Astronomy 142 — Recitation 4

Prof. Douglass

February 9, 2024

Formulas to remember

$$E = mc^2 \tag{1}$$

Starlight heating by a variable-luminosity star

If the average luminosity of a star is L and the average temperature to which this heats a planet is T, then a small variation by $\Delta L < L$ in the star's luminosity leads to a small variation ΔT in the planet's temperature, given by

$$\frac{\Delta T}{T} = \frac{\Delta L}{4L} \tag{2}$$

Maximum fusion tunneling probability and tunneling rate

$$P(T) = De^{-(T_0/T)^{1/3}} \tag{3}$$

where $T_0 = \left(\frac{3}{2}\right)^3 \left(\frac{8\pi q_1 q_2}{h}\right)^2 \frac{m}{k} = 1.565 \times 10^{10}$ K for proton-proton fusion, q_1 and q_2 are the electric charges (in cgs units) of the two fusing nuclei, $m = \frac{m_1 m_2}{m_1 + m_2}$ is the reduced mass of the two fusing nuclei, and D is a constant.

Local sidereal time (LST)

LST = right ascension of celestial objects on the meridian (the meridian = arc through the poles and the zenith); therefore, LST has the same units as right ascension.

The Earth rotates by $360° = 2\pi$ radians in 24 hours of sidereal time; the Earth rotates by $2\pi(1 + \frac{\text{day}}{\text{TY}})$ in one day. 1 day = 24 hours, or 86400 seconds; 1 tropical year (TY) = 365.242189 days. The latter means that the Earth's rotational angular speed is $\Omega_\oplus = 7.292115855 \times 10^{-5}$ radians/s, and that a sidereal day — 24 hours of sidereal time, and the actual rotation period of the Earth — is 23.9344696 hours.

For a given time of day, the corresponding sidereal time advances by 24 hours as the date advances one year.

Workshop problems

Remember! The workshop problems you will do in groups in Recitation are a crucial part of the process of building up your command of the concepts important in ASTR 142 and subsequent courses. Do not, therefore, do your work on scratch paper and discard it. Better for each of you to keep your own account of each problem in some sort of bound notebook.

1. Consider two protons ($q = 4.8 \times 10^{-10}$ esu, $m = 1.67 \times 10^{-24}$ g), separated by 10^{-13} cm.
   (a) Calculate the magnitude of the proton's electric repulsive force and gravitational attractive force. Can gravity hold the protons together?

(b) Calculate the electrostatic and gravitational potential energies. Compare these to the binding energy of the deuteron, 2.2 MeV (3.55×10^{-6} erg). This binding energy is the opposite of the strong-interaction potential energy, and its relatively large value indicates the predominance of the strong interaction on nuclear size scales.

2. Assume that the Sun was initially composed of 70% hydrogen by mass.
   (a) How many hydrogen nuclei, N, were there?
   (b) What is the total nuclear energy supply, $\frac{NE}{4}$, where $E = 0.03 m_p c^2$, if all of the hydrogen could be fused into helium?
   (c) It turns out that only about 13% of the hydrogen in a solar-type star lies within the parts of the star hot enough to participate in fusion during the star's stay on the main sequence. Under these assumptions, what will the Sun's main sequence lifetime be?

3. The flux of pp-chain electron neutrinos from the Sun has been measured to be about half the flux that is expected at the temperature inferred for the Sun's center, $T_{c\odot} = 1.57 \times 10^7$ K. (This is the deficit to which we used to refer as the "Solar neutrino problem.")
   (a) What would the Sun's central temperature have to be in order to produce pp-chain electron neutrinos at half the rate that applies to 1.57×10^7 K?
   (b) Briefly review the means by which we inferred the Sun's central temperature. By how much would the accepted values of other measurable properties of the Sun need to be in error for the temperature to be off by this much? Is it likely that such differences lie within present observational uncertainties?

Learn your way around the sky, lesson 4. (A feature *exclusive* of ASTR 142 recitations.) You may find the lab's celestial globes and the program Stellarium useful in answering these questions about the celestial sphere and the constellations.

4. The vernal equinox is when the Sun reaches the intersection of the celestial equator and the ecliptic, going north; this particular intersection defines the zero of the right ascension scale. This year, the vernal equinox takes place on March 19 at 11:07 PM EDT: 03:07 coordinated universal time (UTC; standard time at zero longitude) on March 20.

   Naturally, this event will take place at different times of day for observers at different longitudes. By convention, longitude, L, is measured in degrees of arc, or degrees-minutes-seconds of arc. We define east to be the positive direction, and $L = 0$ to be on the prime meridian: the arc that runs through the geographic north and south poles, and the Royal Greenwich Observatory in London. Standard time on the prime meridian is the same as UTC.

   Like its cousins the right ascension and azimuthal coordinate ϕ in spherical and cylindrical coordinates, longitude is not an open-ended scale because it is measured on a closed surface. The longitude is therefore within the range $0° - 360°$. Negative values are allowed, as these indicate longitude differences to the west. For example: downtown Rochester has longitude L equal to $282.384° = 282°23'04"$ or $-77.616° = -77°36'56"$.

   As described above, the local sidereal time (LST) is closely related to longitude, except for having the same zero as right ascension and being measured in hours, minutes, and seconds of time instead of angular measures (as with right ascension). LST is always positive, and is between 0 and 24 hours (24^{h}). The zero point, however, is not simply a fixed point in the sky — defining the zero point of LST is what you will explore in this problem.

   (a) At what longitude is the Sun on the meridian at the moment of the vernal equinox? What is the local sidereal time at this location and moment?
   (b) At the moment of the vernal equinox, what is the local sidereal time at zero longitude, and in Rochester?
   (c) Look at the result of an accurate calculation of local sidereal time:

   `https://aa.usno.navy.mil/data/siderealtime`

Calculate the local sidereal time the moment of the vernal equinox, at the longitude calculated in part a, at zero longitude, and at Rochester's longitude. If all has gone well, you will find that your answers differ from the results in parts a and b by the same amount. What do you suppose is the cause of this offset? (That is, what extra offset does the calculator include that we are currently leaving out?)

5. Match the celestial object to the constellation to which it belongs.

Objects	Constellations
Brightest star in the sky	Andromeda
Most massive, luminous star visible to the naked eye	Ara
Nearest region of star formation	Canis Major
Nearest region of massive (O–B) star formation	Centaurus
Nearest open stellar cluster	Dorado
Nearest globular stellar cluster	Ophiuchus
Center of the Milky Way Galaxy	Orion
Nearest spiral galaxy to the Milk Way	Sagittarius
Nearest active galaxy (supermassive black hole, relativistic jets)	Taurus
Nearest rich cluster of galaxies	Virgo

Intro to Python, lesson 4. (A feature *exclusive* of ASTR 142 recitations.)

6. Make a graph of the electrical potential energy between two protons as a function of their separation, r. Let r range from the size of a nucleus to ten times that.