Binary Stars & Stellar Structure

Direct measurements of stellar mass, radius, and temperature in eclipsing binaries
Principles of Stellar Structure
Hydrostatic Equilibrium
Opacity of the Sun

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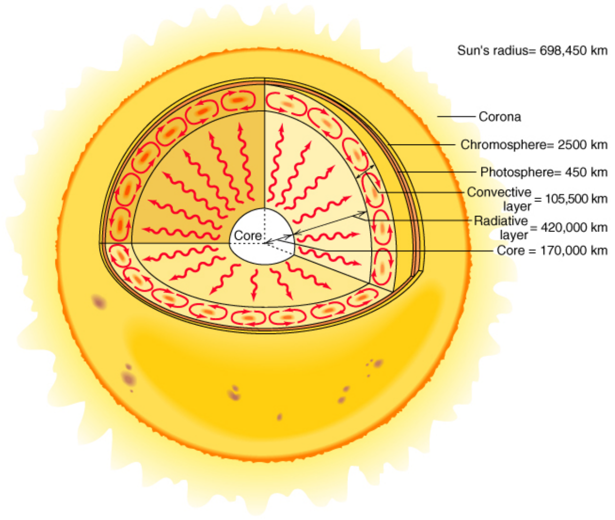
Binary Stars & Stellar Structure

- Eclipsing binaries
- Direct measurements of stellar mass, radius, and temperature
- Principles of stellar structure
- Hydrostatic equilibrium
    - Crude stellar interior models
    - Pressure, density, and temperature in the center (where luminosity is produced)
    - Opacity of the sun: diffusion of light from center to surface

Reading: Kutner Ch. 9.4, Ryden Sec. 14.1, 15.1, & 15.Appendix



Measuring binary star radial velocities

Doppler effect: shift in wavelength of light due to motion of its source with respect to the observer:

$$\frac{\lambda_{\text{obs}} - \lambda_{\text{rest}}}{\lambda_{\text{rest}}} = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{v_r}{c}$$

- Positive (negative) radial velocity leads to a longer (shorter) wavelength than the rest wavelength.
- To measure small radial velocities, a light source with a very narrow range of wavelengths, like a spectral line, may be used.

Measurement of binary star masses

Kepler's Laws

Use radial velocity measurements together with **Kepler's Laws**:

1. All binary stellar orbits are coplanar ellipses, each with one focus at the center of mass.
   - Note: Most binary orbits turn out to have very low eccentricity (they are nearly circular) due to tidal effects.
2. The position vector from the center of mass to either star sweeps out equal areas in equal times.
3. The square of the period is proportional to the cube of the sum of the orbit semimajor axes, and inversely proportional to the sum of the stellar masses:

$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)}(a_1 + a_2)^3$$

Measuring the masses of binary stars

From the definition of the center of mass,

$$m_1 a_1 = m_2 a_2$$
$$\frac{m_1}{m_2} = \frac{a_2}{a_1}$$

From the orbital geometry,

$$2\pi a_1 = v_1 P \qquad 2\pi a_2 = v_2 P$$

Dividing one by the other, we can find an expression for the ratio of the semimajor axes in terms of the orbital velocities.

$$\frac{2\pi a_2}{2\pi a_1} = \frac{v_2 P}{v_1 P}$$
$$\frac{a_2}{a_1} = \frac{v_2}{v_1}$$

So

$$\frac{m_1}{m_2} = \frac{a_2}{a_1} = \frac{v_2}{v_1}$$

Measuring the masses of binary stars

This is great, but we measure v_r, not the orbital velocity. However, because the two stars are always opposite one another, $\phi_1 = \phi_2 = \phi$ and

$$v_{r1} = -v_1 \sin\phi \qquad v_{r2} = -v_2 \sin\phi$$

Therefore,

$$\frac{v_{r2}}{v_{r1}} = \frac{-v_2 \sin\phi}{-v_1 \sin\phi} = \frac{v_2}{v_1}$$

Combining this with our previous result, we find that

$$\frac{m_1}{m_2} = \frac{a_2}{a_1} = \frac{v_2}{v_1} = \frac{v_{r2}}{v_{r1}}$$

So, if you can measure either the (radial) velocities or the semimajor axes, you can determine the mass ratio of the binary system.

Measuring the masses of binary stars

Furthermore, from Kepler's third law,

$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)}(a_1 + a_2)^3$$
$$m_1 + m_2 = \frac{4\pi^2}{GP^2}(a_1 + a_2)^3$$

Remembering that $2\pi a = vP$ for circular orbits, we see that

$$m_1 + m_2 = \frac{4\pi^2}{GP^2}\left(\frac{v_1 P}{2\pi} + \frac{v_2 P}{2\pi}\right)^3$$
$$= \frac{P}{2\pi G}(v_1 + v_2)^3$$

Since $v_{r,\text{max}} = v \sin i$,

$$m_1 + m_2 = \frac{P}{2\pi G}\left(\frac{v_{r1} + v_{r2}}{\sin i}\right)^3$$

Measuring the masses of binary stars

So, for binary stars, we have the relations

$$\frac{m_1}{m_2} = \frac{a_2}{a_1} = \frac{v_2}{v_1} = \frac{v_{r2}}{v_{r1}}$$

and

$$m_1 + m_2 = \frac{P}{2\pi G}\left(\frac{v_{r1} + v_{r2}}{\sin i}\right)^3$$

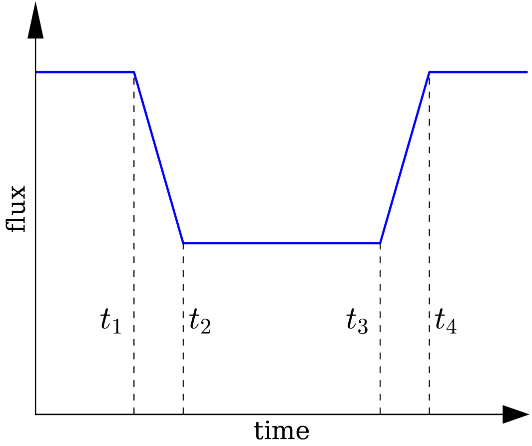
For **unresolved binaries** (the vast majority) we can only measure P and the two velocity amplitudes. This gives *two* equations and *three* unknowns: m_1, m_2, and $\sin i$.

This is why eclipsing binaries are so important. If the system eclipses, we must be viewing the orbital plane close to edge-on. Thus, $\sin i \approx 1$, leaving **two equations with two unknowns**, m_1 and m_2.

Measurement of stellar radius

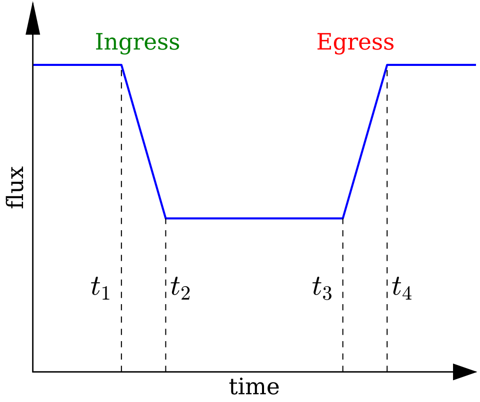
Eclipses do not just turn on and off instantaneously; the transitions are gradual. The shape of the system's **light curve** is sensitive to the sizes of the stars.

- If one star can completely block the light of the other, the "bottom" of the eclipse light curve will be flat.
- The extent of the flat part of the light curve is determined by the radius of the larger companion.
- The slope of the transitions is determined by the radius of the smaller companion.
- If the light curve never flattens out then the eclipses are not "total," i.e., neither star completely blocks the other.



Measurement of stellar radius

- Much can be made of the details of the shapes of the light curve at the onset (ingress) and end (egress) of the eclipse.
- The "corners" are usually labeled as indicated at right.
- During an eclipse, the stars move at speeds v_1 and v_2 in opposite directions perpendicular to the line of sight.
- v_1 and v_2 are measured from orbital periods and radial velocity amplitudes. Thus:



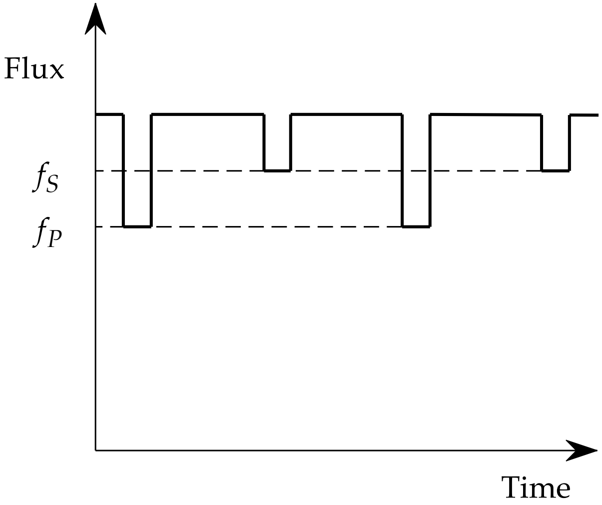
$$2R_S = (v_1 + v_2)(t_2 - t_1) \qquad \text{Diameter of "small" star}$$
$$2R_L = (v_1 + v_2)(t_3 - t_1) \qquad \text{Diameter of "large" star}$$

Measurement of stellar T_e

- A useful measurement of the ratio of the effective temperatures T_e comes from the relative depths of the primary (deeper) and secondary eclipses.
- When the stars are not eclipsed, their total flux is

$$f = \pi R_S^2 I_{0S} + \pi R_L^2 I_{0L}$$

- What configuration defines the primary and the secondary eclipse?



Measurement of stellar T_e

If the **smaller star is hotter**, its surface brightness will be larger, and the primary eclipse is when it is behind the larger star:

$$f_P = \pi R_L^2 I_{0L}$$

Then the secondary eclipse is when the small star is in front:

$$f_S = \pi(R_L^2 - R_S^2)I_{0L} + \pi R_S^2 I_{0S}$$

We have **three** equations and two unknowns, but usually these are used to construct one ratio to remove uncertainties in the distance r:

$$\begin{aligned}
\frac{f - f_P}{f - f_S} &= \frac{\pi R_S^2\, I_{0S} + \pi R_L^2\, I_{0L} - \pi R_L^2\, I_{0L}}{\pi R_S^2\, I_{0S} + \pi R_L^2\, I_{0L} - \pi\left(R_L^2 - R_S^2\right) I_{0L} - \pi R_S^2\, I_{0S}} \\
&= \frac{\pi R_S^2\, I_{0S}}{\pi R_L^2\, I_{0L} - \pi\left(R_L^2 - R_S^2\right) I_{0L}} \\
&= \frac{\pi R_S^2\, I_{0S}}{\pi R_S^2\, I_{0L}} = \frac{I_{0S}}{I_{0L}}
\end{aligned}$$

Measurement of stellar T_e

Note that the way we defined the I_0s is

$$f = \pi R^2 \, I_0 = \frac{L}{4\pi r^2} = \frac{4\pi R^2 \sigma T_e^4}{4\pi r^2} \implies I_0 = \frac{\sigma T_e^4}{\pi r^2}$$

Thus,

$$\frac{f - f_P}{f - f_S} = \frac{I_{0S}}{I_{0L}} = \frac{T_{eS}^4}{T_{eL}^4}$$

If r is known, the luminosity and $T_e = \left(L/4\pi r^2 \sigma\right)^{1/4}$ of the brighter star is determined accurately from the primary minimum flux, and thus T_e of the fainter star from this ratio.

Calculating binary properties

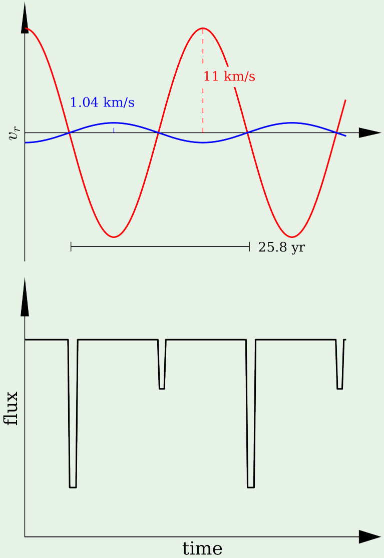
Example

An eclipsing binary is observed to have a period of 25.8 years. The two components have radial velocity amplitudes of 11.0 km/s and 1.04 km/s and sinusoidal variation of radial velocity with time. The eclipse minima are flat-bottomed and 164 days long. It takes 11.7 hr for the ingress to progress from first contact to eclipse minimum.

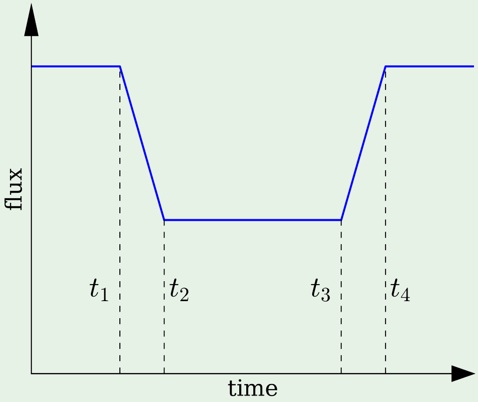
Answer the following questions:

1. What is the orbital inclination?
2. What are the orbital radii?
3. What are the masses of the stars?
4. What are the radii of the stars?

Example



Closeup of primary minimum:



$t_2 - t_1 = 11.7$ hr $\qquad t_3 - t_2 = 164$ days

Example

Answers:

1. Since it eclipses, the orbits must be nearly edge-on. Since the radial velocities are sinusoidal the orbits must be nearly circular.
2. **Orbital radii**:

$$r_S = v_S \frac{P}{2\pi} = 1.42 \times 10^{14} \text{ cm}$$
$$= 9.54 \text{ AU}$$

$$r_L = v_L \frac{P}{2\pi} = 1.34 \times 10^{13} \text{ cm}$$
$$= 0.90 \text{ AU}$$

Therefore, the semimajor radius of the system is $r = r_1 + r_2 = 10.4$ AU.

Example

3. **Masses**: use Kepler's third law

$$\frac{m_L}{m_S} = \frac{v_S}{v_L} = \frac{11}{1.04} = 10.6$$

$$m_S + m_L = \frac{(r/\mathrm{AU})^3}{(P/\mathrm{yr})^2} = 15.2\,M_\odot$$
$$= m_S + 10.6 m_S$$

$$\boxed{m_S = 1.3\,M_\odot, \quad m_L = 13.9\,M_\odot}$$

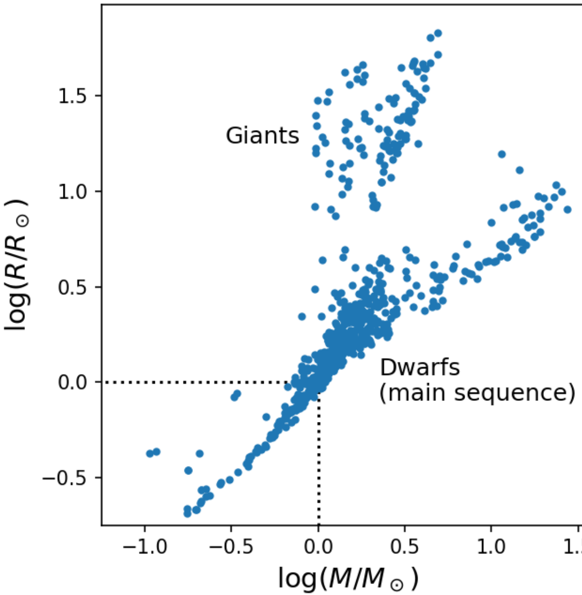
4. **Stellar radii** (note: solar radius $\equiv R_\odot = 6.96 \times 10^{10}$ cm):

$$R_S = \frac{v_S + v_L}{2}(t_2 - t_1)$$
$$= (6.02\ \mathrm{km\ s^{-1}})(11.7\ \mathrm{hr})$$
$$= 7.6 \times 10^{10}\ \mathrm{cm} = 1.1\,R_\odot$$

$$R_L = \frac{v_S + v_L}{2}(t_3 - t_1)$$
$$= 369\,R_\odot$$

Data on eclipsing binary stars

- Those objects for which orbital velocities have been measured comprise the fundamental reference data on the dependences of stellar parameters — effective temperature T_e, radius, and luminosity — on stellar mass.
- A strong trend emerges in plots of mass versus any other quantity, involving all stars which are not outliers in the mass versus radius plot. This trend, in whatever graph it appears, is called the "**main sequence**," and its members are called the dwarf stars.



The H-R diagram

We cannot measure the masses of single stars or binary stars in orbits of unknown orientation. Unfortunately, this includes more than 99% of the stars in the sky.

- We can usually measure the luminosity from bolometric magnitude and the temperature from colors, however. The plot of this relation is called the Hertzprung-Russell diagram (or H-R diagram).
- The H-R diagram for detached eclipsing binaries is very similar to that of stars in general: it corresponds to the main sequence in other samples of stars.
- This correspondence allows us to **infer the masses of single or non-eclipsing multiple stars**.

The H-R diagram