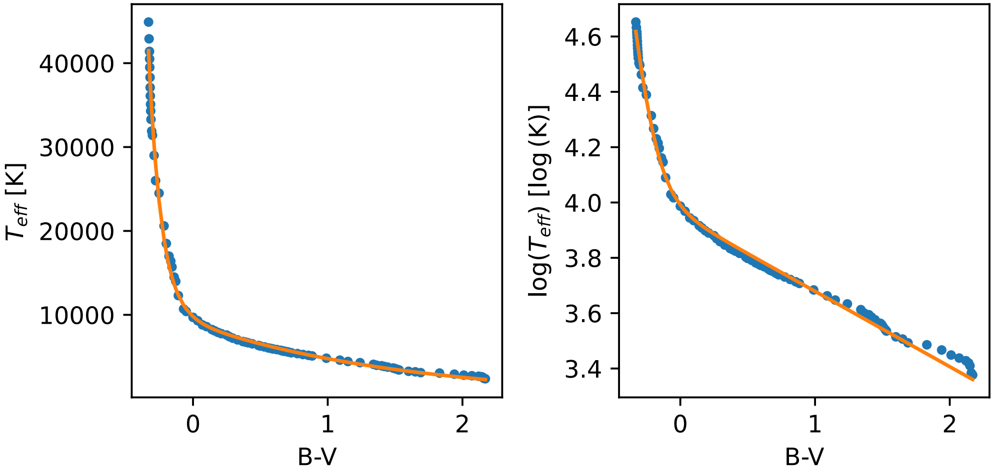
Example

For $T_e < 12,000$ K, and taking $B = V = 0$ at $T = 10,000$ K (like Vega),

$$B - V \approx -0.93 + \frac{9000\text{ K}}{T_e}$$

See pg. 317 of Ryden for details.

Color and effective temperature



Empirical relation for real stellar spectra, from Pecault & Mamajek (2013).

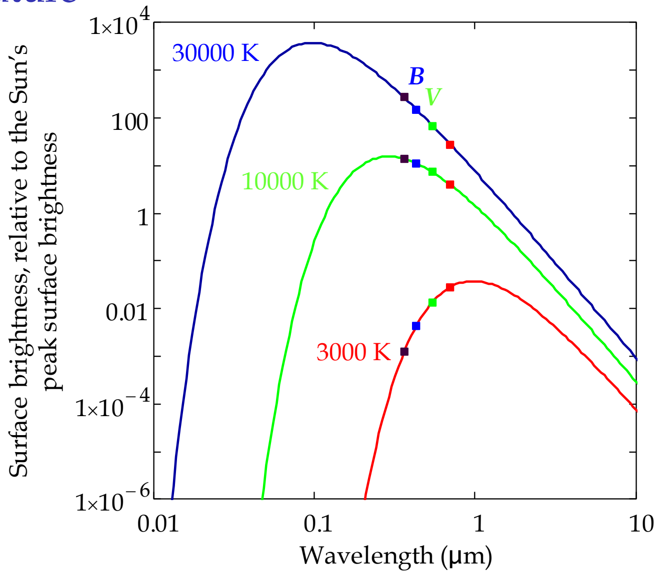
Color and effective temperature

Example

For these spectra (which are perfect blackbodies, not real stars), the visible colors are

$$B - V = 2.5 \log\left(\frac{f_V}{f_B}\right)$$
$$= -0.85$$
$$= -0.46$$
$$= +1.23$$

from top to bottom.



Color, magnitude, and bolometric correction

Once the *shape* of a star's spectrum (i.e., its temperature) is determined from the colors, the total flux is also determined.

- In magnitude terms: the ratio of the *total flux* to the *flux within one photometric band* is expressed as a **bolometric correction** to the star's magnitude in that band (usually V):

$$m = m_V + BC$$

- In blackbody terms: once the temperature of the body is known, so is the total flux ($f = \sigma T^4$). If stars really were blackbodies, the bolometric correction would be

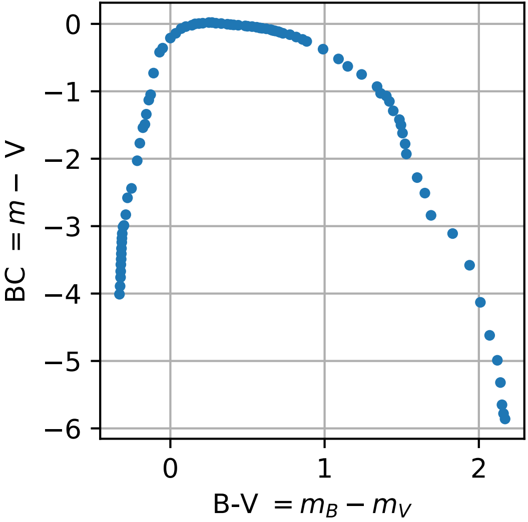
$$BC = m - m_V = 2.5 \log\left(\frac{f_V}{f}\right) = 2.5 \log\left(\frac{B_\lambda(\lambda_V, T)\,\Delta\lambda_V\,\Delta\Omega}{\sigma T^4}\right)$$

Bolometric correction

- The modern BC scale is set by defining the absolute bolometric magnitude scale to a luminosity in Watts:

$$M_{\text{bol}} = 0 \rightarrow L = 3.0128 \times 10^{28}\ \text{W}$$

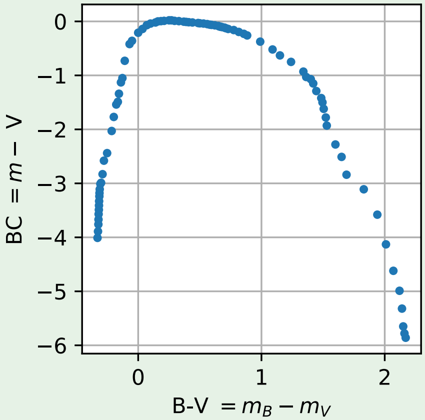
- With $L_\odot = 3.828 \times 10^{26}$ W, $M_{\text{bol}} = 4.74$ for the Sun. Since $M_{V\odot} = 4.86$, $BC_{V\odot} \approx -0.12$.



Empirical bolometric correction for V band, based on real spectra of main-sequence stars (Pecault & Mamajek, 2013).

Color and bolometric correction

Two stars are observed to have the same *apparent magnitude* of 2 in the V band. One of them has a color index $B - V = 0$ and the other has $B - V = 1$. What are their *apparent bolometric magnitudes*?



From the previous plot, $BC = -0.21$ and -0.375 in these cases, so their *bolometric magnitudes* are practically the same too: 1.79 and 1.625.

- That these magnitudes are both less than the V magnitude is a sign that these stars produce substantial power at wavelengths far outside the V band. The bluer star (with $B - V = 0$) turns out to be brightest at ultraviolet wavelengths; the other one is brightest at red and infrared wavelengths.

Measurement of stellar mass and radius

Radius of isolated stars: Stars are so distant compared to their size that normal telescopes cannot make images of their surfaces or measurements of their sizes, though stellar *interferometry* can be used for some large and/or nearby stars.

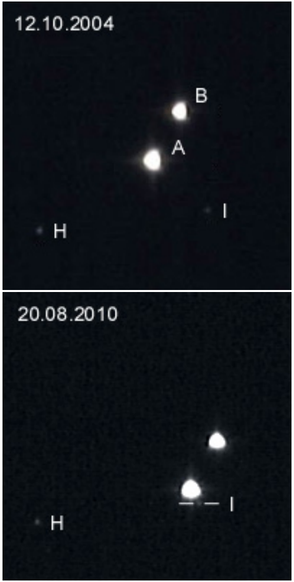
Mass: We cannot measure the mass of an isolated star: we need a test particle in "gravitational contact" with it.

The most helpful test particles are **binary star systems**, though in principle any multi-star system could be probed for the radial velocities and periods required to measure masses.

- Observations of certain binary star systems can also determine the **radius and temperature** of each member.

There are enough nearby binary stars to do this for the full range of stellar types, though more low-mass ones are needed!

Types of binaries



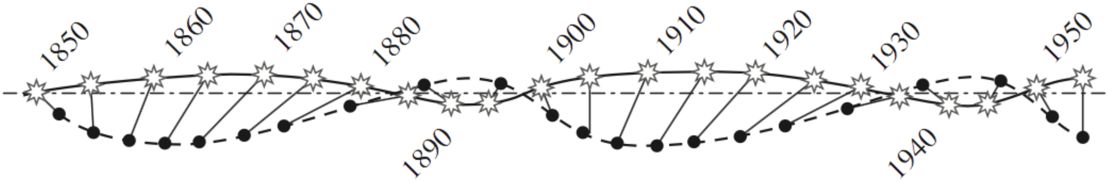
Resolved visual binaries Stars can be seen separately, and the orbital axes and radial velocities can be directly measured. There are not many of these; the rest are **unresolved**.

- At right: 61 Cygni (Schlimmer 2009). Note common apparent angular motion (**proper motion**).

Astrometric binaries Only the brighter member is seen, with periodic wobble in the track of its proper motion. The first system known to be binary, Sirius, was detected in this way by Bessel in 1844. The companion of Sirius was imaged in 1862.

Binaries

Sirius



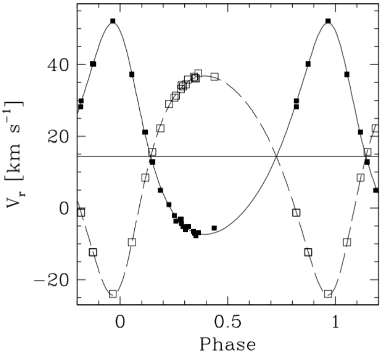
Apparent long-term motion of the Sirius system, showing Sirius A (white star) and Sirius B (black circle), 1844-1950.

Types of binaries

Element (units)	Value	σ
P (d)	90.617	0.007
T (JD-2400000)	45025.79	0.51
e	0.287	0.007
γ-velocity (km s^{-1})	14.35	0.09
ω (°)	22.8	1.3
K_1 (km s^{-1})	29.51	0.25
K_2 (km s^{-1})	30.54	0.26
$a_1 \sin i$ (Gm)	35.23	0.38
$a_2 \sin i$ (Gm)	36.45	0.39
σ(O-C) (km s^{-1})	0.60	...
n_{obs}	22	...

Spectroscopic binaries Unresolved binaries told apart by periodically oscillating Doppler shifts in spectral lines. Periods from days to years.

- **Spectrum binaries** Orbital periods longer than period of known observations.
- **Eclipsing binaries** Orbits seen nearly edge-on such that the stars eclipse each other. Most useful type!



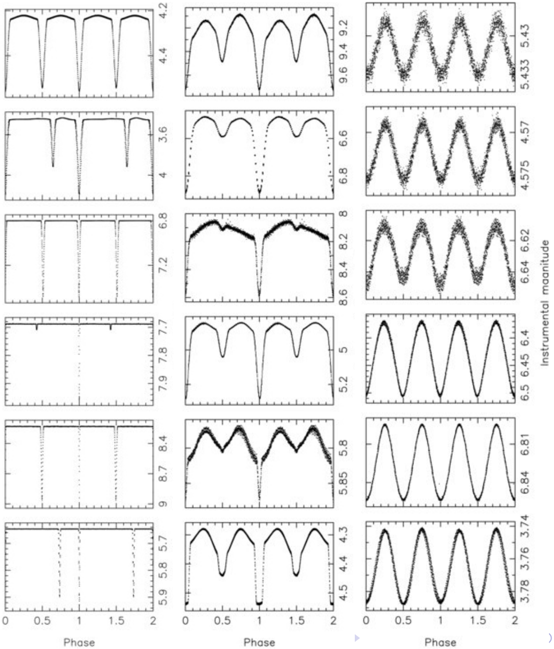
Spectroscopic binary (Platais et al. 2007)

Types of binaries

Binaries for which the separation is clearly larger than the stars are called **detached**. In **semidetached** or **contact** binaries, mass transfer may have modified the stars.

Kepler light curves (Prša et al. 2011; observed flux as a function of time) for eclipsing binaries:

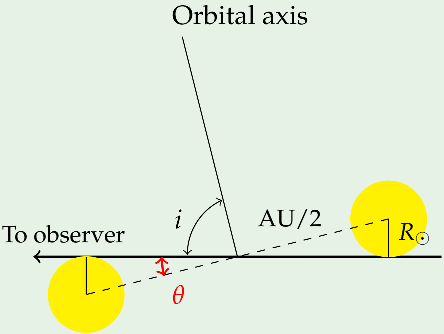
- detached (left)
- semidetached (center)
- contact (right)



Eclipsing binary stars & orientation

If the apparent separation between members of binary systems is small compared to their radii (typical), then the orbital axis must be close to 90°.

Consider two Sun-like stars orbiting each other 1 AU apart, viewed so that they barely eclipse each other in the view of a distant observer. What is their orbital inclination angle?



$$\theta = \sin^{-1}\frac{2R_\odot}{\mathrm{AU}} \approx \frac{2R_\odot}{\mathrm{AU}}$$

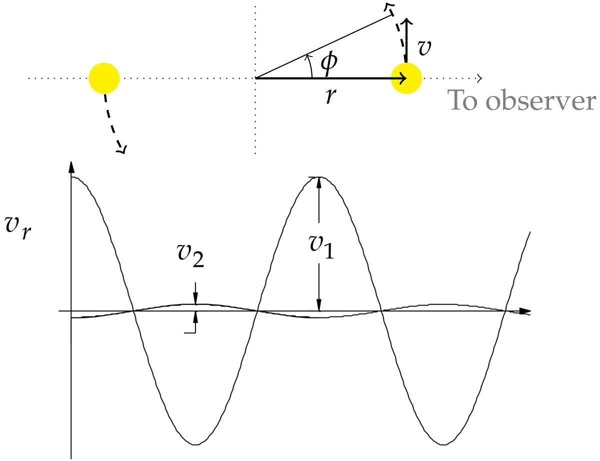
$$i = \frac{\pi}{2} - \frac{2R_\odot}{\mathrm{AU}}$$

$$= 1.561\ \mathrm{rad} = 89.5^\circ$$

$$\sin i = 0.99996$$

Binary star radial velocities

Radial Velocity v_r: the component of velocity along the line of sight.



If the orbits are circular, the radial velocity of each component will be sinusoidal in time:

$$\phi(t) = \omega t = \frac{vt}{r}$$

$$v_x(t) = -v \sin\left(\frac{vt}{r}\right) \equiv v_r$$

$$v_y(t) = v \cos\left(\frac{vt}{r}\right)$$

The radial velocities of the two stars are equal during eclipses.