Astronomy 142 — Recitation #4

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Formulas to remember

Pulsating stars

Pulsation period, fundamental mode, uniform-density star:

$$\Pi = 4\int_0^R \frac{dr}{v_s} = \sqrt{\frac{6\pi}{\gamma G\rho}} \tag{1}$$

Special relativity

Lorentz factor

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \tag{2}$$

where v is the relative velocity between two inertial reference frames.

Time dilation

$$\Delta t = \gamma \Delta\tau \tag{3}$$

where $\Delta\tau$ is the "proper time" interval between two events in the same location (like the ticks of a clock) as measured by an observer at rest with respect to that location, and Δt is the time interval between those same two events as measured by an observer to whom the first observer seems to be moving at relative speed v.

Degenerate matter, white dwarfs, and neutron stars

Electron degeneracy pressure

$$P_e = \begin{cases} 0.0485\frac{h^2}{m_e}\left(\frac{Z}{A}\right)^{5/3}\left(\frac{\rho}{m_p}\right)^{5/3} & \text{(non-relativistic)} \\ 0.123hcn_e^{4/3} & \text{(relativistic)} \end{cases} \tag{4}$$

Radius v. mass, non-relativistic limit

$$R = \begin{cases} 0.114\frac{h^2}{Gm_e m_p^{5/3}}\left(\frac{Z}{A}\right)^{5/3} M^{-1/3} & \text{(electron degeneracy)} \\ 0.0685\frac{h^2}{Gm_p^{8/3}}M^{-1/3} & \text{(neutron degeneracy)} \end{cases} \tag{5}$$

Central density and temperature, non-relativistic limit, white dwarf

$$P_C = 0.77\frac{GM^2}{R^4} \qquad \rho_C = 1.43\frac{M}{R^3} \tag{6}$$

Maximum mass of an electron-degeneracy-pressure-supported object

$$M_{SAC} = 0.2 \left(\frac{Z}{A}\right)^2 \left(\frac{hc}{Gm_p^2}\right)^{3/2} m_p = 1.44 M_\odot \text{ for } \frac{Z}{A} = 0.5 \tag{7}$$

Black holes

Event horizon (Schwarzschild singularity) coordinate radius

$$R_{\text{Sch}} = \frac{2GM}{c^2} \tag{8}$$

The coordinate radius r is a distance measured by a distance observer from the center of the black hole.

Event-horizon temperature of a black hole with mass M

$$T = \frac{hc^3}{16\pi^2 kGM} \tag{9}$$

Workshop problems

Remember! The workshop problems you will do in groups in Recitation are a crucial part of the process of building up your command of the concepts important in ASTR 142 and subsequent courses. Do not, therefore, do your work on scratch paper and discard it. Better for each of you to keep your own account of each problem in some sort of bound notebook.

1. Newton worked on sound as well, but he turned out to be slightly off about how it propagated. He thought it was an isothermal pressure disturbance (constant temperature as pressure changes) instead of an adiabatic one (no heat flows from the pressure fluctuations). In modern terms, Newton's formula for the (isothermal) speed of sound would be

$$v_{s,\text{iso}T} = \sqrt{\frac{kT}{\mu}} \tag{10}$$

   where μ is the average mass of particles (atoms or molecules) in the gas. (Newton did not know about Boltzmann's constant or molecular masses, so he was unaware of what the proportionality constant should be in the relation he proposed for sound speed.)

   If Newton had been correct in these details, what would be the formula for pulsation periods of a uniform-density star, and by what factor do the periods for adiabatic and isothermal pulsations differ?

2. The quantum-mechanical, or de Broglie, wavelength corresponding to a particle with momentum $p_x = mv_x$ is $\lambda = \frac{h}{p_x}$.
   (a) Present a brief argument to show that degeneracy pressure is important when a particle is confined to a space similar in size to, or smaller than, its de Broglie wavelength.
   (b) Show from the relative sizes of their de Broglie wavelengths that electron degeneracy pressure can be larger than electron thermal pressure (so electrons are degenerate) while the nuclei that are mixed in with the electrons still behave like an ideal gas.

Learn your way around the sky (A feature *exclusive* of ASTR 142 recitations.) You may find the lab's celestial globes and the program Stellarium useful in answering these questions about the celestial sphere and the constellations.

3. In Recitation #1, you considered a star with right ascension and declination α and δ, drew a Cartesian coordinate system with the x-axis pointing through $\alpha = \delta = 0$ and the z-axis pointing toward $\delta = 90°$, and derived an expression for a unit vector pointing at the star, in terms of the Cartesian unit vectors $\hat{x}$, $\hat{y}$, and $\hat{z}$. You got

$$\hat{n} = \cos\delta\cos\alpha\hat{x} + \cos\delta\sin\alpha\hat{y} + \sin\delta\hat{z} \tag{12}$$

   (a) In the same coordinate system, construct a unit vector that points toward the zenith and depends upon LST and latitude.

   (b) Derive an expression for the angle between this new unit vector and the old one above. Look for a simplification that allows you to express the result in terms of the difference between LST and the right ascension of the object. This new angle is called the **zenith angle** (ZA); the right-angle complement, the altitude, is the angular distance between the star and the horizon. The difference between LST and RA is called the **hour angle** (HA).

Intro to Python (A feature *exclusive* of ASTR 142 recitations.)

4. Show the relationship between a neutron star's mass and its size, average density, gravitational acceleration at the surface, and the rate of change of that acceleration, dg/dR.